# A Computational Approach to Existence Verification and Construction of Robust QFT Controllers

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**Abstract.** Horowitz's quantitative feedback theory (QFT) (Horowitz, 1993) approach to robust control has been gaining popularity in the control literature for design of robust feedback systems. A central problem in QFT consists of proving the *existence* (or *non-existence*) of a QFT controller solution to a given design problem. In this paper, we propose a novel method based on interval analysis (Moore, 1979) to computationally verify the existence (or non-existence) of a controller solution, for a specified controller structure and an initial domain of controller parameter values. A feature of our proposed method is that it is a constructive existence method, i.e., if a solution of the specified structure exists for the given parameter domain, then all controller solutions lying in the domain are generated with our method. Essentially, the proposed method uses successive partitioning of the parameter domain and controller feasibility tests. We demonstrate the proposed method through a benchmark example.

**Keywords:** Control Synthesis, Interval Analysis, Quantitative Feeback Theory, Robust Control, Robust Synthesis.

## 1. Introduction

A versatile and practical engineering approach to the robust control problem is based on quantitative feedback theory (QFT) of Horowitz (Horowitz, 1991; Horowitz, 1993). The design is quantitative in the sense that the feedback is directly related to the amount of uncertainty and external disturbance. QFT has evolved with techniques to deal with single-input single-output (SISO) as well as multi-input multi-output (MIMO) cases, for linear and nonlinear, lumped and distributed parameter, time varying and time invariant systems. The QFT technique has been successfully applied to several practical problems with large plant uncertainty.

Consider a linear time invariant plant with parametric uncertainty given by $P\left(s,\lambda\right)$, where

$$\lambda = \left(\lambda_1, \ldots, \lambda_l\right)^T \in \mathbb{R}^l$$

is a plant parameter vector, which varies over a box $\boldsymbol{\lambda} = \left[\underline{\lambda}, \overline{\lambda}\right]$ consisting of two real column vectors $\underline{\lambda}$ and $\overline{\lambda}$ of length $l$ with $\underline{\lambda} \leq \overline{\lambda}$. This gives rise to the parametric plant family or set

$$\mathbb{P} \equiv \left\{ P\left(s,\lambda\right) : \lambda \in \boldsymbol{\lambda} \right\}$$

with the nominal plant $P(s,\lambda_0)$ corresponding to an arbitrary nominal $\lambda_0 \in \boldsymbol{\lambda}$.

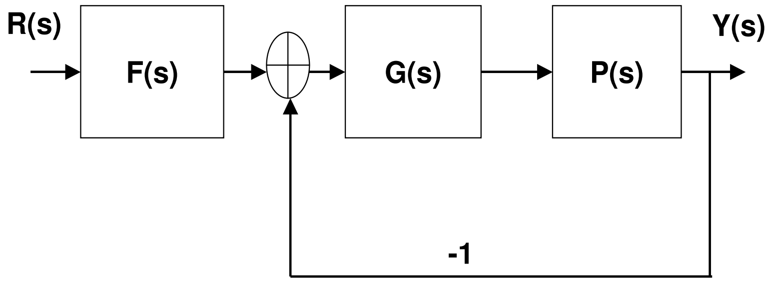


*Figure 1.* The two degree-of-freedom structure used in QFT

To achieve various specifications (specs), generally, the plant $P(s,\lambda)$ is embedded in the two degree-of-freedom feedback structure of QFT formulation as shown in Fig. 1, where $G(.)$ and $F(.)$ are transfer functions for the controller and prefilter, respectively. The controller $G(s,x)$ can be represented in the gain-pole-zero form as

$$G(s,x)=\frac{k_G\prod_{i=1}^{n_z}(s+\widetilde{z}_i)\prod_{i=1}^{n'_z}(s^2+2\zeta_i\upsilon_i s+\upsilon_i^2)}{\prod_{k=1}^{n_p}(s+p_k)\prod_{k=1}^{n'_p}\left(s^2+2\xi_k\vartheta_k s+\vartheta_k^2\right)} \tag{1}$$

