Online Implementation of a Robust Controller using Hybrid Global Optimization Techniques

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Abstract. In this work, we report the experimental implementation of a Quantitative Feedback Theory (QFT) based robust controller, designed *online* using hybrid global optimization and constraint propagation techniques. The hybrid global optimization combines interval global optimization and nonlinear local optimization methods. The constraint propagation techniques are very effective in discarding infeasible controller parameter regions in the optimization search. The obtained experimental results show the effectiveness of hybrid global optimization for the *online* design of robust control systems.

Keywords: global optimization, interval analysis, robust control

1. Introduction

Most of the practical system consists of uncertainties in the form of disturbances, measurement noise and unmodelled or imprecisely modeled dynamics. Therefore the design has to seek a control system that functions adequately over a wide range of uncertain parameters. Such a system is said to be *robust* when, it has low sensitivities, is stable over a wide range of parameter variations, and the performance stays within prescribed limit bounds in the presence of parameter variations. Sometimes the parameter variations are beyond the uncertainty bounds, then there is need of retuning (*adaptation*) of controller parameters online.

Quantitative Feedback Theory (QFT), developed by Horowitz (1993) is a frequency domain based technique for robust controller design. It converts the design specifications of a closed loop system and plant uncertainties into robust stability bounds and performance bounds on the open loop transmission of the nominal system and then synthesize a controller by using the gain-phase loop shaping technique. Traditionally, this synthesis was done *manually* by the designer, relying on design experience and skill. Recently, several researchers have attempted to *automate* this step, see, for instance, (Ballance and Gawthrop, 1991; Bryant and Halikias, 1995; Chait et al., 1999; Gera and Horowitz, 1980; Thompson and Nwokah, 1994)

The main drawback of the approaches cited above lies in attempting to solve a complicated nonlinear optimization problem using convex or linear programming techniques, which generally leads to conservative designs. To overcome these difficulties, Chen *et al.* (1998) reformulated the problem as one of parameter optimization of a fixed order controller and used genetic algorithms for obtaining the solutions. However, it is well known that with genetic algorithms one may obtain a *local* minimum instead of the *global* minimum (Dallwig et al., 1997). Moreover, genetic algorithms tend to become slower as one tries to increase the probability of success.

PSfrag replacements
$R(s)$
$F(s)$
$G(s)$
$P(s)$
$Y(s)$
$+$
$-$

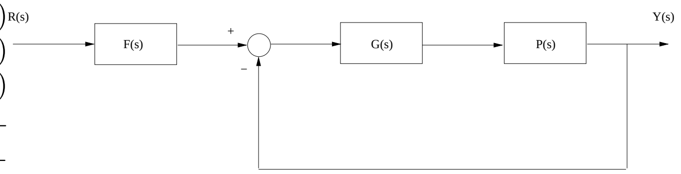


Figure 1. Two Degree of Freedom Structure for QFT.

In this paper, we used an efficient method for automatic loop shaping in QFT, proposed in (Nataraj and Kubal, 2005). The QFT controller synthesis problem is posed as a constrained optimization problem, where the objective function is the high frequency gain of the controller, and the constraint set for the optimization is the set of possibly nonconvex, nonlinear magnitude-phase QFT bounds at the various design frequencies. The method uses hybrid optimization techniques and constraint propagation ideas to solve the optimization problem. The hybrid optimization part efficiently combines interval global optimization (Moore, 1979; Ratschek and Rokne, 1988; Hansen, 1992; Kearfott, 1996) and nonlinear local optimization methods. The method supplement the optimization tools with a new so-called *quick solution* approach, developed based on ideas of constraint propagation techniques. The quick solution approach can quickly discard sizable portions of the infeasible controller parameter regions using simple arithmetic calculations.

In the present work, automatic loop shaping using *hybrid global optimization and constraint propagation* is used for the experimental implementation of QFT based robust adaptive controller on a coupled tank system.

The paper is organized as follows: Section 2 deals with the background of QFT. Problem formulation is given in Section 3. Section 4 give details of hybrid global optimization and constraint propagation. Case study of coupled-tank system is described in Section 5.

