



MODELING UNCERTAINTIES IN SEISMIC VULNERABILITY AND RISK ASSESSMENT

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Seismic vulnerability and risk assessment

Need for improved practices

- New building systems
- Demands for performance beyond building code minimums
- Perception of increasing building risk
- Public awareness of building performance and demands for safety



Performance-based earthquake engineering

Concept

An engineering approach that is based on

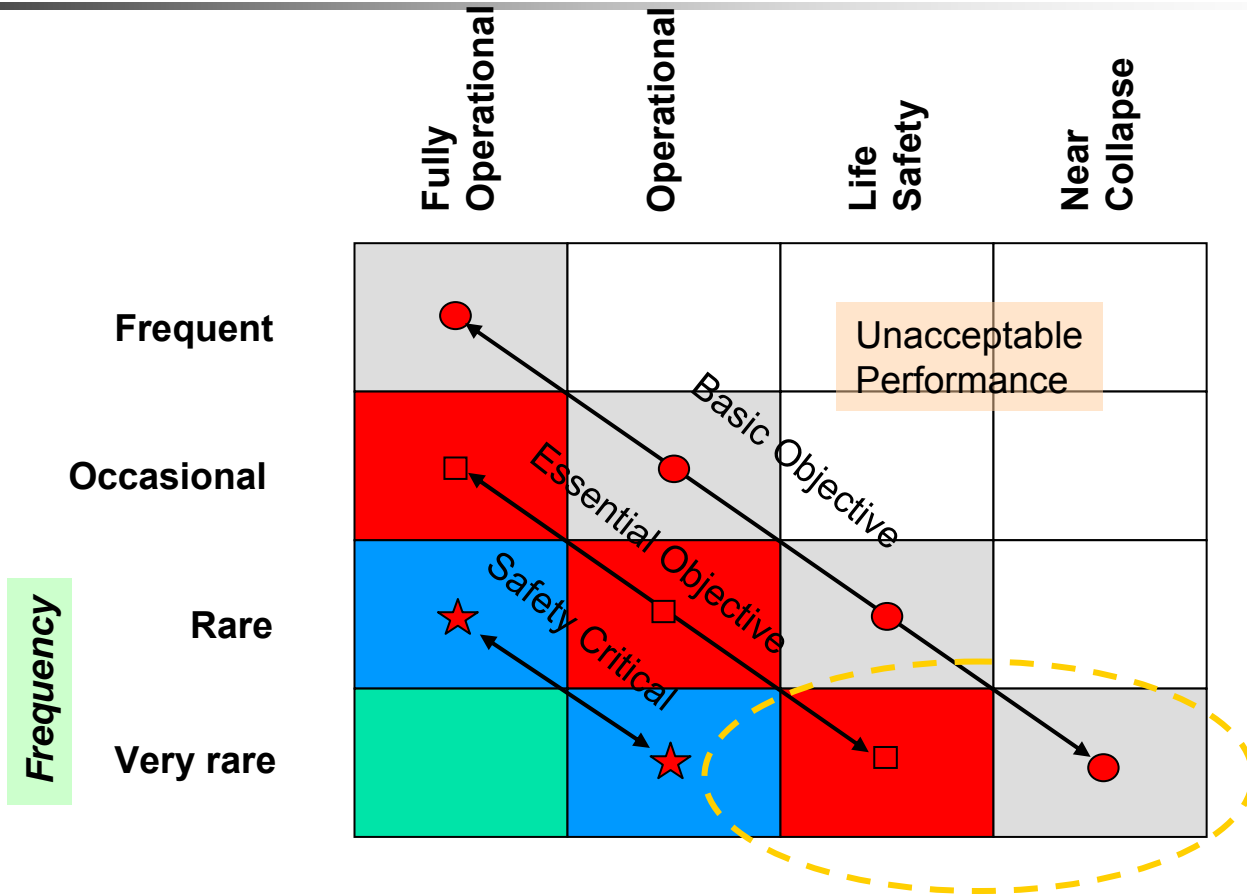
- Specific performance objectives and safety goals
- Probabilistic or deterministic evaluation of hazards
- Quantitative evaluation of design alternatives against performance objectives

but does not prescribe a specific technical solution



PERFORMANCE OBJECTIVE

SEAOC Vision 2000



Consequence-based Engineering

Guiding principle for MAE Center research

A new paradigm for seismic risk assessment and reduction across interconnected systems or regions, enabling effects of uncertainties and benefits of alternate seismic risk management strategies to be assessed in terms of their impact on the population and built environment





MAE Center goals in uncertainty modeling

- To develop a framework for systematic treatment of uncertainty in all aspects of damage synthesis modeling
- To incorporate uncertainties in all steps of seismic risk assessment and the CBE paradigm
- To guide the Center program of research to invest where the return is highest in terms of quantifying and minimizing uncertainty





Overview of presentation

- Risk and uncertainty
- Aleatoric, epistemic uncertainties
- Risk assessment framework
- Illustrations of uncertainty analysis in CBE
 - Building fragility/limit state probabilities
 - Building damage and loss estimation
- Risk-informed decision-making for facility design and evaluation
- Research challenges





Ingredients of risk

- Probability of occurrence
 - Hazard
 - System response, damage states
- Consequences
 - Deaths
 - Dollars
 - Downtime
- Context – who is the decision-maker?





Uncertainty and risk in CBE

- Sources of uncertainty
 - Seismic hazard
 - Ground motions – synthetic or natural accelerograms
 - Construction practices and in material and system properties for steel, concrete, masonry, timber construction -
 - Structural and non-structural component modelling
 - Quantitative definition of performance levels and limit states
 - Damage and loss estimation for individual facilities
 - Damage and loss aggregation for inventories
 - Social impact and social vulnerability
- Uncertainty leads to **risk**
- Risk can be managed by targeting investments to achieve maximum benefits in risk reduction, but risk cannot be eliminated.





Classification of uncertainty

- **Inherent randomness (aleatoric)**
Uncertainty that is explicitly recognized by a stochastic model
- **Knowledge-based (epistemic)**
Uncertainty in the model itself and in its descriptive parameters





Aleatoric (inherent) uncertainty

Experiment: Tensile test of A992 steel

$$F_{y,nom} = 50 \text{ ksi (345 MPa)}$$

Outcome: We cannot predict the result of any test with certainty. However, if we repeat the test a large number of times, we'll find that the mean and coefficient of variation are about 56 ksi and 0.06 (**statistical regularity**).

These values change little at the customary scales of modeling.



Epistemic (knowledge-based) uncertainty

- 2D representations of 3D structures
- Earthquake capability of a fault
 - “The fault is capable of generating an earthquake with $M > 7.5$.” The statement either is true or is not, and a probability must be assigned to the truth of the statement.
- Geologic profile at a site
- Selection of earthquake ground motion model
 - Atkinson/Boore and Frankel models for Memphis, TN are both plausible, but lead to different ground motions.

We can reduce this uncertainty by investing in knowledge creation



Framework for risk assessment

Loss metric: $P[\text{Loss} > 25\% \text{ replacement cost}]$, $E[\text{Loss/yr}]$, etc.