where the controller parameter vector $x$ is

$$\begin{aligned} x=(\quad & k_G,\widetilde{z}_1,\ldots,\widetilde{z}_{n_z},\zeta_1,\ldots,\zeta_{n'_z},\upsilon_1,\ldots,\upsilon_{n'_z}, \\ & p_1,\ldots,p_{n_p},\xi_1,\ldots,\xi_{n'_p},\vartheta_1,\ldots,\vartheta_{n'_p}) \end{aligned} \tag{2}$$

The open loop transmission function is defined as

$$L(s,x,\lambda)=G(s,x)P(s,\lambda)$$

and the *nominal* open loop transmission function as

$$L_0(s,x)=G(s,x)P(s,\lambda_0)$$

The magnitude and angle functions of $L_0(s,x)$ are defined as

$$L_{0\_mag}(\omega,x)=|L_0(s=j\omega,x)|;\; L_{0\_ang}(\omega,x)=\angle L_0(s=j\omega,x) \tag{3}$$

Typically, following specifications are to be met for all $P(s,\lambda)\in\mathbb{P}$ and $\omega\in[0,\omega']$.

– Robust stability margin spec:

$$\left|\frac{L(j\omega,x,\lambda)}{1+L(j\omega,x,\lambda)}\right|\le w_s \tag{4}$$

– Robust tracking performance spec:

$$|T_L(j\omega)| \leq \left| F(j\omega) \frac{L(j\omega, x, \lambda)}{1 + L(j\omega, x, \lambda)} \right| \leq |T_U(j\omega)| \quad (5)$$

– Robust input disturbance rejection performance spec:

$$\left| \frac{G(j\omega, x)}{1 + L(j\omega, x, \lambda))} \right| \leq w_{d_i}(\omega)$$

– Robust output disturbance rejection performance spec:

$$\left| \frac{1}{1 + L(j\omega, x, \lambda)} \right| \leq w_{d_o}(\omega)$$

The QFT design procedure begins with the generation of the plant *template*, which is nothing but the value set of plant at a design frequency $\omega$, given as

$$\mathcal{P}(\omega) \ = \{P(s = j\omega, \lambda) : \lambda \in \boldsymbol{\lambda}\}$$

This is followed by the QFT bound generation step. At each design frequency $\omega_i$, the plant template $\mathcal{P}(\omega_i)$ is used to translate the given performance and stability specs into regions in the Nichols chart where the nominal loop transmission $L_0(j\omega_i, x)$ is allowed to lie. The composition of all such bounds at $\omega_i$ is referred to as the *bound* on $L_0(j\omega_i, x)$ at $\omega_i$ and is denoted as $B\left(L_{0\_ang}(j\omega_i, x), \omega_i\right)$ or simply as $B(\omega_i)$. For example, the bounds at various $\omega_i$ are plotted in Fig. 4 along with the so-called universal high frequency bound (UHFB) valid for all $\omega \geq \omega_h$, where $\omega_h$ is some sufficiently "high" frequency. At any given $\omega_i$, the magnitude of the bound generally varies with the phase $L_{0\_ang}(j\omega_i, x)$; while some bounds are single-valued upper or lower bounds, the others are multiple-valued.

The objective of the QFT procedure is to synthesize $G(s, x)$ that satisfies the bounds $B(\omega_i)$ at all the design frequencies, and then synthesize a prefilter $F(s)$ which places the allowable variation in magnitude of the closed loop system, inside the respective tracking bounds. The details of the QFT design procedure can be found in (Horowitz, 1993).

