Why are intervals and imprecision important in engineering design?

J. M. Aughenbaugh and C. J. J. Paredis

Systems Realization Laboratory
Georgia Institute of Technology
Atlanta, GA 30308
www.srl.gatech.edu
jasona@gatech.edu or chris.paredis@me.gatech.edu

Abstract: It is valuable in engineering design to distinguish between two different types of uncertainty: inherent variability and imprecision. While variability is naturally random behavior in a physical process or property, imprecision is uncertainty that is due to a lack of knowledge or information. There are many sources of imprecision in design. Sequential decision making introduces imprecision because the results of future decisions are unknown. Statistical data from finite samples of environmental factors are inherently imprecise. Bounded rationality leads to imprecise subjective probabilities. Expert opinions and judgments often are imprecise due to a lack of information or conflict. Behavioral simulations and analysis models are imprecise abstractions of reality. Knowledge of a decision maker's preferences may be imprecise due to bounded rationality or other constraints. Consequently, the engineering design community needs efficient computational methods for interval data and imprecise probabilities in order to support decision making in the design process. This paper introduces these sources and needs, with the aim of forming a foundation for future collaboration with the reliable engineering computing community.

Keywords: imprecision, imprecise probabilities, probability boxes, p-boxes, uncertainty, engineering design, intervals

1. Introduction

The goal of this paper is to introduce the needs of the engineering design community for computations with intervals and imprecise probabilities to the reliable engineering computing community. Earlier work has demonstrated the value of using imprecise probabilities in engineering design (Aughenbaugh and Paredis 2005), the role of imprecise probabilities in applying information economics (Aughenbaugh, Ling et al. 2005), and the elimination of decision alternatives using interval comparisons (Rekuc, Aughenbaugh et al. 2006). However, significant computational challenges are faced in implementing these methods in applied problems.

Consequently, the established expertise of the reliable engineering community in these areas could be very valuable in engineering design. By introducing the needs and context of engineering design problems, we hope to foster future collaboration between the design community and the reliable engineering computing community.

Section 2 provides an overview of the design process, including its structure and challenges. The third section describes sources of interval data and imprecise probabilities, together referred to as *imprecision*, in engineering design. The fourth section provides a brief overview of the computational challenges faced in engineering design due to imprecision.

2. The engineering design process

Design is a process of converting information about customer interests and requirements into a specification of a product. This process involves searching through a very large, unstructured space of solutions (Tong and Sriram 1992) based on vague and uncertain knowledge about possible solution alternatives (Gupta and Xu 2002), their physical behavior (Aughenbaugh and Paredis 2004), their cost (Garvey 1999), and the decision maker's preferences (Kirkwood and Sarin 1985; Otto and Antonsson 1992; Carnahan, Thurston et al. 1994; Seidenfeld, Schervish et al. 1995). In order to guide engineers through this process, several approaches have been developed. In this paper, we introduce the general model of *systematic design* described by Pahl and Beitz (1996).

2.1. Systematic design

In systematic design, the design process is broken into four main phases, as summarized in *Table 1*. In the *product planning and clarification of task phase*, a need for a product is determined and described. Product planning is mostly in the domain of corporate strategy and marketing; a company's situation and market condition are analyzed, profitable product ideas sought, and a product proposal made. The next step is to clarify the task by refining the product proposal and creating a detailed requirements list for the product. These requirements tell engineers what a product should be, should not be, and what it must be (at a minimum) in order to be successful. Once a list of requirements and objectives is created, conceptual design can begin.

The conceptual design phase takes the list of requirements and objectives and determines the principle solution structures to be pursued in embodiment design. To some, this is where traditional engineering begins. First, designers distill the problem down to its core, asking *what are we really trying to build?* Then they identify what functions (for example in a car design, functions such as *move person, protect person*, *monitor performance*) the design must perform and how these functions interact at a high level, such as transfers of energy, mass, and information. All of this information is combined into a function structure. Next, designers seek to enumerate possible physical implementations, or working principles, for each function. For example, three working principles for the function *mark a piece of paper* could be *deposit* *material by friction* (e.g. a pencil), *melt material onto paper* (e.g. laser jet printing), or *burn away material* (e.g. scorching the paper with a laser). Since in general there are multiple functions, each with multiple working principles, they can be combined into an overall product in many different ways, or solution variants. Finally, these solution variants must be evaluated and a principal solution *concept* chosen. This concept forms the foundation for embodiment design.

In embodiment design, designers develop the design concept in more detail by considering additional technical and economic criteria. Essentially, embodiment design takes the working principles and concepts developed in conceptual design and develops an actual design specification, at which point detail design can lead directly into production. During detail designthe arrangement, dimensions, materials, and production methods of all parts of the product are finalized and documented.

Table 1. Systematic Design Phases

Phase	Main tasks
Planning and clarifying the task	Investigation into the economic and technical viability of creating a given product, and the definition of the exact requirements of a system and the criteria surrounding its functioning.
Conceptual design	Development of function structure and the evaluation of different solution variants to this problem.
Embodiment design	Conversion of a conceptual working structure to a specification of layout.
Detail design	Finalization of the design and production details.