LS: structural limit state (e.g., $\theta > \theta_{\text{limit}}$)

DS: damage state (relates structural response to loss metric)

$$P[\text{Loss}=c] = \sum_s \sum_{\text{LS}} \sum_d P[\text{Loss}=c | \text{DS}=d] P[\text{DS}=d | \text{LS}] P[\text{LS} | S_a = x] P[S_a = x]$$

or

$$P[\text{Loss}=c | \text{Scenario}] = \sum_{\text{LS}} \sum_d P[\text{Loss}=c | \text{DS}=d] P[\text{DS}=d | \text{LS}] P[\text{LS} | \text{Scenario}]$$

$P[S_a = x]$ = Seismic intensity

$P[\text{LS} | S_a = x]$ or $P[\text{LS} | \text{Scenario}]$ = system response, capacity

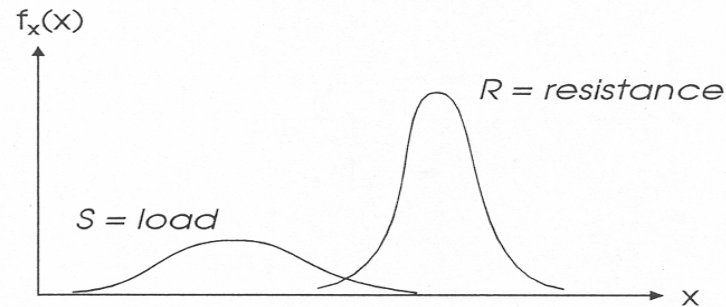
$P[\text{DS} | \text{LS}]$ = Damage state probability

$P[\text{Loss}=c | \text{DS}]$ = Conditional probability of loss



Measuring risk using reliability theory

R , S are random variables describing capacity and demand



Limit state: $Z = R - S < 0$

$$P[LS] = P_F = P [R - S < 0]$$

If R and S are lognormal,

$$P_F = \Phi [- \ln (m_R/m_S) / \sqrt{(V_R^2 + V_S^2)}] = \Phi [- \beta]$$

m_R, m_S = medians; V_R, V_S = logarithmic standard deviations



Role of fragility

The limit state (LS) probability can be decomposed:

$$P[\text{LS}] = \sum_x P[\text{LS} | S_a = x] P[S_a = x]$$

$P[S_a = x]$ = probability that demand S_a equals a specific level x

$P[\text{LS} | S_a = x]$ = conditional limit state probability (**fragility**)

LS = $\{\theta > \theta_{\text{limit}}\}$, $\{\ddot{a} > \ddot{a}_{\text{limit}}\}$, etc.



Lognormal model of fragility

Fragility often is modeled by a lognormal CDF

$$\text{Fragility} = F_R(x) = \Phi\left[\frac{\ln(x/m_R)}{\beta_R}\right]$$

m_R is median (50th percentile) capacity

β_R is standard deviation of $\ln(R)$

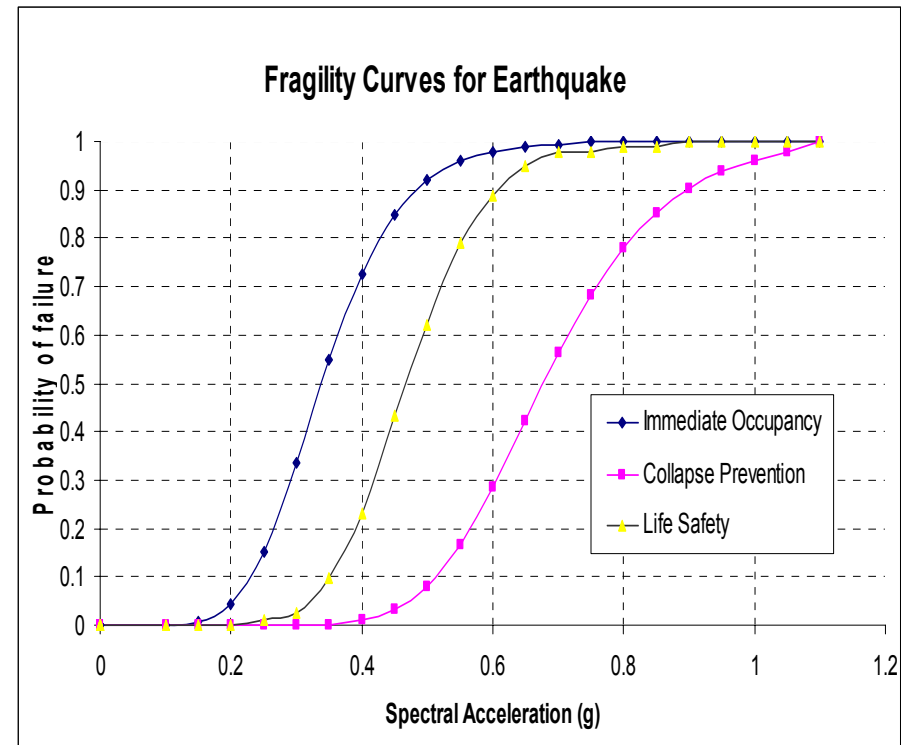


Illustration of fragility analysis

Seismic fragility of steel frame

Basic elements:

- Seismic hazard
- Seismic demand on system
- Structural system response
- Limit states

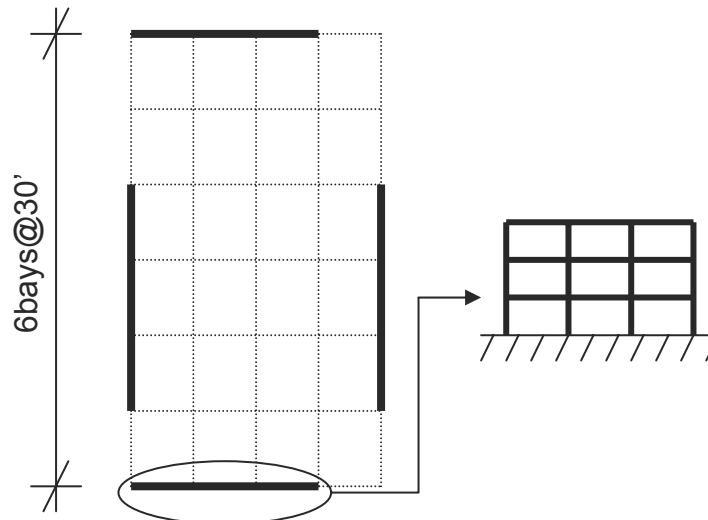
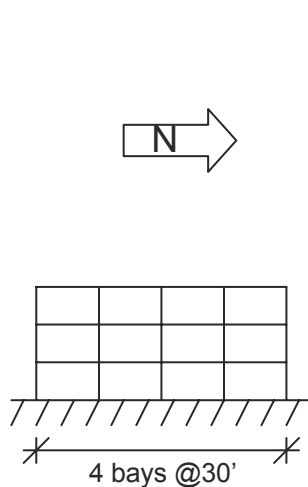
$$P[\text{Loss}=c] = \sum_s \sum_{LS} \sum_d P[\text{Loss}=c | DS=d] P[DS=d | LS] P[LS | S_a = x] P[S_a = x]$$



Three-story steel moment frame in CEUS

NS Moment Resisting Frame

Story	COLUMNS		GIRDERS
	Exterior	Interior	
1	W14X74	W14X99	W21X62
2	W14X74	W14X99	W21X57
3	W14X74	W14X99	W18X35