A central problem in QFT consists of proving the *existence* of a controller solution to a given design problem. Any arbitrary design specs cannot be achieved by a specified controller transfer function structure, particularly for the plants with uncertainty. In certain cases, e.g., for non-minimum phase plants with uncertainty, one can analytically verify the non-existence of controller solution, as demonstrated by Horowitz (Horowitz, 1993). But this argument cannot be generalised, and a great deal of expertise would be required for commenting on the existence of the controller solution. This motivates us to propose a method to computationally verify the *existence* of a controller solution. In this paper, we propose a novel method based on interval analysis (Moore, 1979) to computationally verify the *existence* (or *non-existence*) of a controller solution, for a specified controller structure and an initial domain of controller parameter values.

## 2. Existence Verification

We propose an algorithm to computationally verify the existence of a controller solution for an uncertain plant transfer function, given certain performance and stability specs. The proposed existence verification method is constructive in its approach, i.e., if a solution of the specified structure exists for the given parameter domain, then *all* controller solutions lying in the domain are generated with the method.

Using the QFT formulation, the given specs are converted to the constraints satisfaction problem. The set of bounds $B(\omega_i)$ as described in section 1, gives rise to the set of constraints in the existence verification problem. The tracking and disturbance bounds at design frequency $\omega_i$ are to be respected, leading to the following type of constraints:

– single-valued upper bound :

$$h_i\left(x\right) = \left|B\left(L_{0\_ang}(j\omega_i, x), \omega_i\right)\right| - L_{0\_mag}(j\omega_i, x) \leq 0 \tag{6}$$

– single-valued lower bound :

$$h_i\left(x\right) = L_{0\_mag}(j\omega_i, x) - \left|B\left(L_{0\_ang}(j\omega_i, x), \omega_i\right)\right| \leq 0 \tag{7}$$

To ensure the nominal closed loop stability of the system, the nominal loop transmission is forced to lie on the right side of the respective stability bounds. Thus, the multiple valued stability bounds give rise to additional constraints of following type:

$$\begin{aligned} h_i^s\left(x\right) = \angle B(L_{0\_mag}(j\omega_i, x), \omega_i) - L_{0\_ang}(j\omega_i, x) \leq 0, \\ \text{for } L_{0\_mag}(j\omega_i, x) \in \left[\min\left|B(\omega_i)\right|, \max\left|B(\omega_i)\right|\right] \end{aligned} \tag{8}$$

With these bounds, the constraint satisfaction problem can be given as

$$\begin{aligned} \text{find all} \quad & x \in \mathbf{x} \\ \text{such that} \quad & H(x) \leq 0 \end{aligned} \tag{9}$$

where, $H\left(x\right) = \{h_i(x), h_i^s(x)\}$ is the set of bound constraints in (6,7) and (8), and $\mathbf{x}$ is the bound constrained controller parameter domain. The parameter domain is either user specified, or is constructed based on the given structure of the controller transfer function. To verify if the given constraints can be satisfied with a pre-specified controller structure, we suggest the following procedure to construct the controller parameter domain:

– The upper bound for the interval values of the corner frequency of the poles and zeros of the controller is set to $10^a\omega_h$, where, for instance, $a \approx 1$ or 2. This sets the cutoff frequency for the poles and zeros to a few decades beyond the high frequency $\omega_h$. The upper bound for the interval value of the high frequency gain of the controller can be set to a large value. To avoid the internal stability problem and the RHP pole/zero cancellation of the design, the lower bound for the interval values of the corner frequency of poles and zeros is set to zero.

Finding the solution over a bounded domain is as good as testing the feasibility of infinite combinations of controller parameter values. The most efficient and reliable techniques to do this are based on the rigorous search using interval analysis. Hence, we next present an algorithm based on interval analysis to solve this constraints satisfaction problem for existence verification of controller solution.

### 2.1. The Proposed Algorithm

The proposed algorithm uses successive partitioning of the given search domain, and the range inclosure property of the interval arithmetic

$$\text{range}\,(f, \mathbf{z}) \subseteq f\,(\mathbf{z})$$

where, $f\,(\mathbf{z})$ is the natural interval evaluation of a function $f$ over the box $\mathbf{z}$.

