Randomness and Extreme Events in Oceanic Turbulence

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Abstract

Waves in open ocean are usually modeled as Gaussian seas by the linear superimposition of a very large number of elementary waves having amplitudes related to a given spectrum and random phases. If the elementary waves exchange energy by nonlinear wave interaction, then the dynamics of the wave field is governed by the well known Zakharov equation for wave turbulence (Zakharov 1999).
In this paper, a theory of stochastic wave groups is proposed to explain the occurrence of extreme events in wave turbulence.

In Gaussian seas, the occurrence of a large wave of amplitude h is due to the dynamics of a stochastic wave group (Fedele 2006, Fedele & Tayfun 2006) whose leading order structure is described by the space-time covariance of the sea surface (Lindgren, 1972; Boccotti 2000). Drawing on the theory of quasi-determinism of Boccotti (2000) and the regression approximation approach of Rychlik (1987), we derive a new coherent structure consisting of a large stochastic wave group of O(h), evolving non-interactively on a finite random background of O(1).

The nonlinear dynamics of this wave structure in oceanic wave turbulence is investigated by both considering second order interactions due to bound waves and third order effects due to four wave quasi-resonance interactions. As a corollary, new theoretical upper bounds for the statistical distribution of crest heights and crest-to-trough heights over large waves are derived.

Comparisons based on the nonlinear simulations of the Zakharov equation (see also Socket-Juglard et al. 2005), with wave data collected at the Tern platform in the northern North Sea during an extreme storm, and the results of the wave tank experiment of Onorato et al. (2006), show good agreement with the proposed theoretical wave distributions.

The numerical simulations showed that the time evolution of an initial Gaussian random field consists of three distinct phases: an initial phase where the field is weakly non-Gaussian and nonlinearities are not developed yet. This phase is followed by a strong non-Gaussian transient, during which the wave field becomes non-Gaussian, indicating the failure of the central limit theorem by the building up of space-time correlation. Stronger deviations from Gaussianity occur, an evidence of the strong intermittency characteristic of the transient field. Then, a third and final phase kicks in, during which the wave field reaches a steady state. In particular, for initial wave fields consisting of narrow-band seas (unrealistic oceanic conditions), the steady state is non-Gaussian with kurtosis >3. Very narrow band initial fields yield more intense modulation instability and very strong transients which are followed by

a less intense steady state. Instead, moderately narrow band initial wave fields, tend to a steady state monotonically with a very smooth transient.

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