2. Overview of QFT

Consider a two degree freedom feedback system configuration (see Fig 1), where $G(s)$ and $F(s)$ are the controller and prefilter respectively. The uncertain plant $P(s)$ is given by $P(s) \in \{P(s,\lambda) : \lambda \in \boldsymbol{\lambda}\}$, where $\lambda \in \mathbb{R}^l$ is a vector of plant parameters whose values vary over a parameter box $\boldsymbol{\lambda}$

$$\boldsymbol{\lambda} = \{\lambda \in \mathbb{R}^l : \lambda_i \in [\underline{\lambda_i}, \overline{\lambda_i}],\ \underline{\lambda_i} \le \overline{\lambda_i},\ i = 1, ..., l\}$$

This gives rise to a parametric plant family or set

$$\mathcal{P} = \{P(s,\lambda) : \lambda \in \boldsymbol{\lambda}\}$$

The open loop transmission function is defined as

$$L(s,\lambda) = G(s)P(s,\lambda) \tag{1}$$

and the nominal open loop transmission function is

$$L_0(s) = G(s)P(s,\lambda_0) \tag{2}$$

The objective in QFT is to synthesize $G(s)$ and $F(s)$ such that the various stability and performance specifications are met for all $P(s) \in \mathcal{P}$. In general following specifications are considered in QFT:

1. Robust stability margin
$$\left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq \omega_s$$
2. Robust tracking performance
$$|T_L(j\omega)| \leq \left|\frac{F(j\omega)L(j\omega)}{1+L(j\omega)}\right| \leq |T_U(j\omega)|$$
3. Robust input disturbance rejection performance
$$\left|\frac{G(j\omega)}{1+L(j\omega)}\right| \leq \omega_{d_i}(w)$$
4. Robust output disturbance rejection performance
$$\left|\frac{1}{1+L(j\omega)}\right| \leq \omega_{d_o}(w)$$

In practice, the objective is to satisfy the given specifications over a finite design frequency set Ω. The main steps of QFT design specifications are

1. **Generating templates**: For a given uncertain plant $P(s) \in \mathcal{P}$, at each design frequency $\omega_i \in \Omega$, calculate the value set of the plant $P(j\omega_i)$ in the complex plane.
2. **Computation of QFT bounds**: At each design frequency ω_i, combines the stability and performance specifications with the plant templates which results in the stability margin and performance bounds. The bound at ω_i is denoted as $B_i(\angle L_0(j\omega), \omega_i)$ or simply B_i
3. **Design of Controller** : Design a controller $G(s)$ such that
   - The bound constraints at each design frequency ω_i are satisfied.
   - The nominal closed loop system is stable.
4. **Design of Prefilter**: Design a prefilter $P(s)$ such that the robust tracking specifications are satisfied.

3. Problem Formulation

We consider the controller structure in the gain-pole-zero form as

$$G(s,x) = \frac{k_G \prod_{i_1=1}^{n_z} (s+z_{i_1})}{\prod_{k_1=1}^{n_p} (s+p_{k_1})} \tag{3}$$

where

$$x = (k_G, z_1, ...z_{n_z}, p_1, ..., p_{n_p}) \tag{4}$$

is the controller parameter vector. The magnitude and phase functions of $G(s,x)$ are defined as

$$G_{mag}(\omega, x) = |G(s,x)|;\ G_{ang}(\omega, x) = \angle G(s,x) \tag{5}$$

Now, the QFT controller synthesis problem can be formulated as: Given the QFT bounds and the nominal plant, develop a controller automatically which provides nominal closed loop stability, satisfies all the bound constraints, with minimum high frequency controller gain k_G. Minimization of the high frequency gain of the controller tends to reduce the amplification of the sensor noise in the high frequency range, as shown in (Horowitz, 1993).