In this strategy of the proposed algorithm, at each iteration the controller parameter box $\mathbf{z}$, currently under process, is split into two subboxes, and tested for its feasibility. Any subbox which does not satisfy the constraints is discarded. The subbox which satisfy the constraints is added to a solution list $\mathcal{L}^{sol}$, and the remaining subbox(s) are added to a stack list $\mathcal{L}^{stack}$ for further processing. This process is recursively carried out till the stack list $\mathcal{L}^{stack}$ is exhausted (emptied), i.e., till the given search domain is completely processed.

The proposed algorithm essentially consists of five major components: a feasibility test, list handling, initialization, a termination criterion, and a bisection strategy.

The **Feasibility test** determines if a box $\mathbf{z}$ of controller parameter values satisfies the QFT bound constraints. Evaluation of the natural interval extensions of the nominal loop transmission magnitude and angle functions on $\mathbf{z}$ at some $\omega_i$ gives an angle-magnitude rectangle $\{L_{0\_ang}(j\omega_i, \mathbf{z}), L_{0\_mag}(j\omega_i, \mathbf{z})\}$, in the Nichols chart. This rectangle is called as the $L_0$ box at $\omega_i$. Based on the relative location of this rectangle w.r.t. the bound $B(\omega_i)$, the parameter box $\mathbf{z}$ is determined as *feasible*, *infeasible*, or *ambiguous* at $\omega_i$. The overall feasibility of box $\mathbf{z}$ is decided based on its feasibility at each of the design frequencies. A flag variable $\mathit{flag}_z$ represents the feasibility of box $\mathbf{z}$. The details for the feasibility test are given in sec. 2.1.1.

**List handling**: A stack list $\mathcal{L}^{stack}$ and a solution list $\mathcal{L}^{sol}$ is maintained to save the boxes generated during the partitioning process. The boxes which are determined as *feasible* are put into the solution list $\mathcal{L}^{sol}$, and the *ambiguous* boxes, which need further processing, are put into the stack list $\mathcal{L}^{stack}$. Since the whole stack list $\mathcal{L}^{stack}$ is to be processed, any box from this list can be picked up for further processing, but for convenience, the first box of the stack list $\mathcal{L}^{stack}$ is taken up as current box $\mathbf{y}$ for processing in the next iteration.

**Initialization** (step 1 in the algorithm): The current box under process denoted as $\mathbf{z}$ is set to the initial search box $\mathbf{z}^0$, and the feasibility test is done for $\mathbf{z}$. If $\mathbf{z}$ is *infeasible*, then by the inclusion property of interval analysis there is no feasible solution $\forall z \in \mathbf{z}$; hence, the algorithm exits, declaring that no feasible solution exists in the given initial search box. Else, a stack list $\mathcal{L}^{stack}$ is initialized with the box $\mathbf{z}$, and the solution list $\mathcal{L}^{sol}$ is initialized as an empty list.

**Termination** (step 2 in the algorithm): Since the objective is to find out all the solutions of the constraints satisfaction problem, the algorithm should terminate only after the given initial search domain is completely processed. The stack list $\mathcal{L}^{stack}$ holds the boxes (i.e., part of initial search domain) which are neither fully acceptable nor rejectable as controller solutions. Thus, the algorithm can terminate when all such boxes are completely processed or in other words when the stack list $\mathcal{L}^{stack}$ is emptied. The termination condition is given as

$$\mathcal{L}^{stack} = \varnothing \tag{10}$$

**Bisection** (step 4 in the algorithm): If the above termination condition is not met, i.e., box $\mathbf{z}$ is *ambiguous*, then $\mathbf{z}$ is split along the maximum width direction into two subboxes $\mathbf{v}^1$ and $\mathbf{v}^2$.

**Feasibility check for new subboxes** (step 5 in the algorithm): The feasibility check is performed on each of these two subboxes, any infeasible subbox(s) which does not satisfy the constraints is discarded and the feasible one is added to the solution list $\mathcal{L}^{sol}$.