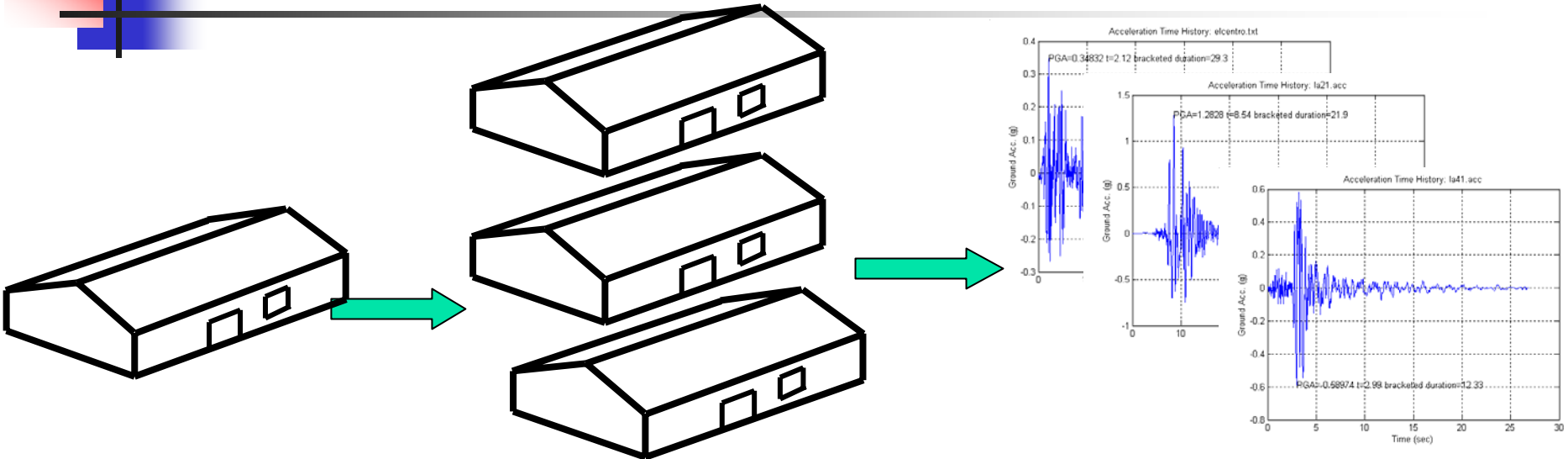
MRF

$$T_1 = 1.89s$$

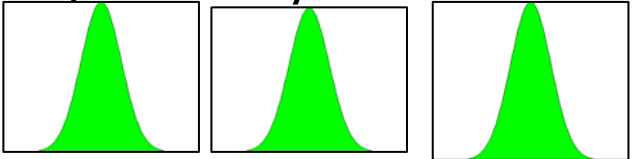
$$\text{Damping} = 2\%$$



Development of structural fragilities

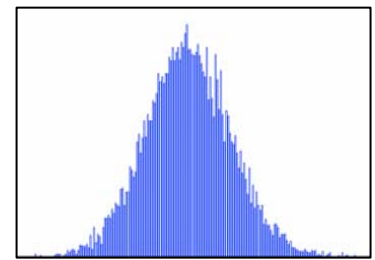


- Random Variables
 - mean value
 - standard deviation
 - probability distribution