The QFT synthesis problem can be posed as a constrained optimization problem

$$\begin{aligned} \min_{x \in \mathbf{x}} f \ &= \ k_G \\ \text{subject to } H(x) \ &\leq \ 0 \end{aligned} \tag{6}$$

- x is the vector of controller parameters, $\mathbf{x}$ is some suitably specified initial search box of controller parameter values.

- $H(x) = \{h_i(x)\}$ is set of bound constraints at each design frequency ω_i

  $$\text{single valued upper bound constraint} : h_i^u(x) = |L_0(j\omega_i, x)| - B_i(\angle L_0(j\omega_i, x), \omega_i) \leq 0 \tag{7}$$

  $$\text{single valued lower bound constraint} : h_i^l(x) = B_i(\angle L_0(j\omega_i, x), \omega_i) - |L_0(j\omega_i, x)| \leq 0 \tag{8}$$

  A multiple valued bound constraint denoted as h_i^{ul} can be split into a single-valued upper bound constraint h_i^u and a single-valued lower bound constraint h_i^l, and then the condition of both the bounds consider together.

- The bound constraint on the controller parameter vector, i.e. the controller parameter values should lie in the initial search region.

- The nominal closed loop stability test is based on finding out the zeros of $1 + L(s, z_0, \lambda_0)$ for some $z_0 \in \mathbf{z} \subseteq \mathbf{x}$

4. Hybrid Global Optimization

Let $\mathbf{z} = (\mathbf{k}_G, \mathbf{z}_1, ..., \mathbf{z}_{n_z}, \mathbf{p}_1, ..., \mathbf{p}_{n_p})$ be the controller parameter box. Let $G_{mag}(\omega_i, \mathbf{z})$ and $G_{phase}(\omega_i, \mathbf{z})$ denote the natural interval extensions of controller magnitude and phase functions respectively. The natural interval extensions of nominal open loop transfer function magnitude and phase are defined as

$$L_{0mag}(\omega_i, \mathbf{z}) = |L_0(j\omega_i, \mathbf{z})| = G_{mag}(\omega_i, \mathbf{z})\ |P(\omega_i, \lambda_0)| \tag{9}$$

$$L_{0phase}(\omega_i, \mathbf{z}) = \angle L_0(j\omega_i, \mathbf{z}) = \ G_{phase}(\omega_i, \mathbf{z}) + \ \angle P(\omega_i, \lambda_0) \tag{10}$$

The evaluations of the natural interval extensions at a given frequency ω_i give magnitude and phase intervals that define a box-like region in the Nichols chart. This is called as the L_0 box at ω_i.

The algorithm proposed in (Nataraj and Kubal, 2005) mainly consists of seven major components: a quick solution approach, feasibility test, local optimization call, initialization, list sorting and handling, a bisection strategy and a termination criteria.

1. **Feasibility test**: Based on the location of the L_0 box w.r.t. bounds B_i, the parameter box $\mathbf{z}$ is determined as feasible, infeasible or indeterminate at ω_i, see Fig. 2. The $flag_z$ represents the feasibility of parameter box $\mathbf{z}$ The details for the feasibility test are given in sec. 4.1.

2. **Quick Solution approach**: The quick solution approach discards the portion of the controller parameter box $\mathbf{z}$ based on the location of the L_0 box w.r.t. the bounds (for details see sec. 4.3).

3. **Initialization**: The current processing box $\mathbf{z}$ is assigned to the initial search box. The quick solution and feasibility test is done for $\mathbf{z}$. If $\mathbf{z}$ is infeasible, then by the inclusion property of interval analysis, there is no feasible solution $\forall \tilde{z} \in \mathbf{z}$, hence, the algorithm exits and print the message 'No solution exist in the given initial search box'. Else, a list L is initialized with triple ($\mathbf{z}$, z, $flag_z$), where $z = \inf \mathbf{z}(1)$ is the minimum value of the high frequency gain based on the current parameter box $\mathbf{z}$.

