A METHOD FOR OUTER INTERVAL SOLUTION OF PARAMETRIZED SYSTEMS OF LINEAR EQUATIONS

Iwona Skalna

Department of Applied Informatics, Faculty of Management
University of Science and Technology
Poland, Cracow, Gramatyka Street 10
skalna@galaxy.uci.agh.edu.pl

This paper deals with the problem of computing sharp bounds for the solution of a system of linear interval equations whose coefficients are affine functions of interval parameters.

Consider the family of linear algebraic systems of the following type

$$A(p)x = b(p), \tag{1}$$

with

$$a_{ij}(p) = \omega(i,j)^T\, p, \tag{2}$$

$$b_j(p) = \omega(0,j)^T\, p, \tag{3}$$

$p \in [p] \in \mathrm{R}^k$ [1]. Such systems are encountered in many practical applications, e.g. in structure mechanics [2].

The family of systems (1) is usually written in the form

$$A([p])x = b([p]). \tag{4}$$

The united solution set of the system (4) is defined as

$$\Sigma(A([p]), b([p])) = \{x:\ \ A(p)\,x = b(p),\ p \in [p]\} \tag{5}$$

If the set (5) is bounded then the interval hull for (5) exists.

In this paper a direct method for computing a tight enclosure for (5) is proposed. This method is based on the following inclusion

$$\Diamond\{\Sigma(A([p],\ b([p])\} \subseteq \widetilde{x} + [-1,\ 1]\langle [D] \rangle |[Z]| \tag{7}$$

with $R = \breve{A}^{-1}$, $\widetilde{x} = R\breve{b}$, $[D]_{ij} = \left(\sum_{k=1}^{n} R_{ik}\omega(k,j)\right)^{\mathrm{T}}[p]$, $[Z]_i = \left(\sum_{j=1}^{n}\sum_{k=1}^{j-1} R_{ij}\left(\omega(0,j) - \widetilde{x}_k\omega(j,k)\right)\right)^{\mathrm{T}}[p]$.

Some numerical examples are provided to demonstrate the usefulness of the method. It is also proved that in case of $[A] = A$ the inclusion in (7) is an equality. Finally for systems with $[A] = A$ an explicit formula for the hull of the solution set is given.

References

[1] Rump, S.M., *Verification methods for dense and sparse systems of equations*, Elsevier, 5:63-135, The Netherlands, Amsterdam, 1994.

[2] Iwona Skalna, *Methods for solving systems of linear equations of structure mechanics with interval parameters*, CAMES, 10(3) 2003.