Real Time Collaboration and Sharing

National Science Foundation Industry/University Cooperative Research Center for *e*-Design: IT-Enabled Design and Realization of Engineered Products and Systems

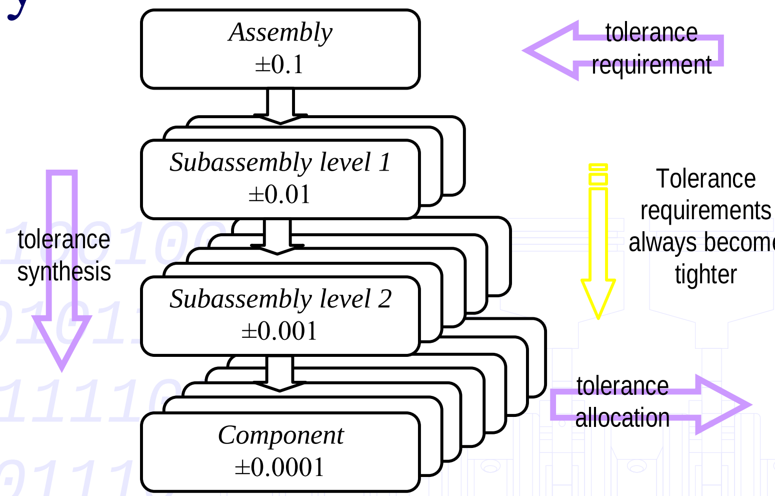
Semantic Tolerance Modeling based on Modal Interval Analysis

Yan Wang, Ph.D.
Department of Industrial Engineering
University of Central Florida

Motivation

- Assumptions of rigid geometry

"The conventional addition theorem of variance is no longer valid for deformable sheet metal assemblies." – Takezawa (1980)



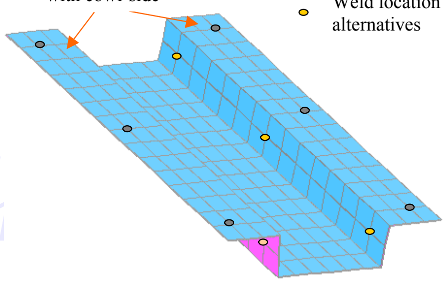
- Semantics of tolerance specifications
  - material difference
  - assembly sequence difference
  - understand numbers
  - "Thou Shallt Not Lie"

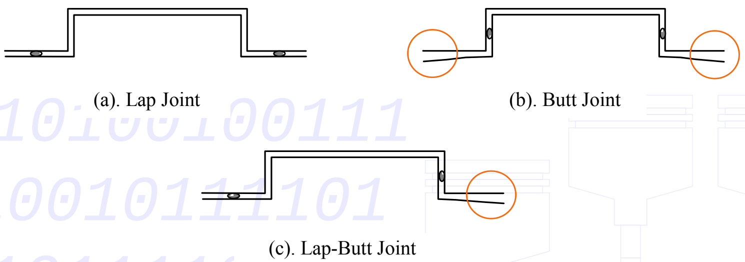
Background - Tolerance Analysis

- Variational estimation
  - 2D/3D geometric tolerance zone
- Statistical approximation
  - root-sum-square, Taylor series
  - adjustment & correction
- Kinematic formulation
  - displacement vectors & matrices
- Monte Carlo simulation
  - most accurate
  - time consuming

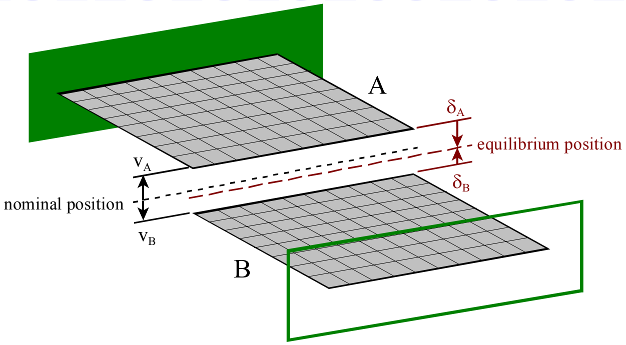
Background - Tolerance Analysis of Flexible Assembly

- Linear FEM structural model [Liu et al., Camelio et al.]





- Linear elastic FEM [Merkley et al.]