2.2. PARTITIONING THE DESIGN PROBLEM

Complex problems can rarely, if ever, be solved globally in one step. Most products have reached a level of complexity at which it is infeasible for one engineer or even engineers from one discipline to design them completely. Instead, the design problem must be broken down into smaller chunks that are designed by separate design teams. The solutions to these sub-problems are then synthesized and integrated into a complete design for the overall system. Systematic design is an appropriate approach for designing a product at one level of detail, but it does not address this higher-level process of decomposing a system into subsystems, concurrently designing subsystems, and subsequently integrating subsystem designs into the overall system. A holistic, hierarchical decomposition approach to the design process that addresses these problems is provided by *systems engineering* (Forsberg and Mooz 1992; Buede 2000; Forsberg, Mooz et al. 2000; Blanchard 2004). This paper will not address systems engineering formally. However, it is useful to consider what happens when the design task is broken into sub-problems.

When the design process is sub-divided, it becomes recursive—the overall design process is a sequence of design sub-problems. For example, consider the design of a car. A car can be broken down into many subsystems (such as engine, drivetrain, wheels, chassis, and so on), and each of these subsystems can be broken down into smaller subsystems, as simply shown in Figure 1. In many cases, a different team of engineers will perform the embodiment of each subsystem. Teams may also work on sub-problems concurrently, rather than sequentially. For example, one team may be designing the drivetrain while another team is designing the engine.

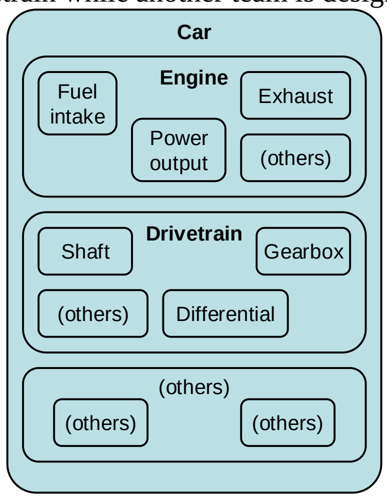

Figure 1. Subsystems of a car

When a team is formed to design the engine, its members first must clarify their task by using their technical expertise to elaborate on the requirements. For example, a particular engine concept is one of the working solutions from the conceptual design (Phase 2) of the car, as shown in Figure 2. Part of this engine design process is subdividing the engine into its subsystems, such as the fuel intake, and so on down to the smallest component of the system. This design process is challenging because the performance of the overall system may be a function of the interactions between sub-systems. Thus, the decisions of one team depend on future decisions and on decisions made concurrently by other design teams. In such situations, engineers can adopt robust design strategies (Chen, Allen et al. 1996) or set-based design approaches (Sobek, Ward et al. 1999; Rekuc, Aughenbaugh et al. 2006). In either case, each team recognizes that the decisions of other teams are uncertain, treating them as random variables, intervals, or sets. The nature of these uncertainties is addressed in Section 3 of this paper. First, the importance of decision making in the design process is discussed.

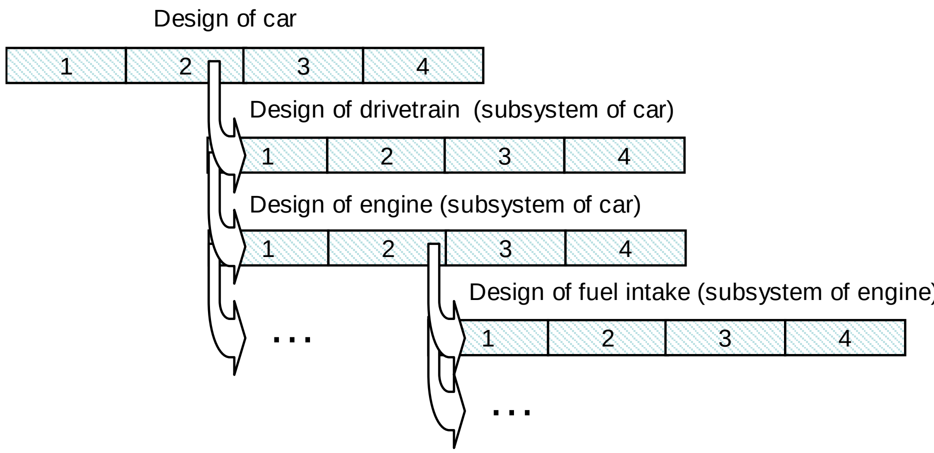

Figure 2. Recursive design process, design phases numbered 1-4

2.3. DECISION-BASED DESIGN

Independent of the design process chosen, designers repetitively must identify problems, search for solutions, evaluate solutions, and choose a final design. Inspired by this process, decision-based design recognizes that the principal role of an engineer in the design process is to make decisions (Mistree, Smith et al. 1990; Hazelrigg 1998; Marston, Allen et al. 2000). This paradigm shifts the emphasis of design research to decision making; one way to improve the design process is to enable engineers to make better decisions.

Engineers must make decisions while subject to many constraints, including limits on human cognitive abilities, or what Herbert Simon describes as bounded rationality (1947). According to Simon, humans cannot simultaneously consider all consequences of every alternative; there is a limit to how much a person can consider at one time. In engineering design, the problems of bounded rationality are exacerbated by the nature of the design process. For example, there are usually multiple people working on a single problem, and these people may be distributed in different geographic locations, organizational divisions, and technical disciplines. Consequently, it is very difficult for the right person to have the right information available at the right time in a format that he or she can comprehend (Cooper 2003). In many cases information, such as future decisions or concurrent decisions made by other design teams, is just inherently unavailable.