- Series of models

- Suite of ground motions

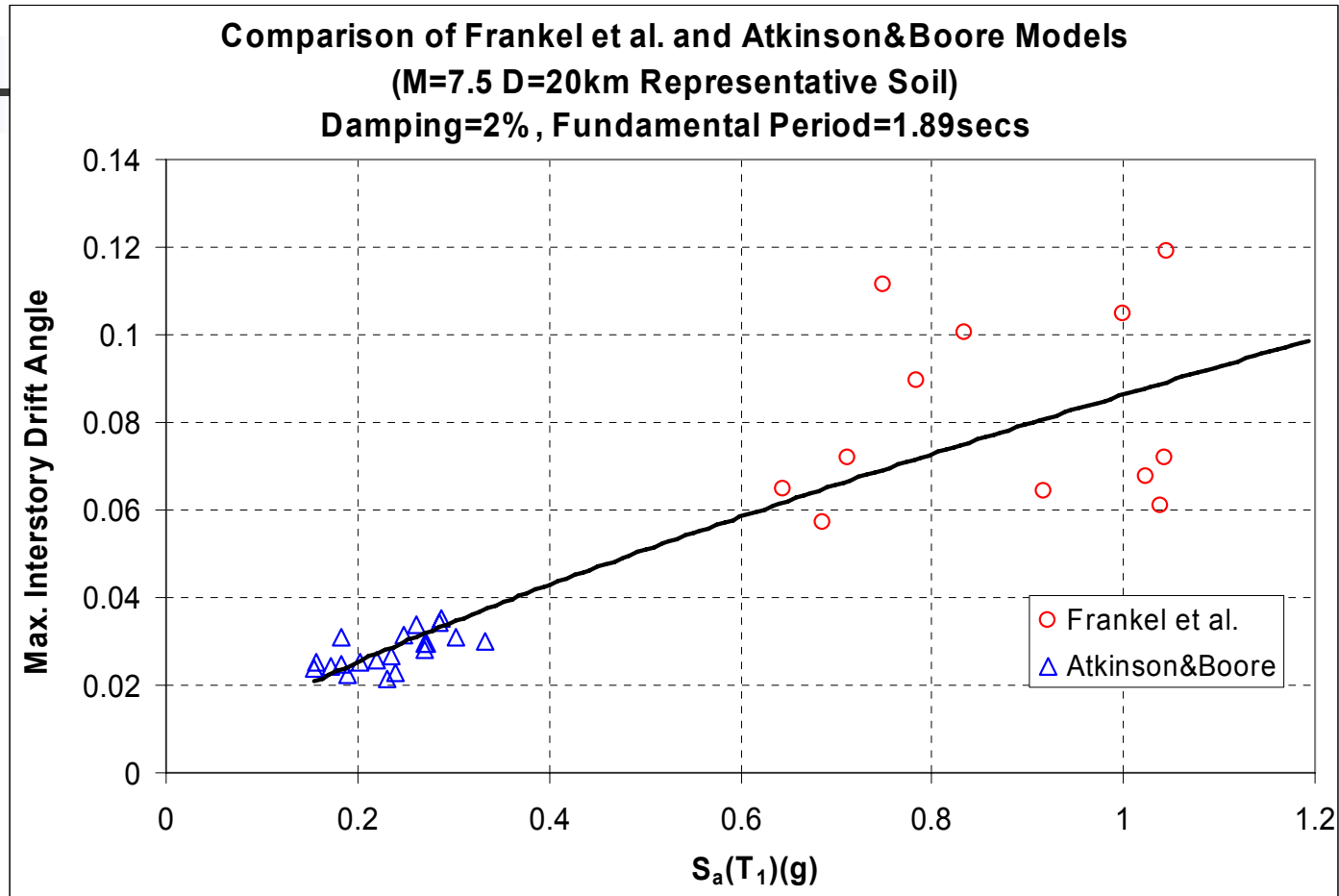


-
- Structural Response



Seismic demand on three-story steel frame in CEUS

Aleatoric vs epistemic uncertainty



Seismic demand model

- Demand: S_a
- Structural response: $\theta = a S_a^b \varepsilon_1$

$$m_\theta = a S_a^b \text{ (median)}$$

ε_1 models scatter about the median

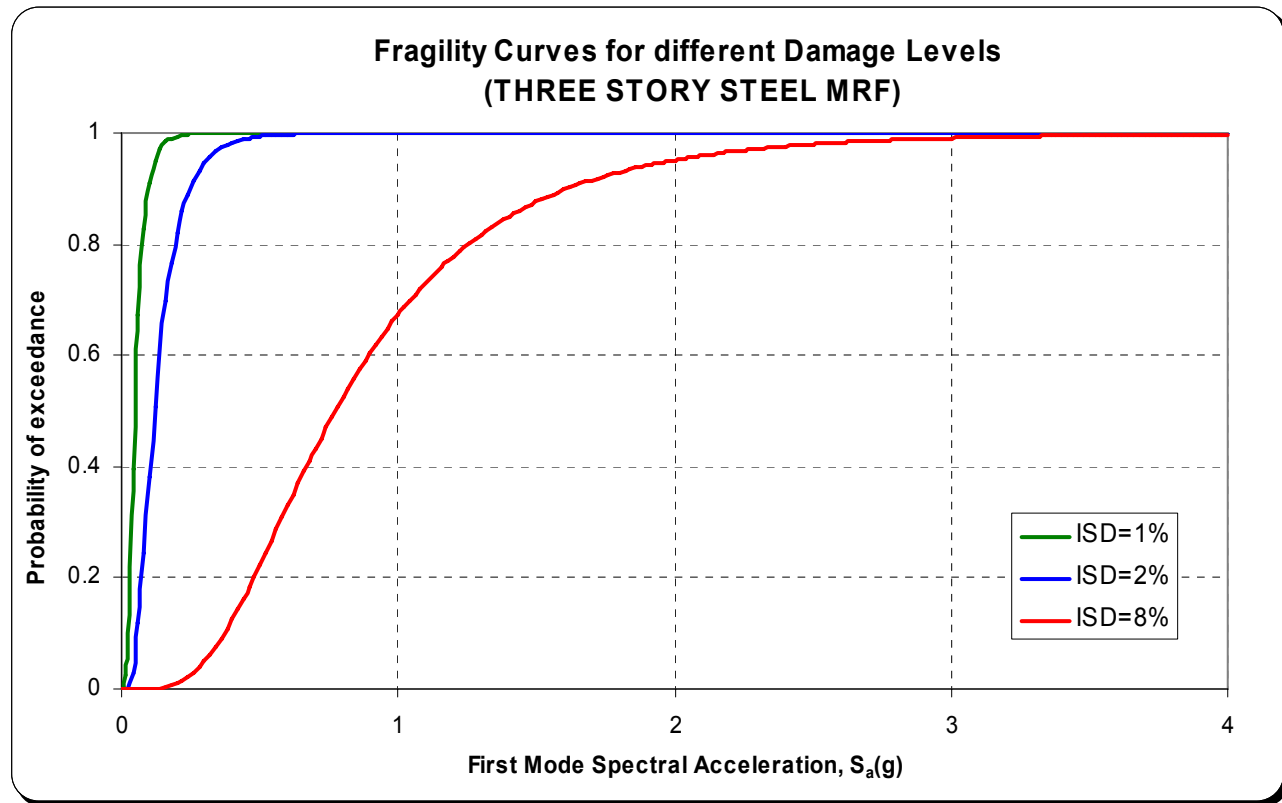
$$SD[\ln \theta | S_a] \approx 0.30$$

Lognormal model for θ (aleatoric)

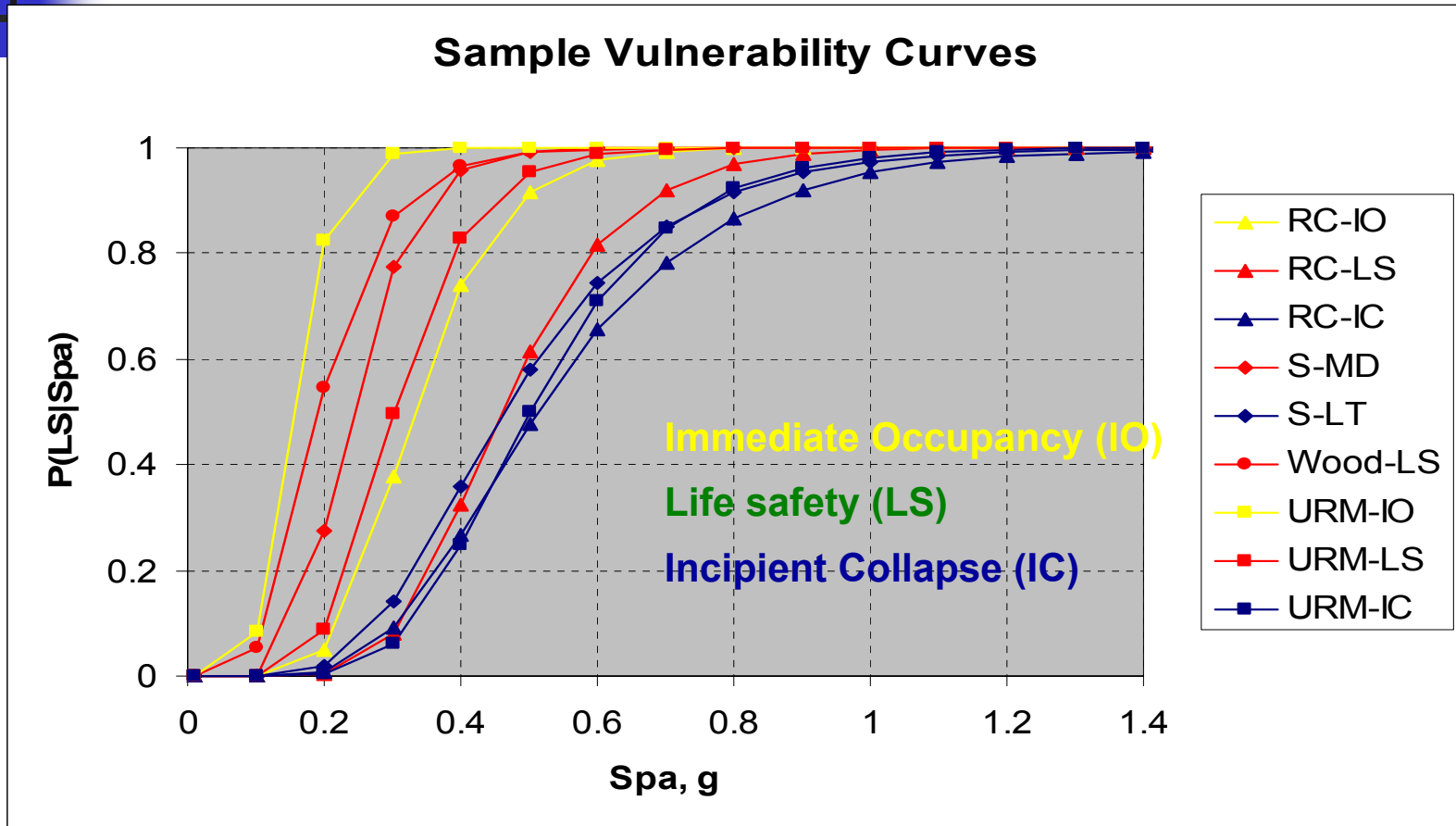
- $P[\text{LS} | S_a = x] = P[\theta > \theta_{\text{limit}} | S_a = x]$



Seismic fragilities based on interstory drift



Fragilities of buildings in the CEUS





Computational challenges

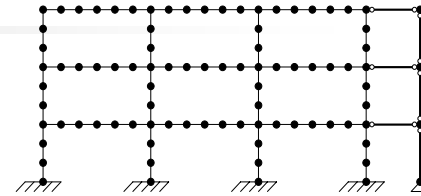
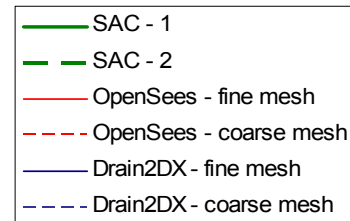
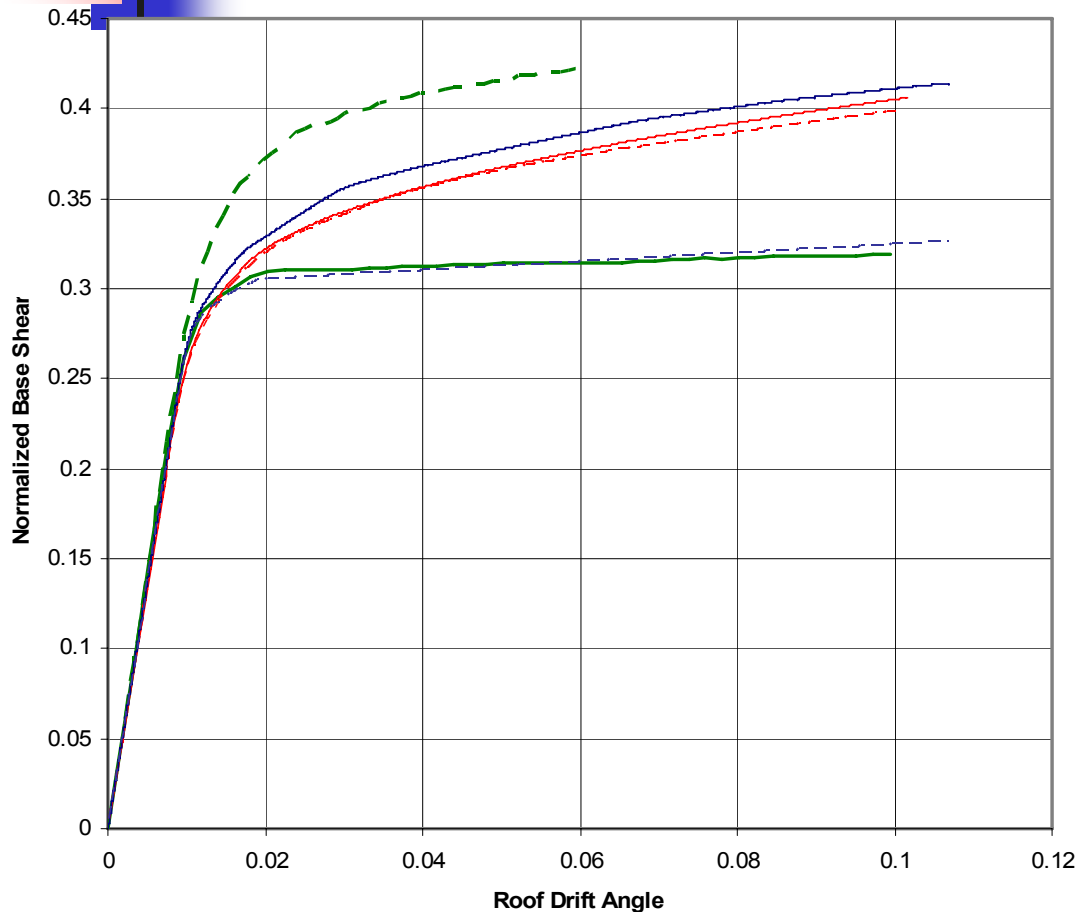
- Stochastic input
- Large nonlinear systems
- Efficient dynamic analysis
- Interpretation of results