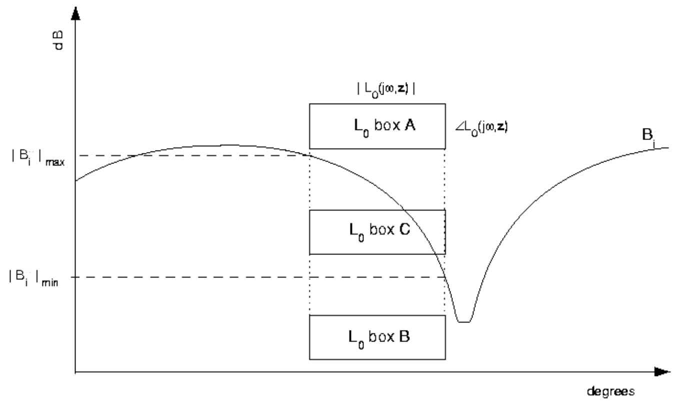
4. **Local optimization call**: A constrained local optimization routine is called to solve the constrained optimization problem (6). For details see sec. 4.2.

5. **Bisection**: At each iteration, the box $\mathbf{z}$ of leading triple is bisected into two subboxes $\mathbf{v}^1$ and $\mathbf{v}^2$

6. **List sorting and handling**: At each iteration, the leading triple is deleted from the list L and the indeterminate bisected triples are added into the list. The list is sorted and arranged in the non decreasing order of the value of objective function.

7. **Termination**:

   a) As the list is sorted and arranged in the non decreasing order of the value of objective function z at each iteration, the leading triple always contains the minimum value of the objective function. Hence, at any iteration, if the box $\mathbf{z}$ of the leading triple is feasible, then the algorithm can be terminated by printing the optimal controller parameter box $\mathbf{z}$



PSfrag replacements

$|B_i|_{max}$

Figure 2. Feasibility conditions for different locations of L_0 box w.r.t. single valued lower bound at ω_i. Box A shows feasible case, box B shows infeasible case and box C shows the indeterminate case.

b) If the relative gain width of the box $\mathbf{z}$ of leading triple is less than a specified relative gain tolerance, and the box $\mathbf{z}$ contains the feasible parameter vector (i.e. feasible local solution) then the algorithm can be terminated by printing the optimal parameter vector z_{local}

4.1. Feasibility check

The feasibility check for a controller parameter box $\mathbf{z}$ consists of checks for the bound constraints satisfaction in (7) and (8) at a given ω_i, $i = 1, ..., n$.

4.1.1. *Feasibility check for bound satisfaction*

Let $|B_i|_{max}$ and $|B_i|_{min}$ be the top most and bottom most value of the single valued lower bound for the entire phase interval $\angle L_0(j\omega_i, \mathbf{z})$. Based on the location of the L_0 box w.r.t. the single valued lower bound one of the following cases arises (see Fig.2)

1. If the entire L_0 box lies on or above $|B_i|_{max}$ (box A in Fig. 2) then h_i^l is satisfied for any controller parameter vector $z \in \mathbf{z}$, so that the entire box $\mathbf{z}$ is feasible at ω_i.
2. If the entire L_0 box lies below $|B_i|_{min}$ (box B in Fig. 2) then h_i^l is not satisfied for any controller parameter vector $z \in \mathbf{z}$, so that the entire box $\mathbf{z}$ is infeasible at ω_i.
3. Else box $\mathbf{z}$ is indeterminate (box C in Fig. 2).

4.2. Local optimization

Local optimization gives an early knowledge of the approximate global minimum. However, the main difficulty is to decide of when to call a local optimization algorithm in a hybrid algorithm. If local optimization is called at each algorithmic iteration, then the computational costs will grow dramatically. Hence, the following decision rule is made regarding when to call the local optimization routine.

- Let z, be any parameter vector belong to the parameter box $\mathbf{z}$.
- z is compared with all previous starting points of local optimization, say z_v,
- If z is sufficiently different (say, for instance more than 10%) from all previous starting points z_v, then call the local optimization routine for z.