Finally, and perhaps most obviously, engineers have finite resources, such as time and money. Consequently, they cannot study every detail of every subsystem extensively. Decisions often are guided with approximate models, expert opinion, rules of thumb, and even pure intuition. One goal of decision-based design is to support these decisions with formal methods. It has been recommended by some researchers in the engineering design community to base these methods on traditional statistical decision theory (Pratt, Raiffa et al. 1995), in which uncertainty is represented using precise probability distributions. However, it is important in engineering design to distinguish between two different types of uncertainty: inherent variability and imprecision (Parry 1996; Nikolaidis, Chen et al. 2004; Aughenbaugh and Paredis 2005).

2.4. VARIABILITY AND IMPRECISION

Variability, also called aleatory uncertainty (from the Latin aleator = dice thrower), is naturally random behavior in a physical process or property (Oberkampf, DeLand et al. 2002; Haukaas 2003). It is also known as objective uncertainty (Ferson and Ginzburg 1996) and irreducible uncertainty (Der Kiureghian 1989). Examples include manufacturing error, errors in communication systems, and radioactive decay. Inherent variability is best represented in stochastic terms, e.g., by a probability density function.

Imprecision, on the other hand, is due to a lack of knowledge or information (Parry 1996) and sometimes is called epistemic uncertainty (from the Greek episteme = knowledge), reducible uncertainty (Der Kiureghian 1989) or subjective uncertainty (Ferson and Ginzburg 1996). Imprecision is generally best represented in terms of intervals (Kreinovich, Ferson et al. 1999; Muhanna and Mullen 2004). While some authors doubt the philosophical distinction between aleatory uncertainty and imprecision, such distinctions are useful in practice (Ferson and Ginzburg 1996; Hofer 1996; Winkler 1996; Aughenbaugh and Paredis 2005).

The role of imprecision in engineering design is often overlooked, at least in part due to practical reasons—engineers do not know how to compute and make decisions effectively with imprecise information. They instead assume away or ignore imprecision. Since methods for representing and computing with imprecise information are research topics in the reliable engineering computing and imprecise probability communities, it is important to demonstrate the need for interval and imprecise methods in engineering design to these communities. Ideally, with a new understanding of the needs of engineers, researchers in these areas can help explain these methods to the design community and work with designers to expand these methods to meet the needs of engineering design practice.

3. Sources of imprecision in engineering design decisions

In this paper, the sources of imprecision in engineering design are considered in the context of the simplified design model illustrated in Figure 3. The partitioning of the design problem into sub-problems results in a sequence of decisions (for simplicity, concurrent decisions are ignored), of which one is shown in detail in Figure 3. In this simple example, a designer, or decision maker (hereafter abbreviated as DM), has two decision alternatives. Based on characteristics of the alternatives and environmental factors, the DM performs multiple simulations (S_i) or other analyses (A_i), including eliciting expert opinion, to study the performance of the alternatives.

Performance attributes are then combined or weighted according to the DM's preferences (perhaps according to utility theory), and the most preferred alternative is selected (or alternatively when there are more than two decisions alternatives, the DM can proceed by selecting a set of the more preferred alternatives (Rekuc, Aughenbaugh et al. 2006)).

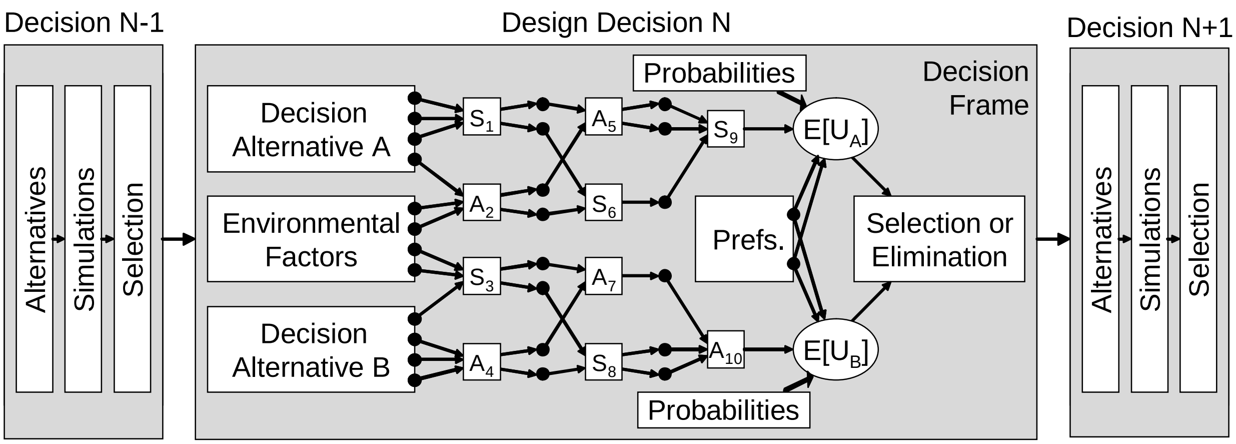

Figure 3. A sequential decision process in simulation-based design.