Background – Interval Analysis in Engineering

- Computer Graphics
- Robust geometry construction & evaluation
- Robot control
- Set-based modeling
- Imprecise structural analysis
- Design optimization
- Finite-element analysis
- Soft constraint solving
- Worst-case tolerance analysis

Background - Modal Interval Analysis (MIA)

- Limitation of traditional IA
  - Linear stack-up based on worst-case scenario
  - Not interpretable
- Modal Interval Analysis

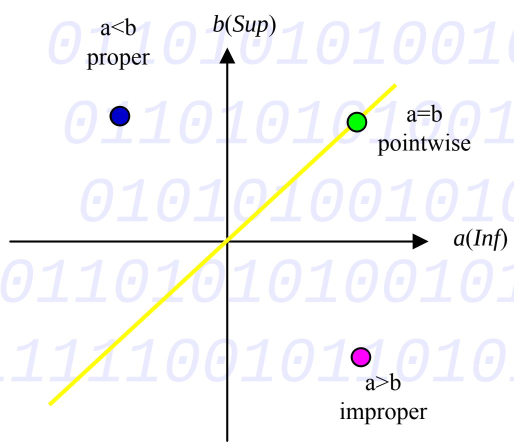
[Gardenes *et al*., 2001; Markov, 2001; Popova, 2001; Shary, 2001 & 2002; Armengol *et al*., 2001;]

  - Semantic extension
  - Tighten range estimation

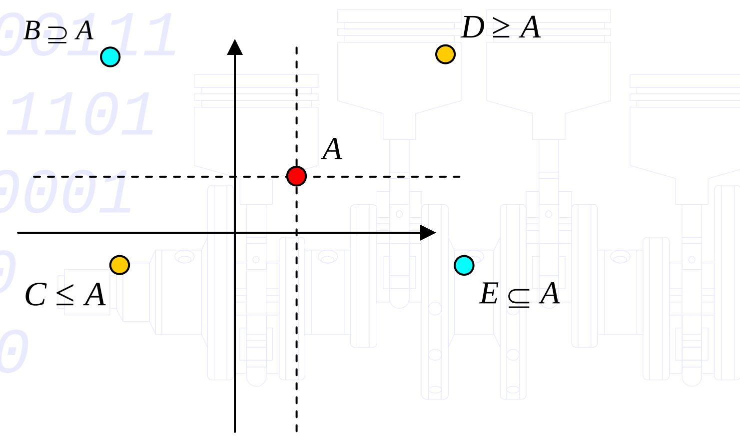
Background - MIA

$$X := (X', QX) \quad X' \in I(\boldsymbol{R}) \quad QX \in \{\mathrm{E}, \mathrm{U}\} = \{\exists, \forall\}$$

$$[a,b] := \begin{cases} ([a,b]', \mathrm{E}) & \text{if } a \le b \quad \textit{existential} \textbf{ or } \textit{proper} \\ ([b,a]', \mathrm{U}) & \text{if } a \ge b \quad \textit{universal} \textbf{ or } \textit{improper} \end{cases}$$



Inf-Sup Diagram



Inf-Sup Diagram

$$[a_1,a_2] \subseteq [b_1,b_2] \Leftrightarrow (a_1 \ge b_1, a_2 \le b_2) \quad A \subseteq B \Leftrightarrow Pred(A) \subseteq Pred(B) \quad \mathrm{Q}(x,A)P(x) \Rightarrow \mathrm{Q}(x,B)P(x)$$

$$[a_1,a_2] \le [b_1,b_2] \Leftrightarrow (a_1 \le b_1, a_2 \le b_2) \qquad dual([a,b]) := [b,a]$$

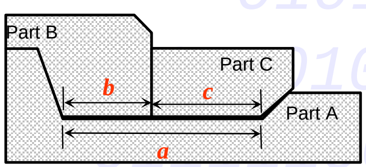
Semantic Tolerance Modeling

- The purpose of semantic tolerance modeling is to capture logical therefore engineering meanings and implications in mathematical representation, which is to build a bridge between mathematic theory and tolerancing practice.

