Applications of fast and accurate summation in computational geometry

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Algorithms that make decisions based on geometric test such as determining which side of a line a point falls on, often fail due to roundoff error. A solution to answer these problems is to use software implementations of exact arithmetic often at great expense. In his reference implementation, Shewchuk [1] uses an arbitrary precision library to obtain fast C code of four geometric predicates, the 2D and 3D orientation and incircle tests. The inputs are single or double precision floating point numbers. The speed of these algorithms is due to the fact that they are adaptive: the running time depends on the degree of uncertainty of the result.

The aim of the paper is to provide fast and accurate algorithms for computing small determinants and robust geometric predicates used in computational geometry. We use a recent algorithm from Ogita, Rump and Oishi [2] that computes accurately the sum of n floating point numbers with a valid error bound. We use this error bound to provide an adaptive algorithm that computes determinants up to a given relative accuracy. It only requires IEEE 754 floating point arithmetic with round to nearest. We apply this algorithm to computed robust geometric predicates and we focus more particularly on the 3D orientation predicate.

Since the algorithms are adaptive, the computing times depend on the condition number of the problem. We compare the different algorithms with respect to condition numbers we introduce. Experimental results show that our algorithm shares the same timings as Shewchuk's predicate in the well-conditioned cases. The ill-conditioned cases represent a very small part of the input in practical applications but are the most time-consuming. For this ill-conditioned instances our algorithm is shown to be over twice as fast as Shewchuk's algorithm. The potential of our algorithms is in providing a fast and simple way to extend the precision of critical predicates in computational geometry. The techniques used here are simple enough to be coded directly in numerical libraries.

References

[1] J. R. Shewchuk, Adaptive precision floating-point arithmetic and fast robust geometric predicates. *Discrete Comput. Geom.*, 18(3):305-363, 1997.
[2] T. Ogita, S. M. Rump, and S. Oishi, Accurate sum and dot product. *SIAM J. Sci. Comp.*, 26(6):1955-1988, 2005.