Almost every aspect of this decision introduces imprecision. More specifically:

- Sequential decision making introduces imprecision because the results of future decisions are unknown.
- Statistical data from finite samples of environmental factors are inherently imprecise.
- Bounded rationality leads to imprecise subjective probabilities.
- Expert opinion and judgments are not precise, due to lack of information or conflict.
- Behavioral simulations and analysis models are imprecise abstractions of reality.
- Preferences may be imprecise due to bounded rationality or non-stationarity.
- Numerical implementation of these models introduces additional imprecision.

In the following sub-sections, we elaborate on how these sources introduce imprecision into the design process.

3.1. SEQUENTIAL DECISION MAKING

As noted earlier, the complexity of the design problem makes it impossible to arrive at an optimal design in one step. Instead, the process is divided into a sequence of decisions. This process is illustrated using a simple design problem with two design variables: vehicle type and engine type. There are two options for vehicle type: car or bike. There are three options for engine type:

gasoline engine, diesel engine, or electric motor. If the DM chooses the design in one step, he or she would choose from the set of six *design alternatives* shown in Figure 4. In the context of this example, each of these design alternatives is a fully detailed design of a final product.

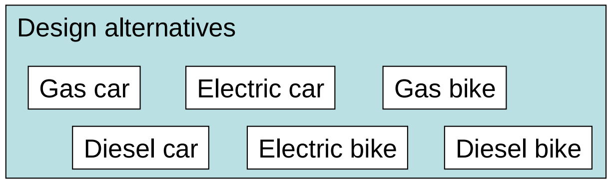

Figure 4. One stage decision

In order to choose the best design out of these six, the DM would need to evaluate and compare all six. While easy in this simple example, it is impractical to enumerate and evaluate all design alternatives by considering all possible combinations of all solution principles for all the subsystems of a complex product. Consequently, the decisions are broken down into sequences to allow for efficient exploration of the design space. For example, in the previous vehicle design example, a DM can follow a sequential approach in which he or she first chooses the vehicle type, and then the engine type, as shown in Figure 5.

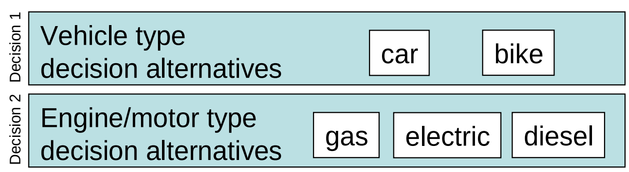

Figure 5. Sequential decisions

Note that it is important here to distinguish clearly between *decision* alternatives and *design* alternatives. A design alternative is one of the possible complete product design specifications (recall Figure 4), while each decision alternative is a specific option for a specific decision and corresponds to a set of design alternatives. For example, when choosing the vehicle type, the DM has two *decision alternatives*: car or bike. Each of these decision alternatives actually corresponds to a *set* of *design alternatives*, as shown in Figure 6.

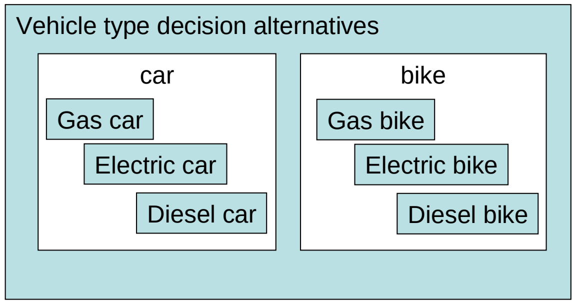

Figure 6. Sets of design alternatives

The choice of decision alternative *car* for vehicle type includes the gas car, diesel car, and electric car design alternatives, because the vehicle type decision will be followed by the engine type decision. Once a decision is made to pursue, for example, a car design rather than a bike, the DM does not need to consider explicitly the design alternatives gas bike, electric bike, and diesel bike; these design alternatives are *eliminated* from consideration.

One limitation of a sequential decision process is that decisions often are coupled. In general, one really needs to know the outcome of future decisions to select the best (or most preferred) decision alternative for the current decision. For example, a fully designed car will have a certain maximum horsepower, but this certain value is unknown when the vehicle type decision is made, because it depends on the future design decision of engine type. The set of car designs in Figure 6 has multiple horsepower maxima, each corresponding to a sub-design (gas car, electric car, and diesel car). Thus, when selecting type *car* rather than *bike*, a DM is not selecting a precisely characterized horsepower, but rather a set or interval of horsepower. In a more complex problem, imprecision will remain once the engine type is chosen because a particular engine type is a set of designs. For example, even if a gas engine is chosen, characteristics such as horsepower, torque, mass, and fuel efficiency will be inherently imprecise because they depend on additional details of the design.

By itself, the inherent existence of sets in sequential decision making demonstrates the need to compute with intervals, sets, or otherwise imprecisely characterized information. However, other sources of imprecision are independent of the existence of sets of design alternatives. These may have different characteristics and may affect the design process differently, as described in the following.