Uni-Incident Interpretability

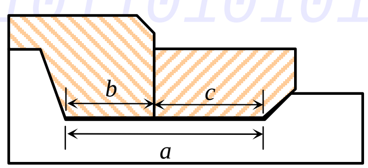
$$f(x, y) = x + y$$

$$[1,3]+[2,5]=[3,8] \quad \forall x \in [1,3], \forall y \in [2,5], \exists z \in [3,8], z = x + y$$
$$[1,3]+[5,2]=[6,5] \quad \forall x \in [1,3], \forall z \in [5,6], \exists y \in [2,5], z = x + y$$
$$[3,1]+[2,5]=[5,6] \quad \forall y \in [2,5], \exists x \in [1,3], \exists z \in [5,6], z = x + y$$
$$[3,1]+[5,2]=[8,3] \quad \forall z \in [3,8], \exists x \in [1,3], \exists y \in [2,5], z = x + y$$



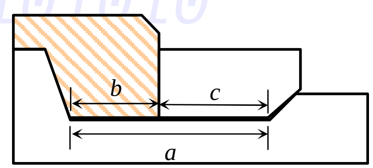
Dimensional relation $a = b + c$

Case 1: given Part **B** and Part **C**, Part **A** needs to fit **B** and **C**.
$\forall b \in B', \forall c \in C', \exists a \in A', a = b + c$



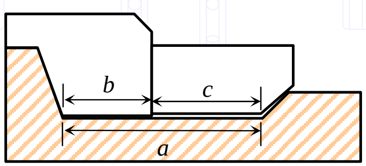
$[2,5]+[1,3]=[3,8]$

Case 2: given Part **B**, Part **A** and Part **C** need to fit **B**.
$\forall b \in B', \exists c \in C', \exists a \in A', a = b + c$



$[2,5]+[3,1]=[5,6]$

Case 3: given Part **A**, Part **B** and Part **C** need to fit **A**.
$\forall a \in A', \exists b \in B', \exists c \in C', a = b + c$



- Manufacturing & Assembly sequence
- Tolerancing intent

Multi-incident Interpretability

$$f(x,y) = xy/(x+y) \quad X = [-1,3] \quad Y = [15,7]$$

$fR(\mathbf{X}) = [-1,3]\times[15,7]/([-1,3]+[15,7]) = [-0.5,1.5]$ is **NOT** interpretable

$fR(\mathbf{XT}^*) = [-1,3]\times[15,7]/([-1,3]+\underline{[7,15]}) = [-1.16667,3.5] \quad \forall x\in[-1,3], \exists y\in[7,15], \exists z\in[-1.16667,3.5], z = xy/(x+y)$

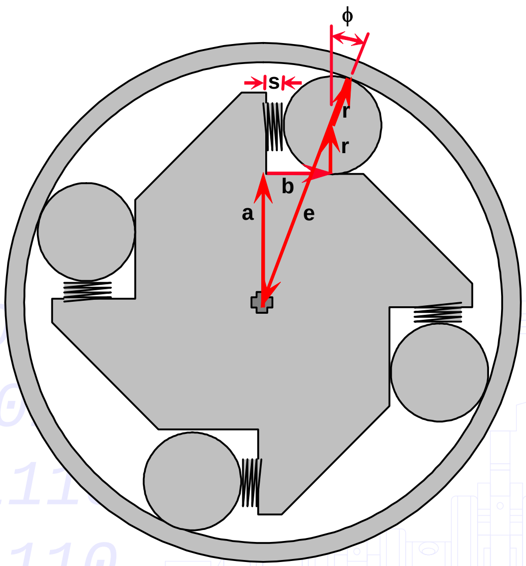
$fR(\mathbf{XT}^*) = [-1,3]\times\underline{[7,15]}/([-1,3]+[15,7]) = [-1.07143,3.21429] \quad \forall x\in[-1,3], \exists y\in[7,15], \exists z\in[-1.07143,3.21429], z = xy/(x+y)$

$fR(\mathbf{XT}^{**}) = [-1,3]\times[15,7]/(\underline{[3,-1]}+[15,7]) = [-0.388889,1.16667] \quad \forall x\in[-1,3], \exists y\in[7,15], \exists z\in[-0.388889,1.16667], z = xy/(x+y)$

$fR(\mathbf{XT}^{**}) = \underline{[3,-1]}\times[15,7]/([-1,3]+[15,7]) = [4.5,-1.5] \quad \forall x\in[-1,3], \forall z\in[-1.5,4.5], \exists y\in[7,15], z = xy/(x+y)$