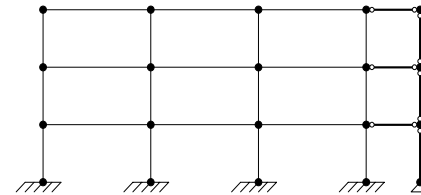
Comparison of FE platforms

Static nonlinear pushover analysis

3 Story Pre-Northridge LA Building (M1 Model with P-Delta Column)
Static Pushover Plot



Fine Mesh



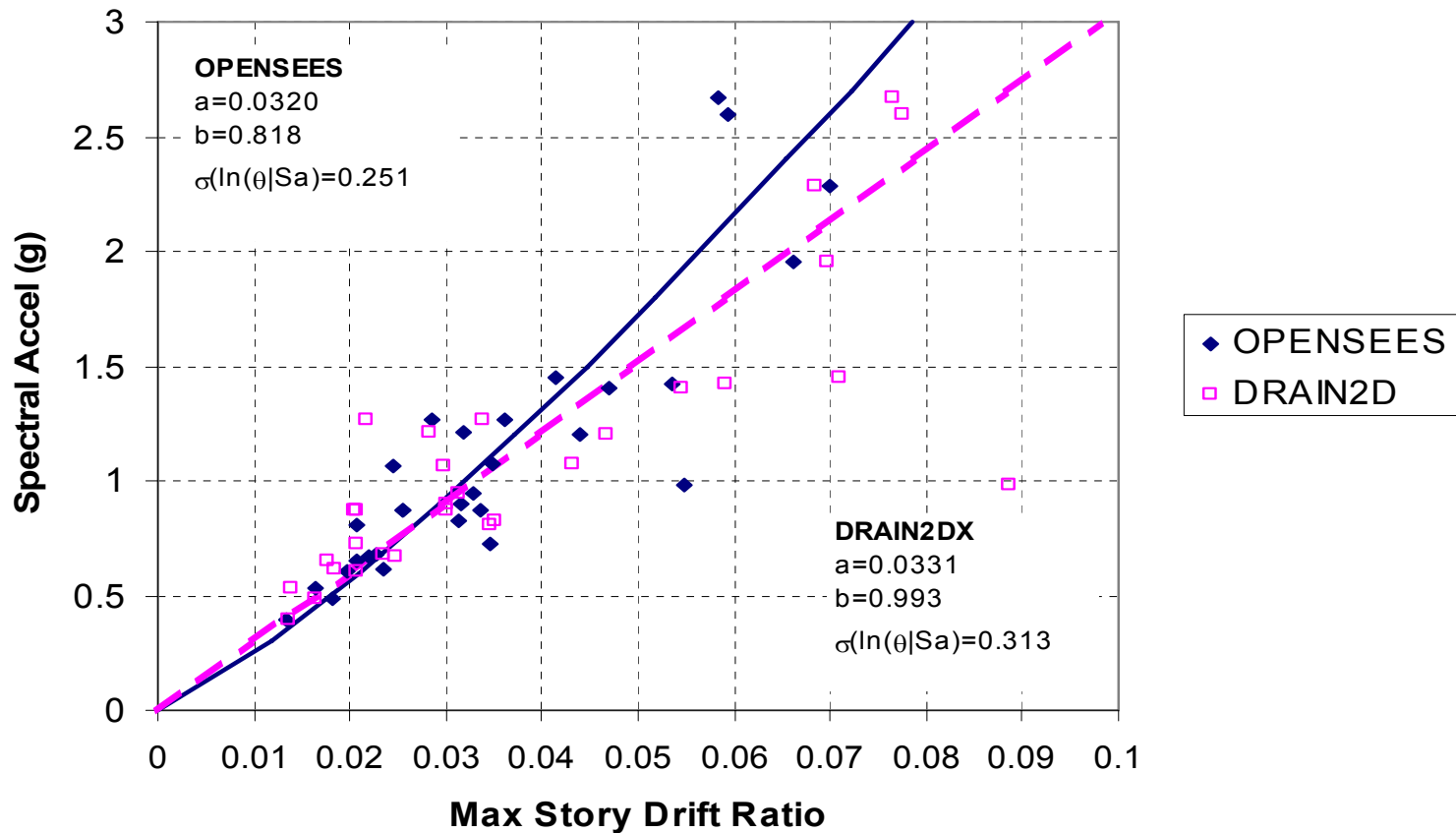
Coarse Mesh



Comparison of FE platforms

Dynamic response - OpenSees vs DRAIN-2DX

LA 3-Story Structure, LA01-LA30 Earthquake Records, 2% Damping Ratio



Limit state probability – point estimate

- Structural fragility: $F_R(x) = \Phi[\ln(x/m_R)/\beta_R]$
- Demand (hazard): $\ln H_Q(x) = k_0 - k \ln x$
- Limit state probability (reflects aleatoric uncertainty):

$$P_{LS} \approx H_Q(m_R) \exp[-1/2(k\beta_R)^2] \quad (\text{acceleration})$$

$$P_{LS} \approx k_0[(m_{\theta C}/a)^{1/b}]^{-k} \exp[-1/2((k\beta_{\theta C}/b)^2) \quad (\text{drift})$$

At Memphis, TN, $k_0 = 7.9 \times 10^{-5}$, $k = 0.981$ and $\beta_C = 0.30$ (typ)

If $m_{\theta C} = 0.036$, $P_{LS} = 2.4 \times 10^{-4}/\text{yr}$





Quantification of uncertainty - data

Probability law $F_X(x; \mu, \sigma)$ models aleatoric uncertainty

$\mu, \sigma =$ parameters of X

Sample: (x_1, x_2, \dots, x_n)

Sample mean and variance of μ, σ from maximum likelihood estimation



Quantification of uncertainty

Encoding expert opinion

Engineering judgment yields range of plausible values, (min, max) – encoded to define epistemic uncertainty

If frequency is symmetric (bell-shaped)

$$\text{Mean} = 0.5 (\text{min} + \text{max}) \quad 0.5 (\text{min} + \text{max})$$

$$\text{SD} = (\text{max} - \text{min})/3.5 \quad (\text{max} - \text{min})/4$$

If frequency is positively skewed,

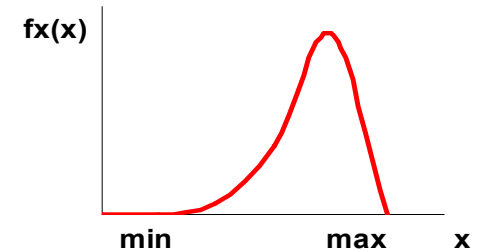
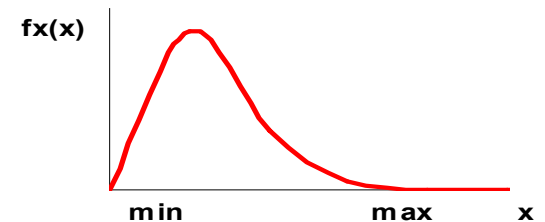
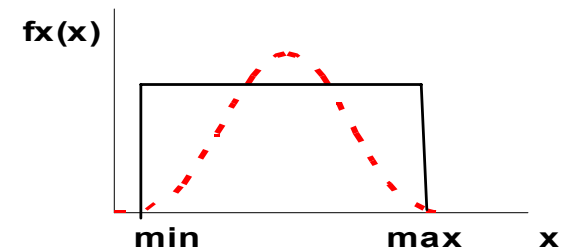
$$\text{Mean} = 0.65 \text{ min} + 0.35 \text{ max}$$

$$\text{SD} = (\text{max} - \text{min})/4$$

If frequency is negatively skewed:

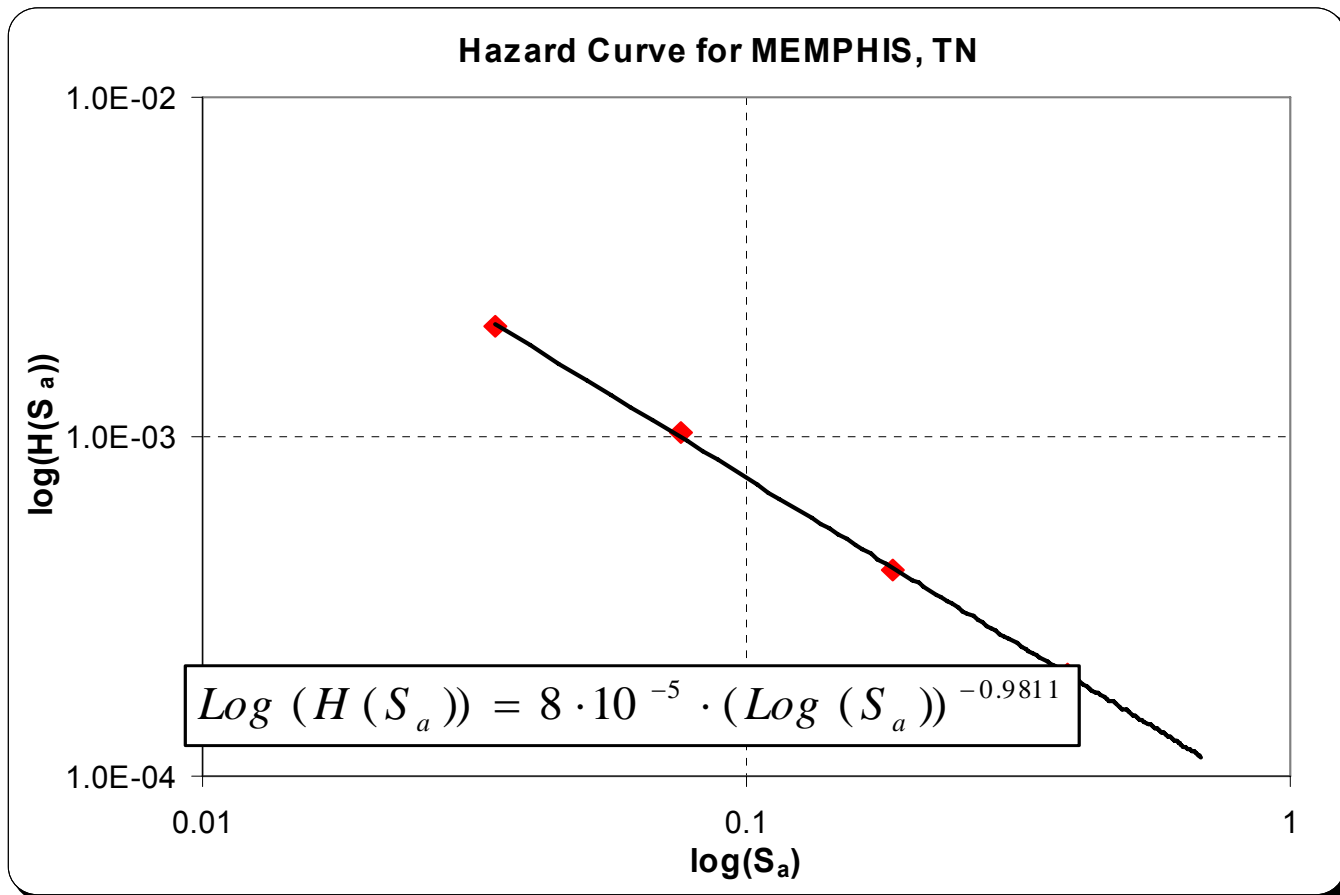
$$\text{Mean} = 0.35 \text{ min} + 0.65 \text{ max}$$

$$\text{SD} = (\text{max} - \text{min})/4$$

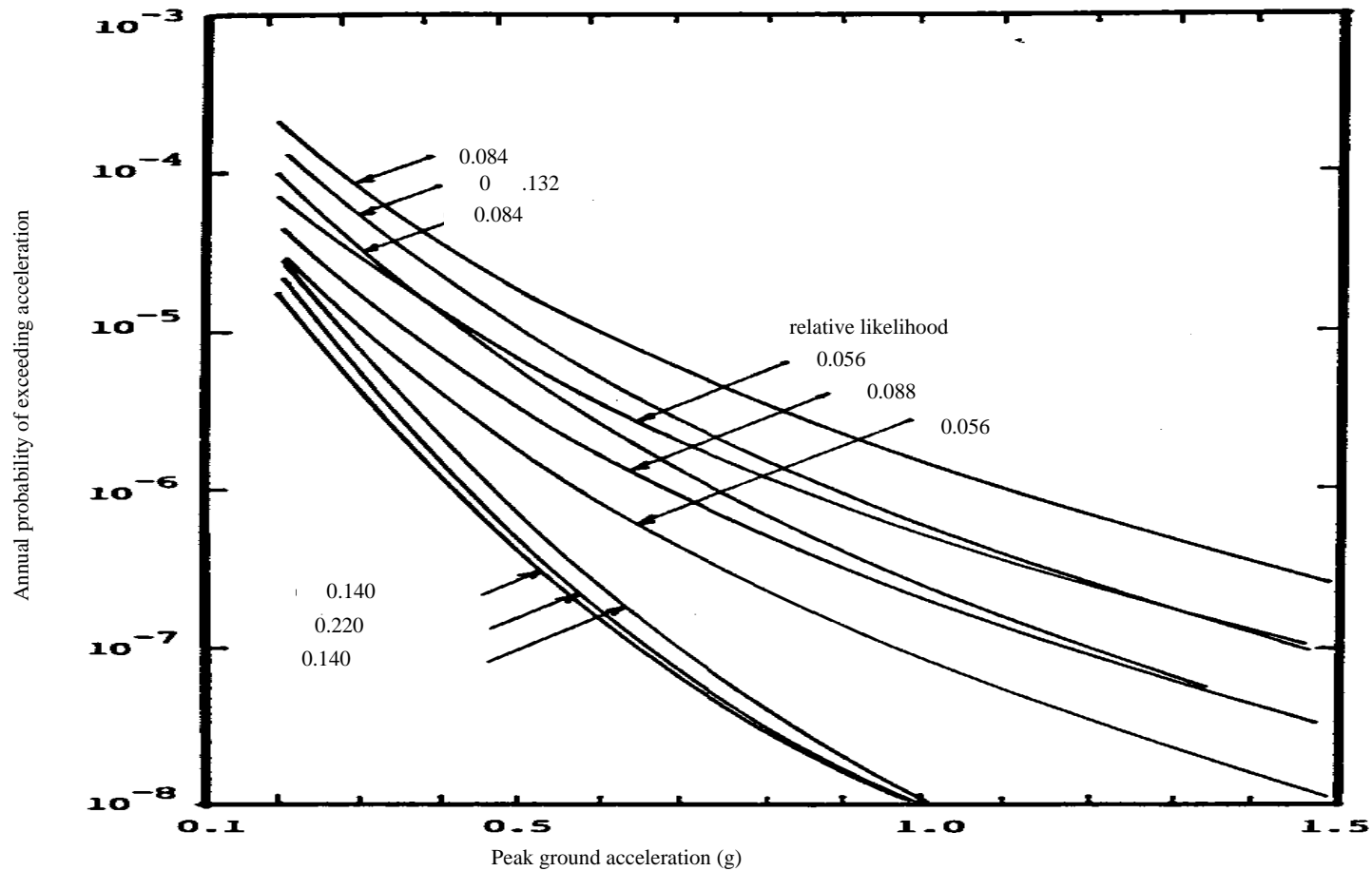


Mean seismic hazard for Memphis, TN

Aleatoric uncertainty: (<http://geohazards.cr.usgs.gov/eq/>)