3.2. Statistical data

Engineers frequently gather statistical data about environmental or other factors to support design decisions. Such quantitative data gives an illusion of being well-characterized, but actually it is inherently imprecise. Assume one needs to design a pressure vessel, and the vessel will be made of a new type of steel for which the yield strength X is not well characterized. Engineers have strong theoretical evidence that the material strength is normally distributed, but they do not know the mean μ or variance σ^2 of the distribution. Because the material is new and testing is relatively expensive, DMs have only measured the yield strength in a set Σ of n independent tension tests, where n is a relatively small number due a high cost of testing. These tests can at best give an estimate of the true distribution, so in addition to inherent randomness (irreducible uncertainty), engineers also face imprecision—they cannot characterize the parameters of the random variable precisely.

For example, assume the engineers have a set of 30 material strength measurements. They could use the 30 samples to estimate the true mean and variance of the distribution using standard statistics. However, these estimates ($\hat{\mu}$ and $\hat{\sigma}^2$) are exactly that—*estimates*. The resulting distribution $X \sim N(\hat{\mu},\hat{\sigma}^2)$ in general is *not* the true distribution. Alternatively, confidence intervals can be constructed on the true mean and variance at the α confidence level as follows, where n is the number of samples and s is the sample standard deviation (Hines, Montgomery et al. 2003):

$$\left[\underline{\mu},\overline{\mu}\right] = \left[\hat{\mu} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}},\ \hat{\mu} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right] \qquad (1)$$

$$\left[\underline{\sigma^2},\overline{\sigma^2}\right] = \left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}},\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right]. \qquad (2)$$

The resulting structure:

$$X \sim N\left(\left[\underline{\mu},\overline{\mu}\right],\left[\underline{\sigma^2},\overline{\sigma^2}\right]\right) \qquad (3)$$

is a probability box, or p-box (Ferson and Donald 1998; Aughenbaugh, Ling et al. 2005). All normal distributions with means and variances given by Equations (1) and (2) are contained inside this p-box. Previous work has suggested that accounting for the imprecision in statistical data with p-boxes will lead, on average, to better design decisions for high-risk application (Aughenbaugh and Paredis 2005). However, there are computational challenges for using p-boxes, or more generally imprecise probabilities (Tintner 1941; Hart 1942; Levi 1974; Walley 1991; Weichselberger 2000), in complex engineering problems, as described in the briefly in Section 4 and elaborated on in detail in another paper in this workshop (Bruns, Paredis et al. 2006).

In this section, we focused on statistical data, emphasizing a rather frequentist interpretation of probability. The *frequentist* interpretation is based on the notion of relative frequencies of outcomes. Under a frequentist interpretation, a probability represents the ratio of times that one outcome occurs compared to the total number of outcomes in a series of identical, repeatable, and possibly random trials. In engineering design, events are not always repeatable. Even assuming some events are essentially repeatable and data can be collected, there is no guarantee that a particular sample is representative of the true relative frequency. Although in theory the relative sample frequency approaches the true relative frequency as the sample size goes to infinity, an infinite sample size is impossible to acquire in practice. Consequently, engineers will always face *imprecision* in their characterizations of the frequentist probabilities. Other times, it is inappropriate to adopt a purely frequentist view of probabilities in engineering design. Often, a *subjective* interpretation is more applicable.

3.3. IMPRECISE SUBJECTIVE PROBABILITIES

Proponents of a *subjective* interpretation of probability assert that there is no such thing as a true or objective probability, but rather probabilities are an expression of belief based on an individual's willingness to bet (de Finetti 1974; Lindley 1982; Winkler 1996). One of the subjectivists' primary arguments against a frequentist perspective is the absence of truly repeatable events, especially in practical problems. For example, the probability that Team A beats Team B in a basketball game has no real meaning under a frequentist interpretation, because that event—that particular game—will occur exactly once. In this context, the notion of a long term frequency, and even random events, is meaningless (de Finetti 1974). However, many people are willing to express their belief of who will win in terms of bets. When framed appropriately, such bets can be taken as subjective probabilities.

We prefer to adopt a loosely subjective interpretation of probability because true relative frequencies cannot be determined with any finite number of data samples, and because a subjective interpretation is applicable to a broader class of problems, as it is not limited to repeatable events. Naturally, subjective probabilities should be consistent with available information, including knowledge about observed relative frequencies (when applicable) and the DM's actual beliefs; such probabilities can be considered *rationalist* subjective probabilities (Walley 1991). Our interpretation is not as strict as the traditional views [see Lindley (1982) for a summary of the strict subjective tradition], because we admit imprecisely known subjective probabilities. The traditional school claims that by definition, subjective probabilities are known to a decision maker, because they *are* his or her beliefs. We prefer an interpretation that acknowledges the practical difficulties in arriving at a precise characterization of such beliefs.

The process of eliciting and assessing an individual's beliefs, or willingness to bet, is resource intensive. Even assuming that precise beliefs—and hence precise probabilities—exist, it will often be impractical to fully characterize them due to constraints such as bounded rationality, time, and computational ability (Weber 1987; Walley 1991; Groen and Mosleh 2005). Consequently, only a partial—and therefore imprecise—characterization of subjective probabilities is normally available.

The notion of imperfectly known probabilities is not new. Decision theory has long differentiated between decision making with known probabilities (*decision making under risk*) and decision making without knowledge of probabilities (*decision making under uncertainty*) (Knight 1921). Since then, researchers have explored the middle ground of incomplete knowledge of probabilities, such as ordered probabilities (Fishburn 1964) and linear constraints on the probabilities (Kmietowicz and Pearman 1984), in addition to the more general imprecise probabilities. The ability to compute with such uncertainties is crucial to the success of engineering design.