Rigidity Interpretability

- Existential interval
  - *"fluctuation" "autonomous"*
- Universal interval
  - *"regulating" "feedback"*



$$R+S=[5.2,5.7]+[2.8,2.1]=[8.0,7.8]=B$$

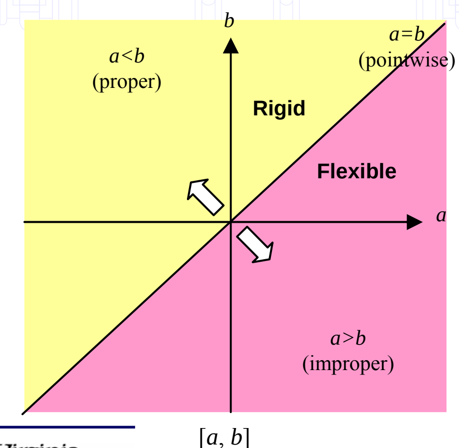
$$\forall r\in[5.2,5.7],\forall b\in[7.8,8.0],\exists s\in[2.1,2.8],r+s=b$$

The spring provides a "cushion" to absorb variance.

$$R+S=[5.2,5.7]+[2.6,2.8]=[7.8,8.5]=B$$

$$\forall r\in[5.2,5.7],\forall s\in[2.6,2.8],\exists b\in[7.8,8.5],r+s=b$$

No flexible material is required to absorb variance.



CENTER FOR *e*-DESIGN

Semantic Tolerancing

Domain	*Existential* or *Proper* category	*Universal* or *Improper* category
Supply management	Pre-determined, Uncontrollable, Supplied	Un-determined, Controllable, Built
Manufacturing sequence	Working dimension, Clearance	Balance dimension, Stock removal
Assembly sequence	Place, Virtual condition size	Fit, Bonus tolerance
Material property	Rigid, Wearable	Flexible, Deformable
Process control	Open loop, Manual mode	Closed loop, Auto mode

Optimality

- Uni-incident optimality
  - If variables **X** have the same modality, *fR*(**X**) is optimal.

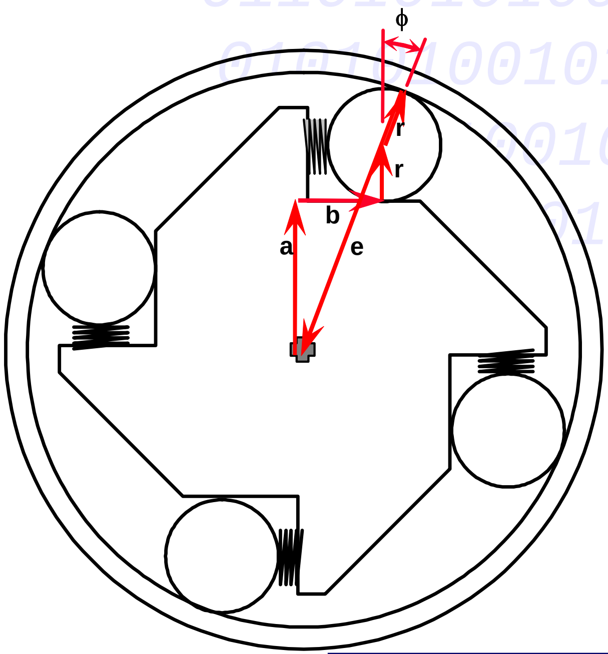
$$f(x, y) = (x + y)^2 \qquad fR(\mathbf{X}) = ([1,3] + [2,5])^2 = [9,64]$$

- Multi-incident optimality
  - If *fR*(**X**) is *totally monotonous* for all of its multi-incident arguments, by transforming, for every multi-incident component, all incidences into its dual if the corresponding incidence has a mononicity sense *contrary* to the global one, *fR*(**XD**) is optimal.

$$f(x, y) = xy/(x + y) \qquad fR(\mathbf{XD}) = [1,3] \times [15,7]/([3,1] + [7,15]) = [0.9375,2.1]$$

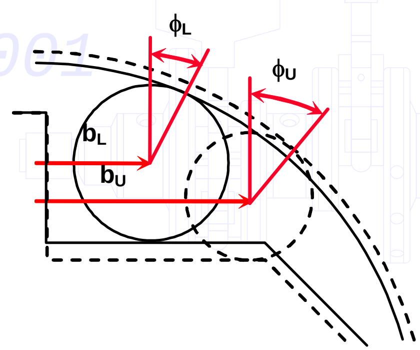
Tolerance Propagation Estimation

Inputs			Output: *Position of roller (b)*		
Name	Lower limit	Upper limit	**Method**	Lower limit	Upper limit
Height of hub (a)	27.595	27.695	**DLM+RSS (CE/Tol®)**	4.3585	5.2625
Radius of ring (e)	50.7875	50.8125	**DLM+WC (CE/Tol®)**	4.1368	5.4842
Radius of roller (r)	11.42	11.44	**MIA**	4.08381	5.44048
			True Range	4.08381	5.44048



