UNTANGLING EQUATIONS INVOLVING UNCERTAINTY: DECONVOLUTIONS, UPDATES, AND BACKCALCULATIONS

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There are multiple ways to solve an equation when the quantities it involves are uncertain. For instance, if a random measurement Y is the sum of a randomly fluctuating true value X and an independent measurement error ε, then probabilistic deconvolution can be used to estimate the distribution of X from the distributions for Y and ε by untangling the equation $Y = X + \varepsilon$. Updating is a related operation used to improve the estimates of one or multiple quantities based on mathematical or mechanistic information about the relationship between them. For instance, separate estimates of the area A, length L, and width W of a rectangle can be considered against their relationship $A = L \times W$ to obtain the best possible estimates that are mutually consistent. Backcalculation is another operation used in problems such as estimating the allowable range of contaminant concentrations C that will ensure a prescribed range of doses D under a prevailing range of intake rates I in a population of exposed individuals so that none receives intolerably large doses. Backcalculation solves the equation $D = C \times I$ but in a way that is decidedly different from either deconvolution or updating. Similar calculations are involved in estimating recommended daily allowances for vitamins that ensure the doses are not too small. Backcalculations are sometimes called tolerance solutions in interval analysis.

Backcalculation, updating and deconvolution all seem to be very different operations, although each can be thought of as an untangling of an equation involving uncertainty. We discuss the connections between the outwardly diverse problems and show how the calculations involved can be understood in a general context. Examples of each of the problems arise in both interval analysis and probability theory. We show that the unification of these two theories in the context of probability bounds analysis (or, more generally, imprecise probabilities) reveals the interrelatedness of these operations. They can be seen as examples of solutions to a general problem and are distinguished by the nature of the uncertainty involved, the dependence among the operands, and the goal of the calculation. We also show how the solutions obtained under the more general theory of uncertainty sometimes make more sense and are more useful than the solutions obtained within the strictures of either more narrow theory.