3.4. Expert opinion

A significant source of information in engineering design are experts who use their knowledge and experience to form judgments, beliefs, and estimates (Cooke 1991; Ayyub 2001). Information from expert opinions is inherently imprecise. First, opinions may not always be cited precisely, especially when expressed in linguistic terms, such as *unlikely*, *large*, or *poor*, a case in which fuzzy set theory has a role (Zadeh 1965; Ayyub 2001). Because an opinion about the world is not necessarily the truth of the world, opinions also can differ from person to person. Often, these opinions will conflict. For example, two experts are asked the probability that a quantity X is below 5; that is, $P\{X < 5\}$. The first expert says that $P\{X < 5\} = 0.3$ (and consequently $P\{X \geq 5\} = 0.7$). The second expert states that $P\{X < 5\} = 0.6$ (and consequently $P\{X \geq 5\} = 0.4$). The combination of such evidence, especially when conflicting, is an important research area, often focused on Evidence Theory (Dempster 1967; Shafer 1976; Yager, Kacprzyk et al. 1994; Oberkampf and Helton 2002; Mourelatos and Zhou 2005). Evidence Theory is a general theory that contains both traditional probability theory and possibility theory as special cases (Klir 1992). Consequently, interpretations of and methods for computing with evidence are of significant interest to engineering designers.

3.5. Imprecise analysis models

An important step in decision making and design is to determine the DM's preferences over design alternatives. As illustrated in Figure 3, this involves the application of multiple models: simulation models that predict the performance of the alternatives, models for the uncertain inputs to these behavioral models, and models of the DM's preferences.

Behavioral models predict the performance of design alternatives in terms of attributes that are important to the DM, such as physical behavior, cost, and reliability. Since these models, like all models, are only an abstraction of reality, they are imprecise. Specifically, although the laws of physics are known very precisely, one often makes significant assumptions when applying the laws of physics to complex geometries, or one omits certain known—but less significant—physical phenomena from the model to reduce the complexity.

For example, a model for an internal combustion engine is often abstracted into an algebraic relationship between engine speed and torque. The detailed physical phenomena (including airflow, gas-mixture combustion, friction, and inertia) are reduced into one simple algebraic relationship. This simple relationship is an idealization that may contain a significant error—the unknown or unmodeled relationships between a variety of parameters that play a role in the engine performance, such as air density, acceleration, or engine temperature. The lack of knowledge of the influence of these parameters on engine performance results in imprecision in the model's results. Since there is no probability distribution associated with such modeling and systematic errors, one cannot express the likelihood of occurrence for a particular error but can at best bound the size of the error, in which case the errors should be represented in terms of interval-based uncertainty.

In addition to the imprecision in the behavioral models themselves, there is often also significant imprecision in the parameter values or inputs to these models. For instance, the air resistance model of a car may include a drag coefficient, which can only be determined precisely through experimentation that is more extensive. Given the limited resources (cost, time, etc.) available for experimentation, the coefficient is only determined up to certain error bounds, which introduces additional imprecision in to the model predictions. There may also be stochastic environmental noise parameters. In this case, the uncertainty in the inputs can be modeled using imprecise probabilities or p-boxes; in addition to the inherent variability of such parameters, they will be imprecisely characterized, as described in the preceding sections for statistical data or subjective probabilities.

3.6. Imprecise preferences

Once the performance attributes of a particular design alternative have been determined, they are combined in a preference model to form a measure (such as expected utility) of the DM's overall preference for the specific alternative, as is illustrated in Figure 3. Keeney and Raiffa (1993) propose a method for developing such a preference model by eliciting preferences with respect to single attributes, expressing the preferences under uncertainty in utils, and then combining the utility functions of the multiple attributes into an overall utility function. Due to resource constraints, such a complete elicitation and precise characterization is unachievable in practice. Instead, the preference model is an imprecise abstraction based on limited preference elicitations. Other literature has examined incomplete or partial information [see (Weber 1987) for a review] in the context of imprecisely characterized preferences (Otto and Antonsson 1992; Carnahan, Thurston et al. 1994; Seidenfeld, Schervish et al. 1995) and unknown weights for tradeoffs between objectives in multi-attribute decision making (Kirkwood and Sarin 1985).

There is also evidence that people cannot express their preferences well in a rational fashion. When presented with choices between which a rational decision maker should be indifferent, even knowledgeable experts with a strong background in decision theory often judge the choices differently (Tversky and Kahneman 1974). This psychological evidence suggests that the environment and manner in which a choice is posed affects the elicited action, and thus choices are not a perfect indication of preference. It is also possible that preferences are non-stationary, meaning they vary over time. Even if they are reasonably stationary over a relevant time horizon, practical and psychological evidence strongly suggest that preferences can only be modeled imprecisely.

