A Computational Approach to Existence Verification and Construction of Robust QFT Controllers

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Outline

- Contribution
- Introduction to QFT
- Problem definition
- Proposed Algorithm
- Example
- Conclusions
Contribution

◆ **Existence verification**

◆ **Constructive approach** –
  
  – Guaranteed existence verification
  
  – Give all solutions, if any solution exist.
Introduction

2-DOF Structure for QFT formulation
Introduction ...

QFT Objective:

Synthesize $K(s)$ and $F(s)$ for the following specifications:

- Robust Stability margin
- Tracking performance
- Disturbance Attenuation
Introduction …

QFT Procedure:

1. Generate the plant template at the given design frequencies $\omega_i$.

2. Generate the bounds in terms of nominal plant, at each design frequency, on the Nichols chart.
Introduction ...

QFT Procedure ...

3. Synthesize a controller $K(s)$ such that
   1. The open loop response satisfies the given performance bounds,
   2. And gives a nominal closed loop stable system.

4. Synthesize a prefilter $F(s)$ which satisfies the closed loop specifications.
Problem Definition

Given an uncertain plant and time domain or frequency domain specification, find out if a controller of specified (transfer function) structure exists.
Feasibility Test

Feasible
Ambiguous
Infeasible

magnitude

phase
Proposed Algorithm

Inputs:
1. Numerical bound set,
2. The discrete design frequency set,
3. Inclusion function for magnitude and angle,
4. The initial search space box $z^0$.

Output:
Set of Controller parameters, **OR**
The message “No solution Exist”.
Proposed Algorithm...

BEGIN Algorithm

1. Check the feasibility of initial search box $z^0$.
   - Feasible $\Leftrightarrow$ Complete $z^0$ is feasible solution.
   - Infeasible $\Leftrightarrow$ No solution exist in $z^0$
   - Ambiguous $\Leftrightarrow$ Further processing required: Initialize stack list $L^{stack}$ and solution list $L^{sol}$. 
Proposed Algorithm ...

2. Choose the first box from the stack list $L^{stack}$ as the current box $z$, and delete its entry from the stack list.

3. Split $z$ along the maximum width direction into two subboxes.
Proposed Algorithm …

4. Find the feasibility of each new subbox:
   - Feasible ⇔ Add to solution list \( L^{sol} \).
   - Infeasible ⇔ Discard.
   - Ambiguous ⇔ Further processing required : Add to stack list \( L^{stack} \).
Proposed Algorithm

5. IF $L^{stack} = \{\text{empty}\}$ THEN (terminate)
   - $L^{sol} = \{\text{empty}\} \Rightarrow \text{“No solution exist”, Exit.}$
   - $L^{sol} \neq \{\text{empty}\} \Rightarrow \text{“Solution set = ” } L^{sol}, \text{ Exit.}$

6. Go to step 2 (iterate).

End Algorithm
Example

◆ Uncertain plant: \( G(s) = \frac{k(1 - \tau s)}{s(1 + \beta s)} \)

◆ Uncertainty: \( k \in [1,3], \beta \in [0.3,1], \tau \in [0.05,0.1] \)

◆ Robust stability spec: \( \omega_s = 1.3032 \)

◆ Tracking spec: \( |T_U(j4)| = 0.5 \quad \text{and} \quad |T_L(j4)| = -3.5 \)
QFT Bounds

Nichols Chart

(UHFB) $\omega > \omega_n = 500$

B(4)

B(45.4)
Example ...

- For first order controller:
  - Parameter vector \( z = \{k, \tilde{z}_1, p_1\} \)
  - Initial search box \( z^0 = (0,10^8],(0,10^4],(0,10^4] \)

- For second order controller:
  - Parameter vector \( z = \{k, \tilde{z}_1, \tilde{z}_2, p_1, p_2\} \)
  - Initial search box \( z^0 = (0,10^8],(0,10^4],(0,10^4],(0,10^4],(0,10^4] \)
Example ...

- **For third order controller:**
  - Parameter vector \( z = \{k, z_1, z_2, z_3, p_1, p_2, p_3\} \)
  - Initial search box
  \[
  z^0 = (0,10^8], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4] 
  \]

- **For fourth order controller:**
  - Parameter vector \( z = \{k, z_1, z_2, z_3, z_4, p_1, p_2, p_3, p_4\} \)
  - Initial search box
  \[
  z^0 = (0,10^8], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4] 
  \]
Results

◆ For the aforementioned structures and the initial search domains, the proposed algorithm terminated with the message: "No feasible solution exists in the given initial search domain".

◆ This finding is in agreement with the analytically found ‘non-existence’ of Horowitz.
Conclusions

- An algorithm has been proposed to computationally verify the existence (or non-existence) of a QFT controller solution.

- Proposed algorithm has been tested successfully on a QFT benchmark example.

- The proposed algorithm is based on interval analysis, and hence provides the most reliable technique for existence verification.
Thank you!