4.3. Quick Solution

We can easily show from (2), (3) that the magnitude and phase of L_0 vary *monotonically* over the gain, zero and pole intervals. Further, from Fig. 3, we also observe that the coordinate $(\inf\angle L_0, \sup|L_0|)$ is contributed by supremum values of gain and zero intervals and infimum values of pole intervals, while the coordinate $(\sup\angle L_0, \inf|L_0|)$ is contributed by infimum values of gain and zero intervals and supremum values of pole intervals

The proposed *quick solution* approach uses these simple observations and a few arithmetic calculations for discarding infeasible parts of gain, pole and zero intervals. In general, optimization techniques alone would take perhaps many iterations to achieve the same.

5. Case study

5.1. Plant Description

The coupled tank system whose schematic is given in Fig. 4 consists of two hold-up tanks which are coupled by an orifice. Water is pumped in to the first tank by variable speed pump. The orifice allows this water to flow into the second tank and hence out to a reservoir. The aim is to control the water level in the second tank by changing the flow rate to the first tank by varying the speed of the pump. The speed of the pump is varied by varying the control voltage (0-10V) to the pump. The liquid level in the tank is measured using a depth sensor whose output is voltage (0-10V), which is proportional to the level.

The input to the plant is the voltage to the variable speed pump and the output is the water level in the second tank in terms of voltage signal.

The control voltage to the pump motor drive is from a digital computer along with the Advantech 5000 series data acquisition system. The mentioned data acquisition comprises 8-channel analog input module and 4-channel Analog output module. The analog input channel accepts the signal of $0-10$ volts. The analog output channels can generate an output of $0-10$ volts. Communication

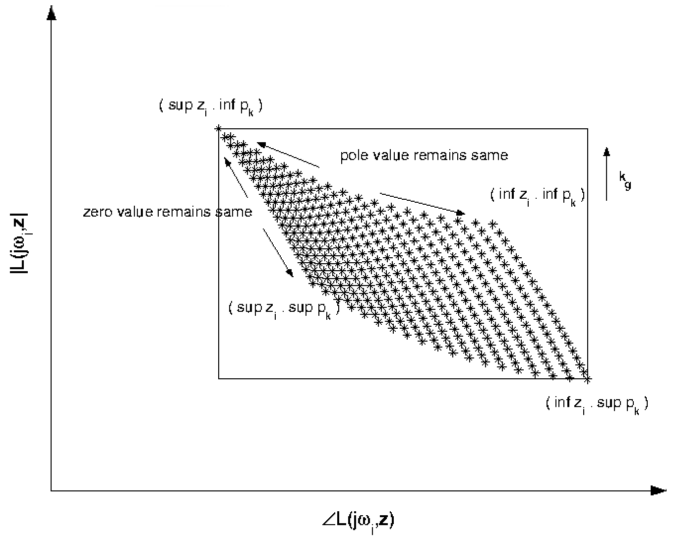


Figure 3. Variation of $|L_0(j\omega_i, \mathbf{z})|$, $\angle L_0(j\omega_i, \mathbf{z})$ w.r.t. gain, zero and pole intervals. The outer rectangle shows the L_0 box.

PSfrag replacements
Tank1
Tank2
Q_i
Q_o
Q_1
h_1
h_2
Variable Speed
Pump

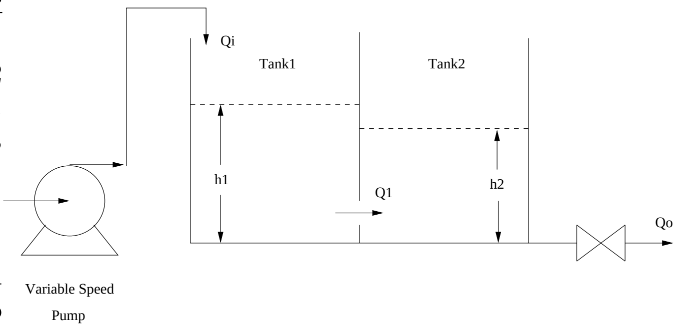


Figure 4. Schematic of Coupled Tank System.

between data acquisition system and the digital computer is via serial port. The control design algorithm is implemented on a PC in Microsoft FORTRAN 95 with interval arithmetic support INTLIB (Kearfott et al., 1994).