The algorithm for existence verification of a controller solution based on the above described strategy is now presented.

**Inputs:** Numerical bound set, the design frequency set $\{\omega_i : i = 1, \cdots, n\}$, expressions for natural interval extensions $L_{0\_mag}(\omega, \mathbf{z})$ , $L_{0\_ang}(\omega, \mathbf{z})$ of the nominal loop transmission magnitude and angle functions in (3), and the initial search box $\mathbf{z}^0$.

**Output:** List of feasible controller parameters or a message ”No feasible solution exists in the given initial search domain”.

**BEGIN Algorithm**

1. Checking the feasibility of initial search box.

   a) Set the current box to the initial search box, i.e., set $\mathbf{z} = \mathbf{z}^0$.

   b) Call *Feasibility* Subroutine to determine if the current box $\mathbf{z}$ is completely infeasible, completely feasible, or an ambiguous case. The feasibility test returns a value for the variable $flag_z$.

   c) Initialization

      *i)* **IF** $flag_z = infeasible$ **THEN** print the message “No feasible solution exists in the given initial search domain”, and **Exit** the algorithm.

      *ii)* **ELSE IF** $flag_z = feasible$ **THEN** print the message “The complete initial search domain is a feasible set of solution”, and **Exit** the algorithm.

      *iii)* **ELSE** initialize the stack list $\mathcal{L}^{stack} \Leftarrow \{\mathbf{z}\}$ and initialize the solution $\mathcal{L}^{sol} \Leftarrow \{\}$.
      **END IF**

2. Choose the first box from the stack list $\mathcal{L}^{stack}$ as current box $\mathbf{z}$, and delete its entry from the stack list $\mathcal{L}^{stack}$.

3. Split the current box $\mathbf{z}$ in the maximum width direction to get two new subboxes $\mathbf{v}^1$ and $\mathbf{v}^2$, such that $\mathbf{z} = \mathbf{v}^1 \bigcup \mathbf{v}^2$ .

4. Call *Feasibility* Subroutine to determine the feasibility of each new subbox, and get the value of $\mathit{flag}_{v^1}$ and $\mathit{flag}_{v^2}$.

5. **DO** for $i = 1, 2$,

   a) **IF** $\mathit{flag}_{v^i} = \mathit{infeasible}$, **THEN** discard the subbox $\mathbf{v}^i$
   b) **ELSE IF** $\mathit{flag}_{v^i} = \mathit{feasible}$, **THEN** add the subbox $\mathbf{v}^i$ to the solution list $\mathcal{L}^{sol}$
   c) **ELSE** add $\mathbf{v}^i$ to the stack list $\mathcal{L}^{stack}$.
   **END IF**

   **END DO**

6. **IF** the termination condition given in (10) holds, **THEN**

   a) **IF** $\mathcal{L}^{sol} = \varnothing$, **THEN** print the message "No feasible solution exists in the given initial search domain" and **Exit** the algorithm.
   b) **ELSE IF** $\mathcal{L}^{sol} \neq \varnothing$, **THEN** print the message "The feasible solutions are:" $\mathcal{L}^{sol}$, and **Exit** the algorithm.
   **END IF**

   **END IF**

7. **Go to** step **6**.

**END Algorithm**

2.1.1. *Feasibility Subroutine*

This subroutine finds the feasibility of the controller parameter box $\mathbf{z}$, and returns the value of $\mathit{flag}_z$, which represents its feasibility.

**Inputs:** Numerical bound set, the design frequency set $\{\omega_i : i = 1, \cdots, n\}$, expressions for natural interval extensions $L_{0\_mag}(\omega, \mathbf{z})$ , $L_{0\_ang}(\omega, \mathbf{z})$ of the nominal loop transmission magnitude and angle functions in (3), and the parameter box $\mathbf{z}$.