$$\phi = \cos^{-1}\frac{a+r}{e-r}$$

$$b = \sqrt{(e-r)^2 - (a+r)^2}$$

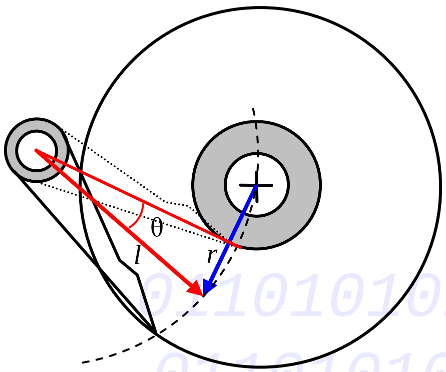


Results compared to Direct Linearization Method (DLM) [Chase *et al.*, 1997]

HD Track Distance Simulation

$$R_{k+1} = R_k \cos\frac{\Delta}{2} + \sqrt{4L^2 - R_k^2}\,\sin\frac{\Delta}{2}$$

$$\boxed{R_{k+1} = R_k \cos\frac{\Delta}{2} + \sqrt{4L^2 - [Dual(R_k)]^2}\,\sin\frac{\Delta}{2}}$$



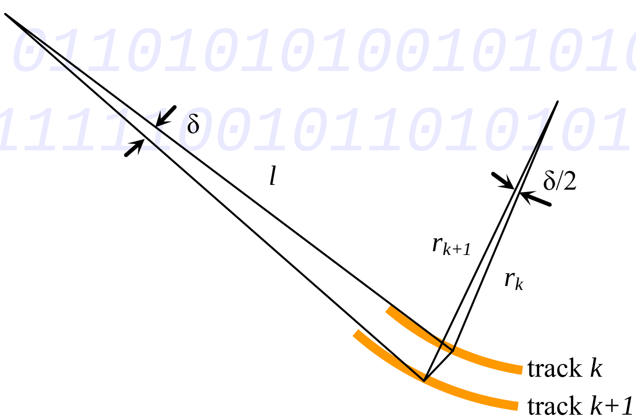
l: arm length
θ: arm angle
r: distance to disk center (track number)

(a) In hard disk, precise arm movement is required to seek tracks

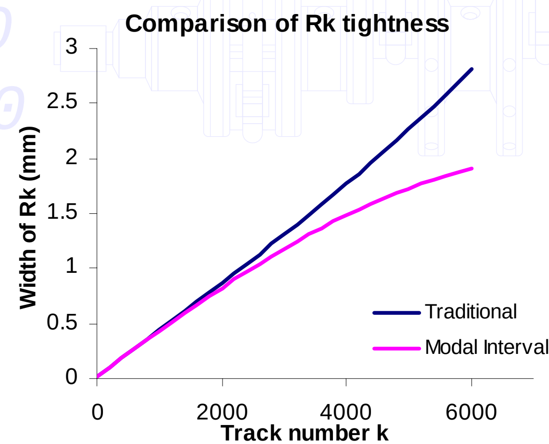
$$r_{k+1} = r_k \cos\frac{\delta}{2} + \sqrt{4l^2 - r_k^2}\,\sin\frac{\delta}{2}$$

r_k: distance from track k to disk center
r_{k+1}: distance from track $k+1$ to disk
δ: arm angle increment for each track

(c) Incremental relation between track distances



(b) Illustration of distance relation between adjacent tracks



(d) Tighter variation estimation based on modal interval compared to traditional worst-case estimation