5.2. Real-time Parameter Estimation

On-line determination of process parameters is a key element in adaptive control system. In the present work recursive least square method is used for parameter estimation. In recursive identification method, the parameter estimates are computed recursively in time. This method has a small requirement on memory since only a modest amount of information is stored. This amount will not increase with time.

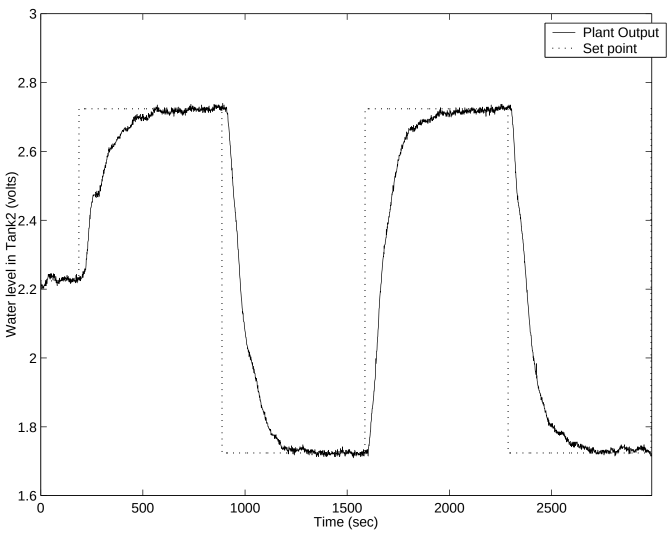


Figure 5. Experimental closed-loop responses to setpoint changes for the coupled-tank system.

5.3. Controller Design and Implementation

For the design of robust QFT controller, the closed-loop specifications include the robust QFT stability margins and tracking performance which are specified by

$$\left|\frac{L(j\omega)}{1+L(j\omega)}\right| \leq 3dB$$

and

$$|T_L(\omega)| \leq \left|\frac{F(j\omega)L(j\omega)}{1+L(j\omega)}\right| \leq |T_U(\omega)|$$

respectively, where

$$T_U(s) = \frac{16.67s+1}{2140s^2+56.44s+1}$$

and

$$T_L(s) = \frac{1}{4.495\times 10^4 s^3+4740s^2+139.2s+1}$$

A second order model structure is selected for the coupled-tank plant, whose parameters are estimated *online* using recursive least square method. The method mentioned in sec. 4 is used to design the controller *online*. The implementation results are shown in the Figs. 5 and 6.

It can be noticed in Fig. 6 that the obtained closed-loop responses satisfy the given time-domain specifications.

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PSfrag replacements
Tu
Tl
Plant output
Time (sec)
Water level in Tank2 (volts)

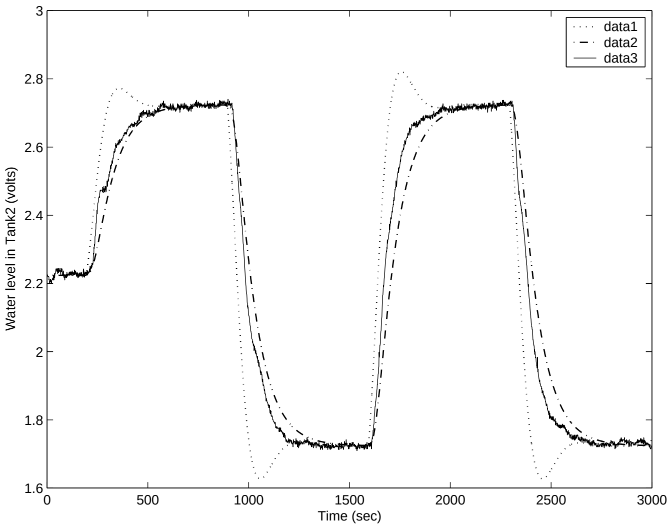

Figure 6. Experimental responses along with the given time-domain specifications.

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