**Output:** Value of $\mathit{flag}_z$.

**BEGIN Subroutine**

1. At every design frequencies $\omega_i$, $i = 1, \cdots, n$, do the following:

   a) Evaluate $L_{0\_mag}(\omega_i, \mathbf{z})$ and $L_{0\_ang}(\omega_i, \mathbf{z})$.
   b) For single valued upper bounds: Over the phase interval $L_{0\_ang}(\omega_i, \mathbf{z})$, find out the maximum and minimum magnitude value of the bound $B(\omega_i)$, and denote it as $B^{\max}_{mag}(\omega_i, \mathbf{z})$ and $B^{\min}_{mag}(\omega_i, \mathbf{z})$, respectively (Fig. 2 explains this notation).
   c) For multiple valued stability bounds:

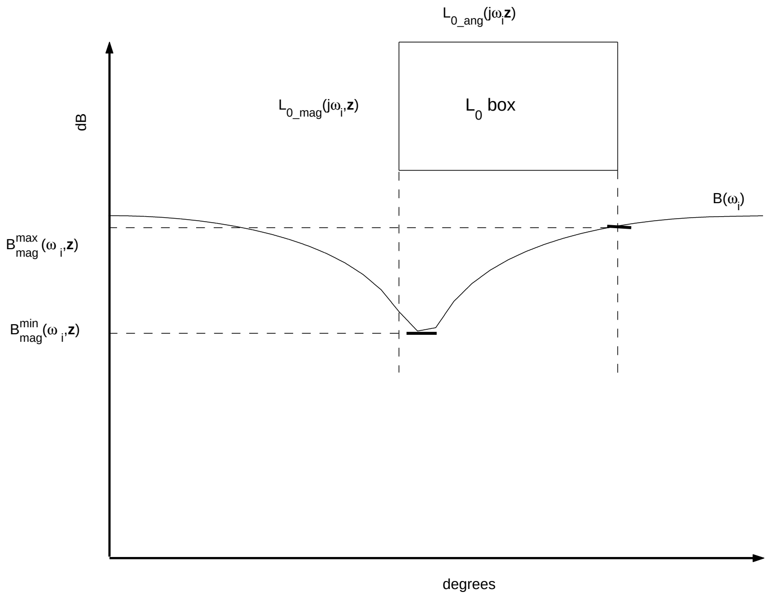


*Figure 2.* Definitions of $B^{\max}_{mag}(\omega_i, \mathbf{z})$ and $B^{\min}_{mag}(\omega_i, \mathbf{z})$.

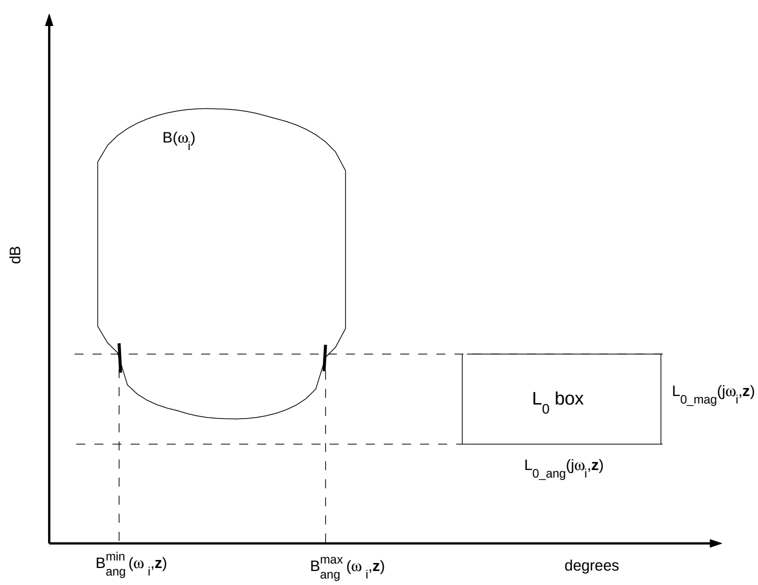


*Figure 3.* Definitions of $B^{\max}_{ang}(\omega_i, \mathbf{z})$ and $B^{\min}_{ang}(\omega_i, \mathbf{z})$