Derivative Process Control Simulation

$$V(k+1) = V(k) + K_d[V_0 - V(k)] - \frac{1}{S}[V(k) - V_a]$$

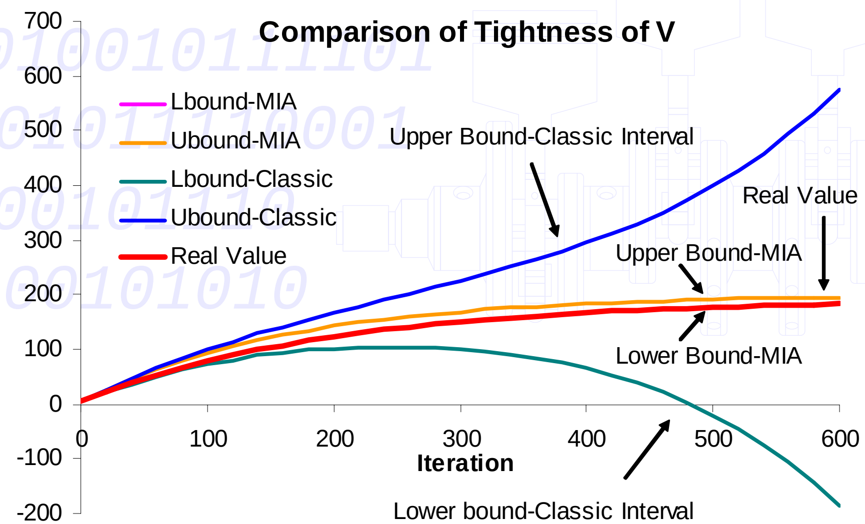
$$\boxed{V(k+1) = V(k) + K_d[V_0 - dual(V(k))] - \frac{1}{S}[dual(V(k)) - V_a]}$$

K_d=[0.004,0.005]
V_a=[2,3]
S=[1000,1001]
V_0=[240,241]
$V(0)$=[3,3]

$$\frac{dv}{dt} = k_d(v_0 - v) - \frac{1}{s}(v - v_a)$$

v: sensored tooling speed
v_0: nominal control speed
k_d: action factor of controller
v_a: sensor shift due to surroundings
s: sensitivity factor of sensor

(a) A simple derivative controller model



(b) Optimal variation estimation based on modal interval compared to classic interval methods

Closeness of MIA Arithmetic Operations

- In traditional IA
  - There is no interval $[x,y]$ such that $[a,b]+[x,y]=0$.

$$A + X = B \qquad X = B - dual(A)$$

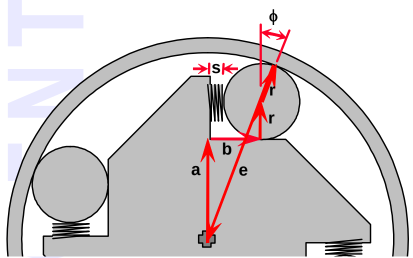
[2,3]+X=[4,7]   X=[4,7]-dual([2,3]) =[4,7]-[3,2]=[2,4]

[2,3]+[2,4]=[4,7]

$$AX = B \qquad X = B / dual(A)$$

[2,3]*X=[6,12]   X=[6,12]/dual([2,3]) =[6,12]/[3,2]=[3,4]

[2,3]*[3,4]=[6,12]



$$f(R_1, R_2, \cdots, R_n) = 0$$

Tolerance Analysis

$$AX = B$$

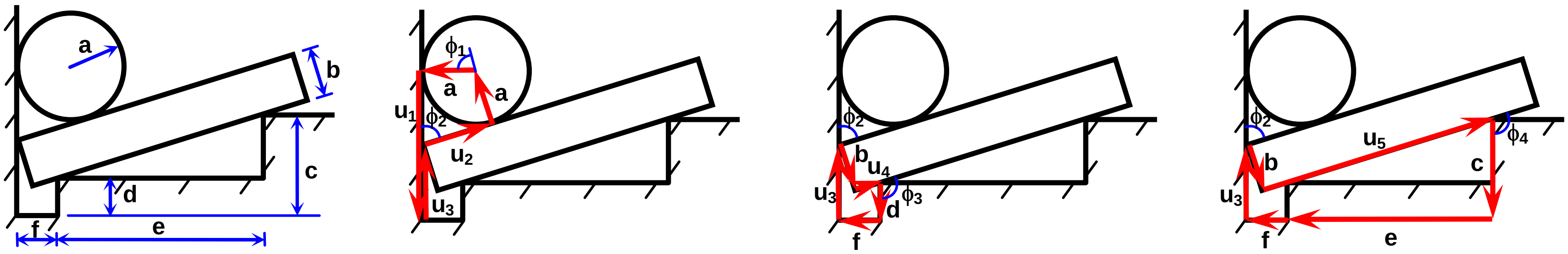
- If A is strictly diagonally dominant
  - Jacobi algorithm [Sainz et al., 2002]

$$\Im(X_i) := \frac{B_i - \sum_{i \neq j} Dual(A_{ij}) \times Dual(X_j)}{Dual(A_{ii})} \quad \left(0 \notin A_{ii} \text{ and } i = 1, \cdots, n\right)$$

$$\mathbf{X}^{(0)} \supseteq \mathbf{X}^{(1)} \subseteq \mathbf{X}^{(2)} \supseteq \cdots \subseteq \mathbf{X}^{(2k)} \supseteq \mathbf{X}^{(2k+1)} \subseteq \cdots$$