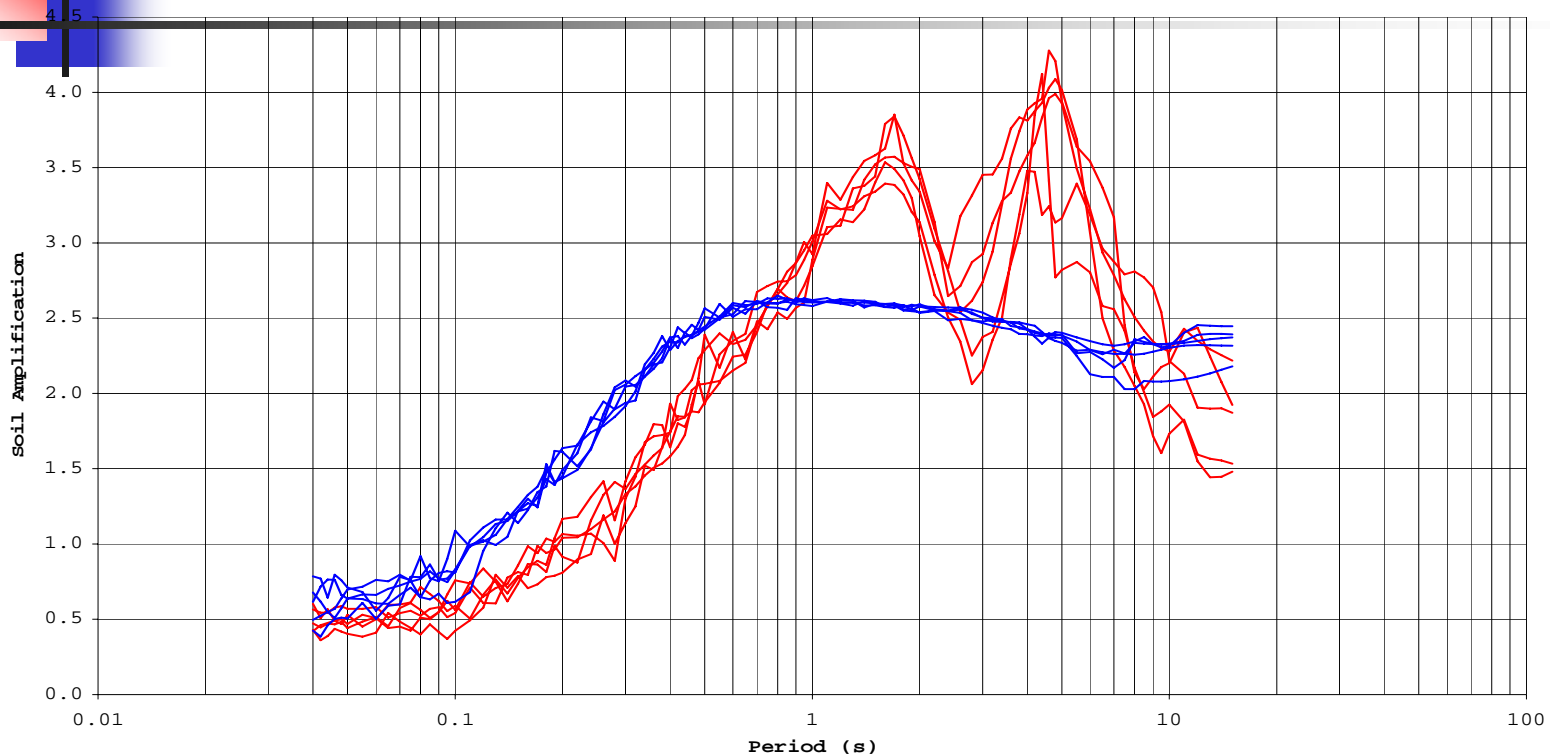
Epistemic uncertainty in seismic hazard due to alternate source hypotheses (β_{UH})



Synthetic earthquake ground motions

Local site amplification

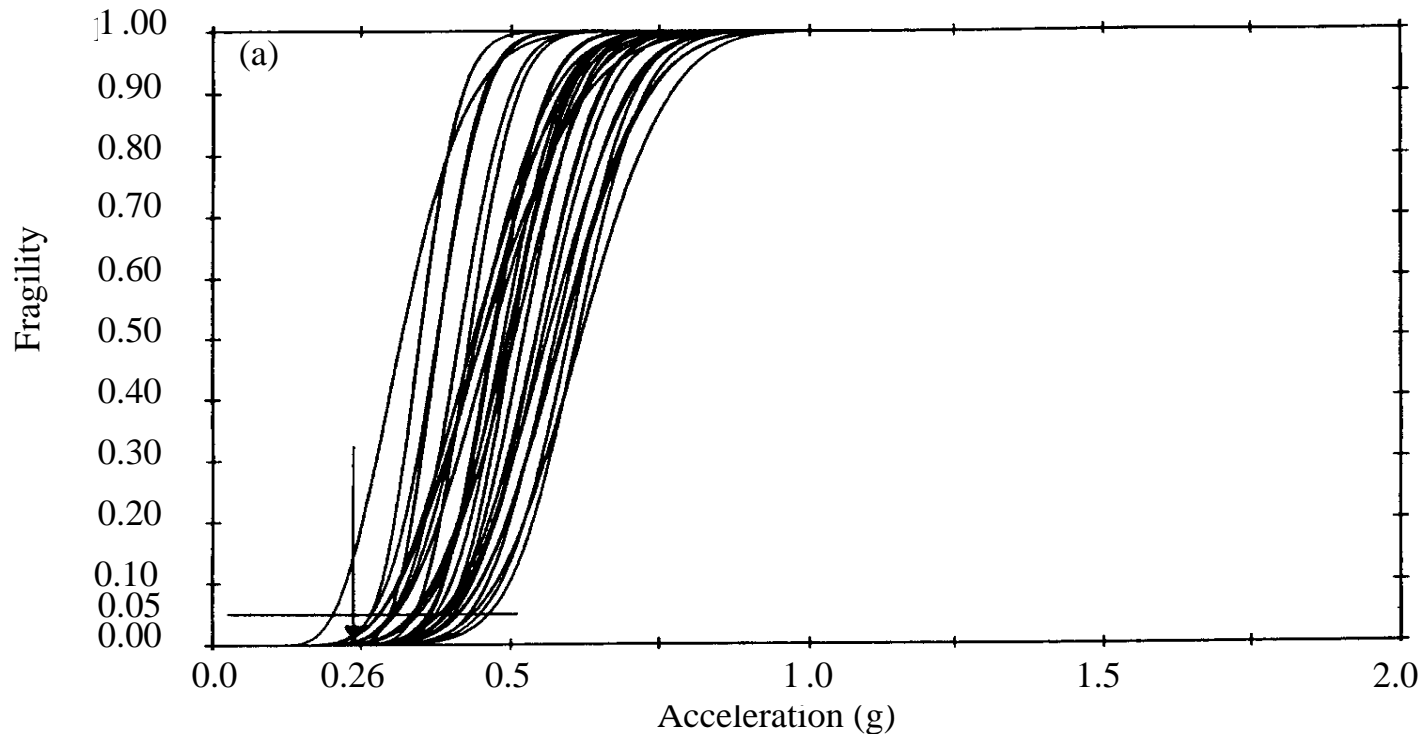
Rix-Fernandez vs Wen-Wu - Memphis, TN



- Rix-Fernandez (M=7.5 D=50 km - nonlinear soil properties included)
- Wen-Wu (quarter-wavelength method)