*i)* **IF** $L_{0\_mag}(j\omega_i, \mathbf{z}) \bigcap [\min|B(\omega_i)|, \max|B(\omega_i)|] \neq \varnothing$ **THEN**

A) Over the magnitude interval $L_{0\_mag}(\omega_i, \mathbf{z})$, find out the maximum and minimum phase value of the bound $B(\omega_i)$, and denote it as $B_{ang}^{\max}(\omega_i, \mathbf{z})$ and $B_{ang}^{\min}(\omega_i, \mathbf{z})$, respectively (Fig. 3 explains this notation).

**END IF**

2. Set the feasibility flag as follows:

a) **IF** for all $\omega_i$, $i = 1, \cdots, n$,

$$\inf\{L_{0\_mag}(\omega_i, \mathbf{z})\} \geqslant B_{mag}^{\max}(\omega_i, \mathbf{z})$$

**AND**

$$\begin{aligned} \{L_{0\_mag}(j\omega_i, \mathbf{z}) \bigcap & \ [\min|B(\omega_i)|, \max|B(\omega_i)|] = \varnothing \\ & \ \textbf{OR} \\ L_{0\_ang}(\omega_i, \mathbf{z}) & \ \geqslant B_{ang}^{\max}(\omega_i, \mathbf{z})\} \end{aligned}$$

**THEN** set the $flag_z$ = *feasible* and **RETURN**.

b) **ELSE IF** for any $\omega_i$, $i = 1, \cdots, n$,

$$\sup\{L_{0\_mag}(\omega_i, \mathbf{z})\} \leqslant B_{mag}^{\min}(\omega_i, \mathbf{z})$$

**OR**

$$\begin{aligned} \{L_{0\_mag}(j\omega_i, \mathbf{z}) \bigcap & \ [\min|B(\omega_i)|, \max|B(\omega_i)|] \neq \varnothing \\ & \ \textbf{AND} \\ L_{0\_ang}(\omega_i, \mathbf{z}) & \ \leqslant B_{ang}^{\min}(\omega_i, \mathbf{z})\} \end{aligned}$$

**THEN** set the $flag_z$ = *infeasible* and **RETURN**.

c) **ELSE** set the $flag_z$ = *ambiguous* and **RETURN**.
**END IF**

**END Subroutine**

Thus, the feasibility subroutine returns $flag_z$ = *infeasible*, *feasible*, or *ambiguous*, depending on whether the parameter box $\mathbf{z}$ is completely infeasible, completely feasible, or an ambiguous case, respectively, w.r.t. the bound constraints.

REMARK 1. ***Convergence and Reliability:*** *The convergence of the proposed algorithm can be easily proved on the lines of interval branch and bound algorithms (*Ratschek and Rokne, 1988*) and the reliability of the algorithm immediately follows from the interval analysis techniques.*