- If A is not strictly diagonally dominant, general interval methods (such as [Neumaier, 1990; Hansen, 1992; Ning and Kearfott, 1997])

Stacked Block Assembly



(a) known parameters (b) vector loop 1 (c) vector loop 2 (d) vector loop 3

Known size variation	$a = 6.62 \pm 0.2$ $b = 6.805 \pm 0.075$ $c = 10.675 \pm 0.125$ $d = 4.06 \pm 0.15$ $e = 24.22 \pm 0.35$ $f = 3.905 \pm 0.125$
Unknown kinematic variation	$u_1 = 18.7181 \pm ?$ $u_2 = 8.6705 \pm ?$ $u_3 = 10.0477 \pm ?$ $u_4 = 2.1894 \pm ?$ $u_5 = 27.2965 \pm ?$ $\phi_1 = 74.7243 \pm ?$ $\phi_2 = -74.7243 \pm ?$ $\phi_3 = -105.2761 \pm ?$ $\phi_4 = -105.2761 \pm ?$
Loop 1	$\begin{cases} F_1 = u_2\cos(90+\phi_2) + a\cos(180+\phi_2) + a\cos(180+\phi_1+\phi_2) = 0 \\ F_2 = u_3 + u_2\sin(90+\phi_2) + a\sin(180+\phi_2) + a\sin(180+\phi_1+\phi_2) - u_1 = 0 \\ F_3 = 90+\phi_2+90+\phi_1+90+90-360 = 0 \end{cases}$
Loop 2	$\begin{cases} F_4 = b\cos(\phi_2) + u_4\cos(\phi_2+90) + d\cos(\phi_2+90+\phi_3) - f = 0 \\ F_5 = u_3 + b\sin(\phi_2) + u_4\sin(\phi_2+90) + d\sin(\phi_2+90+\phi_3) = 0 \\ F_6 = 90+\phi_2-90+90+\phi_3-90+180 = 0 \end{cases}$
Loop 3	$\begin{cases} F_7 = b\cos(\phi_2) + u_5\cos(\phi_2+90) + c\cos(\phi_2+90+\phi_4) - e - f = 0 \\ F_8 = u_3 + b\sin(\phi_2) + u_5\sin(\phi_2+90) + c\sin(\phi_2+90+\phi_4) = 0 \\ F_9 = 90+\phi_2-90+90+\phi_4-90+180 = 0 \end{cases}$

Stacked Block Assembly - Results

$$\left[\frac{\partial F_i}{\partial s_j}\right]_{i\times j}\Delta\mathbf{s}+\left[\frac{\partial F_i}{\partial k_i}\right]_{i\times i}\Delta\mathbf{k}=0$$

- Results compared with DLM [Chase *et al.*, 1997]

MIA Linearization	DLM Worst-Case	DLM Statistical
$\Delta u_1=[0.5420,\ -0.5420]$	$\Delta u_1=[-0.5421,\ 0.5421]$	$\Delta u_1=[-0.2998,\ 0.2998]$
$\Delta u_2=[0.4672,\ -0.4672]$	$\Delta u_2=[-0.3899,\ 0.3899]$	$\Delta u_2=[-0.2725,\ 0.2725]$
$\Delta u_3=[0.3137,\ -0.3137]$	$\Delta u_3=[-0.2942,\ 0.2942]$	$\Delta u_3=[-0.1844,\ 0.1844]$
$\Delta u_4=[0.2729,\ -0.2729]$	$\Delta u_4=[-0.2384,\ 0.2384]$	$\Delta u_4=[-0.1411,\ 0.1411]$
$\Delta u_5=[0.5209,\ -0.5209]$	$\Delta u_5=[-0.5174,\ 0.5174]$	$\Delta u_5=[-0.3836,\ 0.3836]$
$\Delta \phi_1=[0.0228,\ -0.0228]$	$\Delta \phi_1=[-0.8156,\ 0.8156]$	$\Delta \phi_1=[-0.4784,\ 0.4784]$
$\Delta \phi_2=[0.0228,\ -0.0228]$	$\Delta \phi_2=[-0.8156,\ 0.8156]$	$\Delta \phi_2=[-0.4784,\ 0.4784]$
$\Delta \phi_3=[0.0228,\ -0.0228]$	$\Delta \phi_3=[-0.8156,\ 0.8156]$	$\Delta \phi_3=[-0.4784,\ 0.4784]$
$\Delta \phi_4=[0.0228,\ -0.0228]$	$\Delta \phi_4=[-0.8156,\ 0.8156]$	$\Delta \phi_4=[-0.4784,\ 0.4784]$