3.7. NUMERICAL CALCULATIONS ARE IMPRECISE

This source of imprecision is probably the most familiar, but possibly the least significant. As is probably known to most readers, the precision of calculations implemented on a digital computer are only precise up to the machine's numerical precision. In practice, modern computers have a very high precision, and this effect is generally not important, especially in comparison to the other sources of imprecision in engineering design. For example, consider the use of a model to calculate some parameter. It often does not matter whether the numerical solution of this model is within 10^{-10} or 10^{-15} of the model's "true" answer, because the model being used is already imprecise; moving to 10^{-15} accuracy just means that one would know the model's wrong answer better; it provides no further insight into the true answer for the real system.

Imprecision also can arise with the use of numerical methods, which are used to approximate analytical solutions when analytical methods are unavailable. Some of these methods are not guaranteed to converge on the exact solution for certain problems, and thus introduce considerable uncertainty that an analyst must explore. Other methods converge on the true solution, but this convergence is not exact in most algorithms; there is usually a tolerance set in them as a stopping criterion. For example, an iterative method may terminate when the solution changes by less than some small amount over several iterations. Consequently, the solution is known imprecisely. While these computational issues are of some interest, it is again believed that the imprecision they introduce generally is inconsequential compared to imprecision from other sources. In order to provide value in engineering design, research in reliable engineering computing must address these more substantial sources of imprecision.

4. Challenges of designing with imprecise information

The presence of imprecision in engineering design decisions obviously demands methods for making decisions and calculating with imprecise information. The goal of this paper is only to explain the context of engineering design and the sources of imprecision in design problems. A companion paper (Bruns, Paredis et al. 2006) in this workshop addresses the challenges of decision making and computing with imprecise information in detail, and a forthcoming conference paper details decision policies for eliminating alternatives in a set-based approach to design (Rekuc, Aughenbaugh et al. 2006). We conclude this paper with a brief overview of the decision-making problem and some references for computations with intervals and imprecise probabilities for completeness.

4.1. CHALLENGES IN DECISION MAKING

In general, there are three possible scenarios of preference between alternatives A and B. Either A is preferred to B, B is preferred to A, or the DM is indifferent between A and B. When utilities are used to reflect preference, these relationships can be determined by the inequality or equalities of the expected utilities (von Neumann and Morgenstern 1944). However, when imprecision exists, the expected utilities are not known precisely and become intervals, as shown in Figure 7. Consequently, comparisons between the alternatives become more complicated.

For example, consider the intervals of expected utility for two alternatives (A and B) shown in Figure 7(a). In this example, the intervals overlap. Since the true expected utility of B can lie anywhere in the given interval, the point labeled b_1 is possible. Similarly, both a_1 and a_2 are possible true values for the expected utility of A. Notice that a_1 is greater than b_1, but a_2 is less than b_1. Consequently, the available evidence is *indeterminate*; the DM cannot determine which alternative is the most preferred, nor can the DM determine that he or she is definitely indifferent.

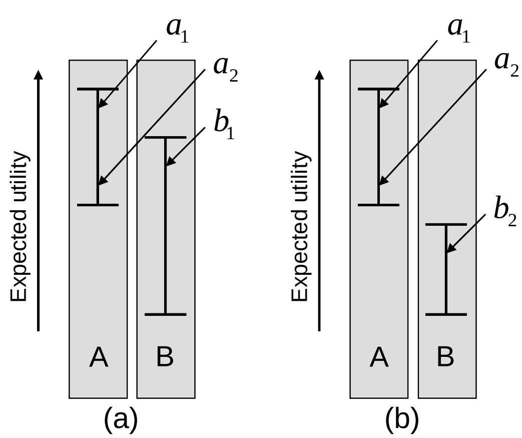

Figure 7. Intervals of expected utility

Given indeterminacy, a DM has two choices: he or she can collect more information in an effort to reduce the imprecision and remove the indeterminacy, or he or she can arbitrarily choose an alternative. *Arbitrary* means not uniquely determined by the DM's preferences, beliefs, and values (Walley 1991), but it does not necessarily imply without guidance or random. Several policies are possible to guide arbitrary choice, including Γ-maximin (Berger 1985) and the Hurwicz-criterion (Arrow and Hurwicz 1972). A Γ-maximin policy says that given indeterminacy in a maximization problem, a DM should select the alternative with the highest lower-bound. This is a conservative policy in that it seeks to mitigate the worst-case. Robust design strategies that choose solutions that are insensitive to imprecision are also applicable at this stage. If the remaining uncertainty is extreme, it may be valuable to consider an alternative approach such as information gap theory (Ben-Haim 2001).

If a DM elects to continue collecting information, he or she will still need to compare imprecise information, such as intervals. When delaying a decision to collect more information, a designer is effectively adopting a set-based approach to design, in which multiple decision alternatives are considered in parallel. In this process, inferior decision alternatives are eliminated from the set under consideration as soon as they are determined to be less preferable than any other alternative. The simplest case of a clear choice between alternatives for which the DM's preferences are characterized by intervals of expected utility is shown in Figure 7(b). In this case, it does not matter where in the given interval the true expected utility of A falls—it will always be greater than any value in the interval for expected utility of B. This illustrates a situation referred to as *interval dominance*, (For a brief synopsis, see Zaffalon, Wesnes et al. 2003).

While interval dominance is simple to understand and implement, it will rarely be sufficient for eliminating alternatives. Instead, a DM must turn to policies such as *maximality* (Walley 1991) or *E-admissibility* (Levi 1974). The use of these policies is explained and demonstrated in much more detail in the forthcoming conference publication (Rekuc, Aughenbaugh et al. 2006). A summary of the topics is given here.

