Computational Methods for Decision Making Based on Imprecise Information

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Abstract

In this paper, we investigate computational methods for decision making based on imprecise information in the context of engineering design. The goal is to identify the subtleties of engineering design problems that impact the choice of computational solution methods, and to evaluate some existing solution methods to determine their suitability and limitations.

The first challenge in developing a feasible method for computing with imprecise information is to determine how to represent it. Since engineering design problems include both imprecision and probabilistic uncertainty, a probability-box (or p-box) is used as a representation [1]. Although not as general as imprecise probabilities, p-boxes are easy to understand and are structured such that they allow for efficient computation.

Assuming that p-boxes are used to represent the uncertainty, computational methods must be chosen for p-box computations that are compatible with the characteristics of engineering design problems. First, such methods need to apply to black-box models. Much of engineering design practice uses pre-existing models that are either impractical or impossible to modify. Therefore, it is important to accommodate engineering models that are available only as black-boxes—i.e. for given (deterministic) inputs, there exists a procedure that generates the outputs, but little or no knowledge is available about the actual mathematical relationships between inputs and outputs. Although, in the future engineering modelers may have software environments (such as Risk Calc [2]) that support analysis with p-boxes, our current problem statement should avoid such assumptions. Second, the methods must be able to handle repeated variables. Often, a difference in expected utility needs to be computed which results in repetition of the parameters in the utility function. Finally, it is important that the method propagate imprecision efficiently and can be integrated with optimization methods. While, for precise information, the output of a design performance model is expected utility, in the

presence of imprecision, performance models result in upper and lower bounds on expected utility. As a consequence, rather than maximizing the expected utility, the focus should be on eliminating the dominated design alternatives. To lead to efficient elimination, the bounds on expected utility must be accurate (i.e. approach the outputs' best-possible bounds) and not hyper-conservative.

Several approaches for propagating imprecise probabilities have been published in the literature. These methods are insufficient for practical engineering analysis. The dependency-bounds approach of Williamson and Downs [3] works well only for open models (that is, models with known mathematical operations). The distribution-envelope approach of Berleant [4] is more accommodating to black-box models, but seems to be prohibitively expensive for problems of large dimensionality. Both of these approaches rely on interval arithmetic and are therefore limited in their applicability by repeated variables.

In an attempt to overcome the difficulties faced by these deterministic methods, we propose an alternative approach that utilizes both Monte Carlo simulation and an optimization algorithm. We have implemented this approach in the design of an automotive gearbox [5]. The Monte Carlo/optimization hybrid approach has its own drawbacks in that it requires the solution of a global optimization problem, and it assumes independence between the uncertain variables.

References

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