If variations of angles are known ➡

Known size variation	$a = 6.62 \pm 0.2$ $\quad b = 6.805 \pm 0.075$ $\quad c = 10.675 \pm 0.125$ $d = 4.06 \pm 0.15$ $\quad e = 24.22 \pm 0.35$ $\quad f = 3.905 \pm 0.125$
Known kinematic variation	$\phi_1 = 74.7243 \pm 0.4281$ $\quad \phi_2 = -74.7243 \pm 0.4281$ $\phi_3 = -105.2761 \pm 0.4281$ $\quad \phi_4 = -105.2761 \pm 0.4281$
Unknown kinematic variation	$u_1 = 18.7181 \pm ?$ $\quad u_2 = 8.6705 \pm ?$ $\quad u_3 = 10.0477 \pm ?$ $\quad u_4 = 2.1894 \pm ?$ $\quad u_5 = 27.2965 \pm ?$
Linear equations	$\begin{cases} -u_1 + u_2 \sin(90+\phi_2) + u_3 + a\sin(180+\phi_2) = 0 \\ u_2 \cos(90+\phi_2) + a\cos(180+\phi_2) - a = 0 \\ u_3 + u_4 \sin(\phi_2+90) + b\sin(\phi_2) - d = 0 \\ u_4 \cos(\phi_2+90) + b\cos(\phi_2) - f = 0 \\ u_5 \cos(\phi_2+90) + b\cos(\phi_2) - e - f = 0 \end{cases}$ $\begin{bmatrix} -1 & [0.2562,0.2707] & 1 & 0 & 0 \\ 0 & [0.9626,0.9667] & 0 & 0 & 0 \\ 0 & 0 & 1 & [0.2562,0.2707] & 0 \\ 0 & 0 & 0 & [0.9626,0.9667] & 0 \\ 0 & 0 & 0 & 0 & [0.9626,0.9667] \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} [-6.1804,-6.5922] \\ [8.6659,8.0652] \\ [10.8602,10.3888] \\ [2.3054,1.9179] \\ [26.8754,25.7879] \end{bmatrix}$
Result of Jacobi algorithm	$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} [18.7024,18.7335] \\ [9.0026,8.34302] \\ [10.2466,9.85174] \\ [2.39497,1.98397] \\ [27.9196,26.6762] \end{bmatrix}$
Interpretation of result	$U(u_1,[18.7024,18.7335]')U(a,[6.42,6.82]')U(b,[6.73,6.88]')U(c,[10.55,10.8]')U(d,[3.91,4.21]')$ $U(e,[23.87,24.57]'U(f,[3.78,4.03]')U(\phi_1,[74.2962,75.1524]')U(\phi_2,[-75.1524,-74.2962]')$ $U(\phi_3,[-105.7042,-104.8480]')U(\phi_4,[-105.7042,-104.8480]')$ $E(u_2,[8.34302,9.0026]')E(u_3,[9.85174,10.2466]')E(u_4,[1.98397,2.39497]')E(u_5,[26.6762,27.9196]')$ $\begin{cases} -u_1 + u_2 \sin(90+\phi_2) + u_3 + a\sin(180+\phi_2) = 0 \\ u_2 \cos(90+\phi_2) + a\cos(180+\phi_2) - a = 0 \\ u_3 + u_4 \sin(\phi_2+90) + b\sin(\phi_2) - d = 0 \\ u_4 \cos(\phi_2+90) + b\cos(\phi_2) - f = 0 \\ u_5 \cos(\phi_2+90) + b\cos(\phi_2) - e - f = 0 \end{cases}$

Concluding Remark

- A semantic tolerance modeling scheme based on modal interval is proposed
- To capture engineering and logic relation between specifications
  - physical property difference between rigid and flexible materials
  - sequence of specification, measurement, and assembly
- To support better design and manufacturing specifications

Yet to be Achieved

- Instead of focusing only on mathematic and numerical convenience, a good mathematic model of tolerance should convey the full semantics of size and geometric tolerances and support analysis and synthesis with a simple yet comprehensive structure.