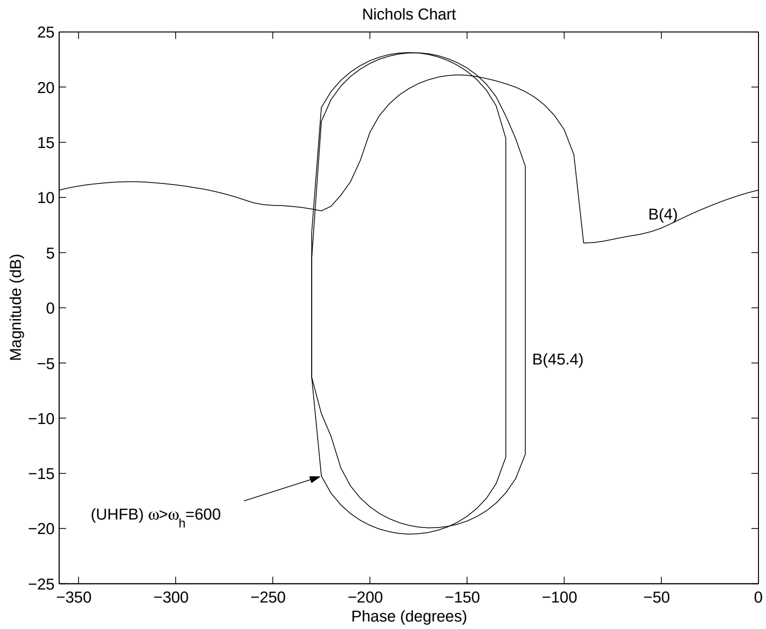


*Figure 4.* Bounds on $L_0$

## 3. Design Example

The proposed algorithm was tested on a QFT benchmark example, so that the results can be compared with that of an existing method. The example chosen is the design of robust controller for a non-minimum phase plant with uncertainty, given by Horowitz (Horowitz, 1993).

The uncertain plant transfer function is given as

$$P(s,\lambda) = \frac{k(1-\tau s)}{s(1+\beta s)} : k \in [1,3], \beta \in [0.3,1], \tau \in [0.05, 0.1]. \qquad (11)$$

The specs are:

- Robust stability margin spec (4): $w_s = 1.3032$
- Tracking spec (5): $|T_U(j4)| = 0.5dB$ and $|T_L(j4)| = -3.5dB$.

With just two design frequencies 4 and 45.4 rad/sec, using the Bode gain-phase relationship, Horowitz (Horowitz, 1993) showed analytically that, for the uncertain plant transfer (11), no controller solution exists for the above given specs.

We use the proposed algorithm to computationally verify the above finding of Horowitz, i.e., of the non-existence of a controller of first and second order for the above specs. We choose the non-minimum phase plant

$$P_0(s) = \frac{(1 - 0.05s)}{s(1 + 0.3s)}$$

as the nominal case. The stability and tracking bounds for these specs are shown in Fig. 4. These bounds $B(\omega_i)$ at each design frequency $\omega_i$ are generated using the QFT toolbox (Borghesani et al., 1995). From the nature of the generated QFT templates, we find that the UHFB frequency $\omega_h \approx 600$ rad/sec. Based on this value of $\omega_h$, the upper bound on the pole/zero can be fixed as 6000 rad/sec. (see sec. 2), but instead we choose an arbitrarily large value of $10^4$ rad/sec. Moreover, using an arbitrarily large upper bound for the gain, the initial search box $\mathbf{z}^0$ is constructed as follows:

- For the first order controller, the parameter vector $z = \{k, \widetilde{z}_1, p_1\}$ is

$$\mathbf{z}^0 = (0, 10^8], (0, 10^4], (0, 10^4]$$

- For the second order controller, the parameter vector $z = \{k, \widetilde{z}_1, \widetilde{z}_2, p_1, p_2\}$ is

$$\mathbf{z}^0 = (0, 10^8], (0, 10^4], (0, 10^4], (0, 10^4], (0, 10^4]$$

For the aforementioned structures and the initial search domains, the proposed algorithm terminated with the message: “No feasible solution exists in the given initial search domain”. Thus, this finding is in agreement with the analytically found ‘*non-existence*’ of Horowitz mentioned above.

## 4. Conclusions

An algorithm has been proposed in this paper to computationally verify the *existence* (or *non-existence*) of a QFT controller solution, for a specified controller structure and an initial domain of controller parameter values. The proposed algorithm has been tested successfully on a QFT benchmark example for cross validating the results.

## Acknowledgment

The financial assistance provided by Council of Scientific and Industrial Research(CSIR), India to Sachin Tharewal for carrying out the research work as SRF is gratefully acknowledged.

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