ON COMPLETION TIMES OF NETWORKS OF CONCURRENT AND SEQUENTIAL TASKS

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Abstract

Determining the completion time of PERT networks is of importance to engineering project management, and is the subject of an extensive and expanding body of works. Activity durations can often only be estimated, since the execution of an activity typically depends on various factors whose details are not knowable in advance. This leads to modeling activity durations with a generalization of the constant, the probability distribution. A constant then becomes a particular type of distribution, an impulse. When activity durations are modeled as distributions, determining the distribution of the completion time of the network is more challenging. Frequent computational tasks on random variables occurring during evaluation of activity networks are addition and maximization (Agrawal and Elmaghraby 2001). Not only must distributions be found for the sum of random variables expressed as distributions describing the durations of activities on a given path, but also various paths may have some chance of being the critical one depending on what the actual activity durations, each a sample of its distribution, turn out to be. This requires calculating distributions of the maximums of random variables, because when computing the time to complete concurrent tasks, the joint completion time is the maximum of the completion time of the concurrent tasks.

The problem of determining the time to complete multiple tasks that may proceed concurrently, sequentially, or both is considered. In the solution offered, each individual task completion time may be described with a number, interval, or distribution function. In the case of distribution functions, two task completion times might be independent random variables, as when the tasks are performed in different environments and proceed independently. Alternatively, completion times might be positively correlated, as when both depend on the quality of management and proceed within the same managerial environment, or they could be negatively correlated, as when resource sharing means that faster completion of one implies slower completion of the other. Finally, various factors might interact to make completion times dependent in a way that is difficult to characterize accurately. The solution offered avoids requiring the assumption that individual task completion times are independent or have any other dependency relationship. One application of the results is in project management, as in the context of PERT (Program Evaluation and Review Technique) diagrams.

References

[1] Agrawal, M.K. and S.E. Elmaghraby, On Computing the Distribution Function of the Sum of Independent Random Variables, Computers & Operations Research, 28, 5, April 2001, 473-483.