Consider five decision alternatives whose utility is expressed as a function of a single shared imprecise parameter (such five car designs whose performances all depend on the ambient air temperature) in Figure 8. The intervals for all of these alternatives overlap except for E and D, and hence only D can be eliminated according to interval dominance. Eliminations will have to be made using other criteria.

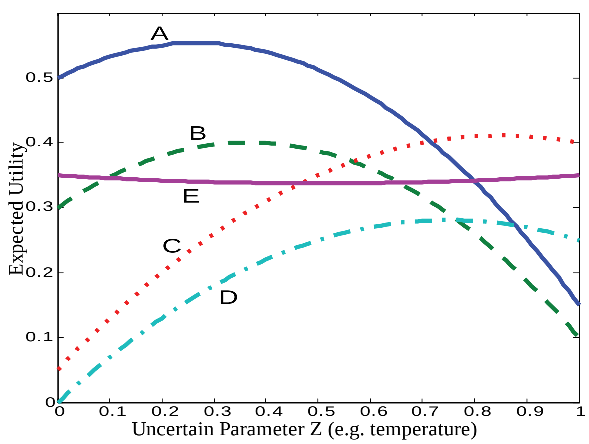

Figure 8. Performance of 5 alternatives influenced by a single uncertain parameter

From Figure 8, one can see that alternative A performs better than alternative B at all temperatures, so A is clearly better than B—an illustration of the criterion of maximality. Similarly, C is always better than D, so D could be eliminated even if it were not interval dominated by E. Consequently, neither alternatives B nor D can be the best decision, so the DM should no longer consider them.

Another way to look at Figure 8 is to ask which alternatives are ever the best, at any temperature. In this case, these are only alternatives A and C. This is an illustration of the E-admissibility criterion, which assumes that eventually all of the imprecision will be eliminated (the temperature will be known exactly), and then only the alternatives that are optimal at some temperature need to be considered. This may be true for some sources of imprecision (such as future decisions), but the DM should carefully consider the tradeoff between the value of obtaining more information and the cost of doing so by applying information economics (Aughenbaugh, Ling et al. 2005). Although the cost of additional investigation is often worth the improved ability to make a more informed decision, the DM will reach a point at which the cost of gathering additional information outweighs the expected benefits. Consequently, imprecision will rarely be eliminated, and the DM must resort to arbitrary choice.

In the case of arbitrary choice, it is desirable that robust alternatives be available. In Figure 8, alternative E is robust to temperature and would be a good arbitrary choice. However, alternative E is eliminated according to E-admissibility since it is never the optimal. Consequently, it appears that maximality is a better criterion than E-admissibility because maximality retains both the possible optimal solutions and the non-dominated robust solutions. In some cases, the imprecision can be reduced through additional analysis and design to the point that the optimal solution can be found, while in other cases a robust or otherwise arbitrary choice will need to be made. The key advantage of considering the alternatives in this manner is that the true optimal solution remains a candidate until late in the process, thus improving the chances of choosing it as the final design. The remaining question is *how can such intervals be calculated, propagated, and compared in a computationally efficient manner*?

4.2. COMPUTATIONAL CHALLENGES

For engineering applications, it is crucial to adopt a mathematical formalism that is convenient and inexpensive for computation and decision making. Various methods have been developed for propagating intervals (Moore 1979; Alefeld and Herzberger 1983; Kearfott and Kreinovich 1996) and imprecise distributions through known algebraic relationships (Springer 1979; Williamson and Downs 1990; Ferson and Ginzburg 1996; Berleant and Zhang 2004; Ferson and Hajagos 2004). However, many engineering models are black boxes—unknown or very complex relationships modeled by simulations or other means—for which algebraic relationships are unavailable. The only existing methods for these types of models are based on brute-force, multi-loop methods that include at least one Monte Carlo sampling loop. These methods are impractical for engineering design because they are prohibitively expensive in terms of computations. Clearly, the methods that have been found effective for algebraic models must be extended, adapted, or replaced for computations in more general engineering design problems. This problem is formulated in substantially more mathematical detail in the companion paper (Bruns, Paredis et al. 2006).

5. Summary

There are many sources of imprecision in engineering design. The sequential nature of design decisions inherently leads to sets and intervals; probabilities and preferences are not known precisely, and models are imprecise approximations of reality. The presence of imprecision can lead to indeterminacy in decisions when traditional statistical decision theory is applied. Consequently, engineering researchers need to explore and develop new decision theories. The use of intervals and imprecise probabilities to capture a decision maker's state of knowledge also leads to new computational challenges. The potential benefit of using such formalisms is clear, but the feasibility of implementing them efficiently in complex design problems has not been proven. The development and application of efficient algorithms for computations with imprecise structures would help advance the state of engineering design significantly. In order to develop such methods, strong collaboration between the engineering design community and the reliable engineering computing community is needed. As a starting point for such collaboration, this paper has outlined the sources and role of imprecision in engineering design.

6. Acknowledgments

Jason Aughenbaugh is supported under a National Science Foundation Graduate Research Fellowship. Any opinions, findings, conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation. This work was supported in part by NSF grant DMI-0522116.

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