

# Recent Advances in the Validated Integration of ODEs

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We discuss the foundations of Taylor Model-based validated integrators. Compared to other validated tools, these methods originally developed in the field of beam physics tools allow the representation of the flow of extended domains with very little overestimation. In particular we discuss various preconditioning methods, including the method of curvilinear coordinates and various blunting approaches. We also discuss methods that lead to simplifications for linear systems or very small boxes. We illustrate the performance with a comprehensive collection of examples, including some systems from the field of beam physics.