Epistemic uncertainty in fragility due to structural behavior modeling (β_{UC})



Limit state probability

Accounting for epistemic uncertainty

- Structural fragility: $F_R(x) = \Phi[\ln(x/M_R)/\beta_R]$
- Demand (hazard): $\ln H_Q(x) = K_0 - k \ln x$
- Limit state probability: $P_F \approx K_0(M_R)^{-k} \exp[(k \beta_R)^2/2]$



Encoding epistemic uncertainty

Assume that

M_R can be estimated to within $\pm 10\%$ with
"certainty" (95% confidence)

....implies $\beta_{UR} \approx 0.06$

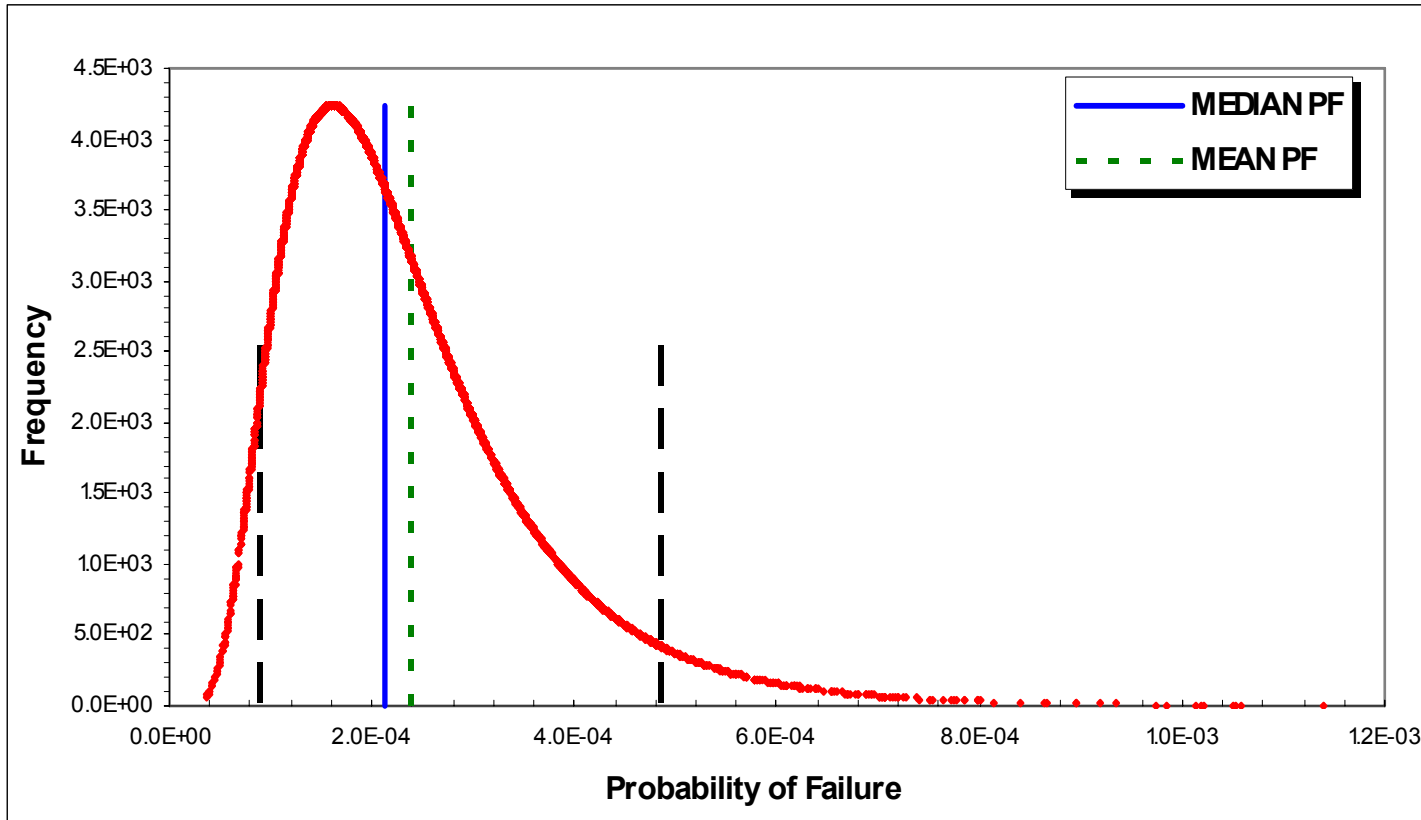
K_o can be estimated to within an order of
magnitude with "certainty" (95%
confidence)

....implies $\beta_{UH} \approx 0.50$



FREQUENCY DISTRIBUTION OF P_{LS} ($m_R = 0.036$)

Role of epistemic uncertainty in risk



Confidence in risk assessment

If the performance limit of the frame is defined by $m_R = 0.036$ from nonlinear FEA:

- A "*point estimate*" of P_{LS} is:
 - 2.4 x 10⁻⁴ (mean)
 - 2.1 x 10⁻⁴ (median) (Reflects aleatoric uncertainty)
- "I am *95% confident* that P_{LS} is *between 0.76 and 5.6 x 10⁻⁴/yr*" (Reflects epistemic uncertainty)
or
- "I am *90% confident* that the limit state probability is less than 1 in 2475/yr (or 2% in 50 yr)" (Reflects epistemic uncertainty)





Damage and loss assessment

Basic elements:

- Seismic demand vs damage states
- Damage state probabilities
- Loss estimation and loss metric

$$P[\text{Loss}=c|\text{Scenario}] = \sum_{LS} \sum_d P[\text{Loss}=c | \text{DS}=d] P[\text{DS}=d | \text{LS}] P[\text{LS}|\text{Scenario}]$$



Earthquake scenario and seismic demand

Scenario earthquake: $M = 7.5$, $R = 50$ km
Frankel ground motion model

Maximum likelihood estimators for θ | Scenario:

$$\ln m_{\theta} = (\sum \ln \theta_i) / 20 \quad \rightarrow \quad m_{\theta} = 0.029$$

$$SD[\ln \theta] = \sqrt{\sum (\ln \theta_i - m_{\theta})^2 / 20} \quad \rightarrow \quad \beta_{th} = 0.22$$



Loss assessment

(Percentage of replacement cost)

$$P[\text{Loss}=c|\text{Scenario}] = \sum_d P[\text{Loss}=c|DS] P[DS|\text{Scenario}]$$

$$P[\text{Loss} < 5\%|\bullet] = 0.01$$

$$P[\text{Loss } 5\text{-}10\%|\bullet] = 0.12$$

$$P[\text{Loss } 10\text{-}25\%|\bullet] = 0.22$$

$$P[\text{Loss } 25\text{-}50\%|\bullet] = 0.34$$

$$P[\text{Loss} > 50\%|\bullet] = 0.31$$

Decision metrics:

$P[\text{Loss} > 25\%] = 65\%$ (50% confidence) (aleatoric uncertainty)

"I am 90% confident that the probability is less than 5% that the loss exceeds 50% of replacement value (epistemic uncertainty)



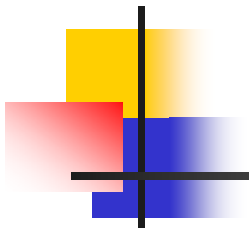
Seismic vulnerability and risk assessment

Research Issues

$$P[\text{Loss}=c] = \sum_s \sum_{LS} \sum_d P[\text{Loss}=c | DS=d] P[DS=d | LS] P[LS | S_a = x] P[S_a = x]$$

- Selection of performance goals
 - Life safety
 - Economic loss
 - Impact on social fabric
- Relation of performance objectives to limit states
- Identification of hazards and hazard levels
- **Efficient structural analysis and simulation tools**
- Quantification of uncertainties
- Acceptable risks and reliability benchmarks





Thank you!

