Modeling Hysteris in CLIP – The Two Tanks Problem

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Abstract

Hickey and Wittenberg [HW04] study the "Two Tanks Problem" described by Stursberg *et al.* [SKHP97] In this paper, we expand on the use of CLIP (a Constraint Logic Programming over Intervals and Functions language) to formally describe more complex systems. We add complexity in several forms. The simplest is to have a larger system. We move from a system with two tanks to one with four tanks, and we add valves to the pipes connecting the tanks. This example easily generalizes to an N-tanks problem where the tanks, connected by pipes, form an arbitrarily complex graph. The more important is the refinement of the model in several places. We use a valve model in which the flow varies exponentially with the valve position over much of the valve's range, and then discontinuously as the valve is almost closed. We introduce hysteresis in our analysis to avoid an infinite loop of zero-time transitions, and we discuss why our techniques should not have trouble with "Zeno" transitions.

An elegant feature of our technique is that while the possibility of Zeno behaviour [ZJLS01] can arise either from physical reasons (a value is near zero, so the sign of the changes is hard to know) or for modeling reasons (the system is near the boundary between two behaviour regimes, and while both regimes describe similar behaviour near the boundary, the model might switch between the two regimes infinitely often in a finite time), we use the same technique of hysterisis in switching from one regime to the other to prevent the Zeno behaviour. This is easily done in CLIP by changing the conditions for a state change from one to the other to include hysterisis.

References

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