Evaluation of Inconsistent Engineering data

Michael Beer
Department of Civil Engineering
National University of Singapore, Singapore

Program

1 Introduction

2 Mathematical Models

3 Quantification Techniques

4 Numerical Processing and Result Interpretation

5 Resumé

General situation

The engineer‘s endeavor

- numerical modeling – structure and environment

➡ design and prognoses – system behavior, safety, robustness, economic aspects, aesthetics

» close to reality
» numerically efficient

Deterministic methods

- deterministic structural parameters
- deterministic computational models

⟷ Reality

Imprecision ? Variation ? Ambiguity ? Vagueness ?

⬇

Problem of uncertainty and its consequences

Uncertainty

Examples:

- Mechanical behavior of novel materials
  - Nanotubes as micro reinforcement
  - Fiber reinforced concrete
  - Textile reinforced concrete

Modeling on Micro, Meso and Macro scale

Distribution and orientation of reinforcement material ?

Interaction ? Failure modes ? Long-term behavior ?

- Risk and hazard analysis – earthquakes and tsunamis
  - Hazard potential and consequences of Sumatra earthquakes

Generation ? Propagation ? Local effects for Singapore ?

- Extreme environmental conditions – hurricanes and ice loads
  - reliability and life-time assessment of offshore structures

Frequency ? Location ? Direction ? Strength ? Effects ?
Damage and residual safety ?

Uncertainty

Resumé

- different engineering fields

- common problem: uncertainty / lack of information

inconsistency of data / information:
stochastic and non-stochastic characteristics simultaneously

» small samples
» imprecise sample elements
» changing environmental conditions
» linguistic assessments
» experience, expert knowledge

➡ appropriate mathematical modeling and quantification

Modeling of uncertainty

Probabilistic uncertainty models

- traditional stochastic models
- subjective probabilities and BAYES'ian approach

Non-probabilistic uncertainty models

- intervals
- convex models
- fuzzy sets

Mixed probabilistic/non-probabilistic uncertainty models

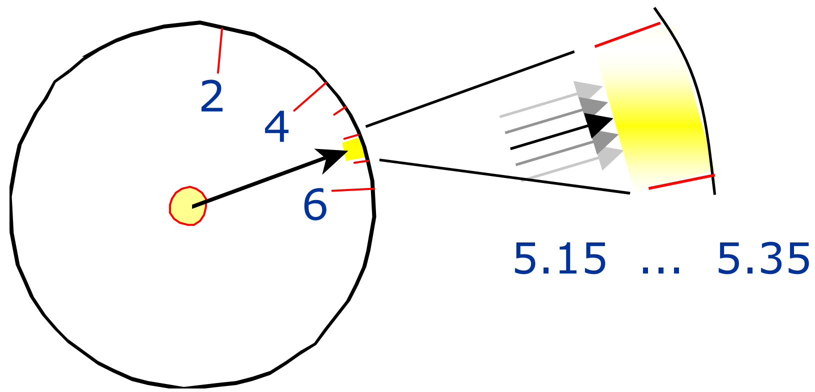
- interval probabilities
- sets of probabilities / p-box approach
- random sets
- fuzzy random quantities / fuzzy probabilities
- evidence theory
- imprecise probabilities

➡ appropriate model choice in each the particular case depending on the available information

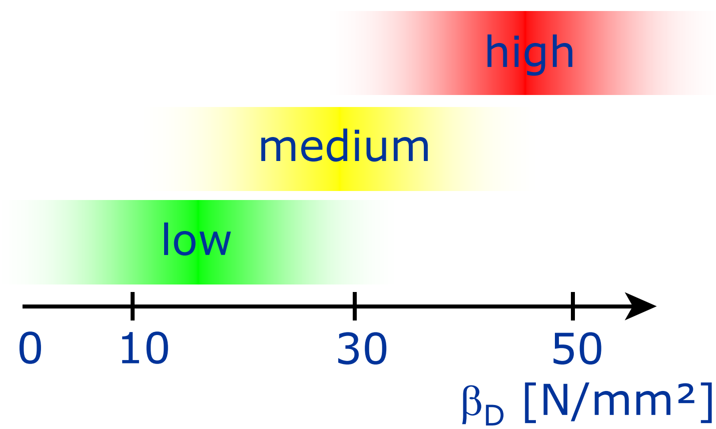
➡ application of different uncertainty models in parallel

Sources of imprecision

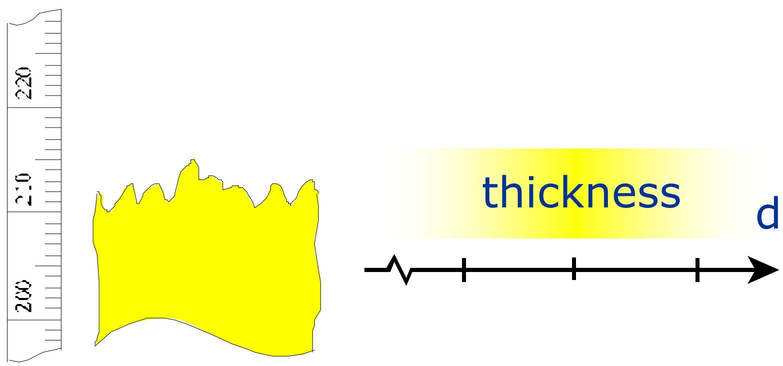
- imprecision of measuring devices



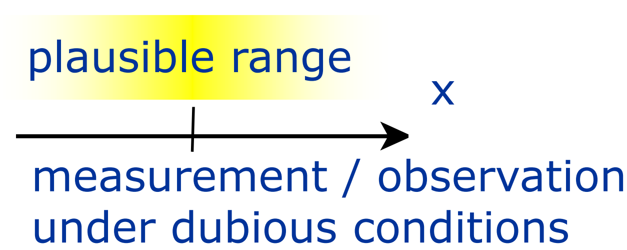
- linguistic assessments



- imprecise measuring points



- expert assessment / experience



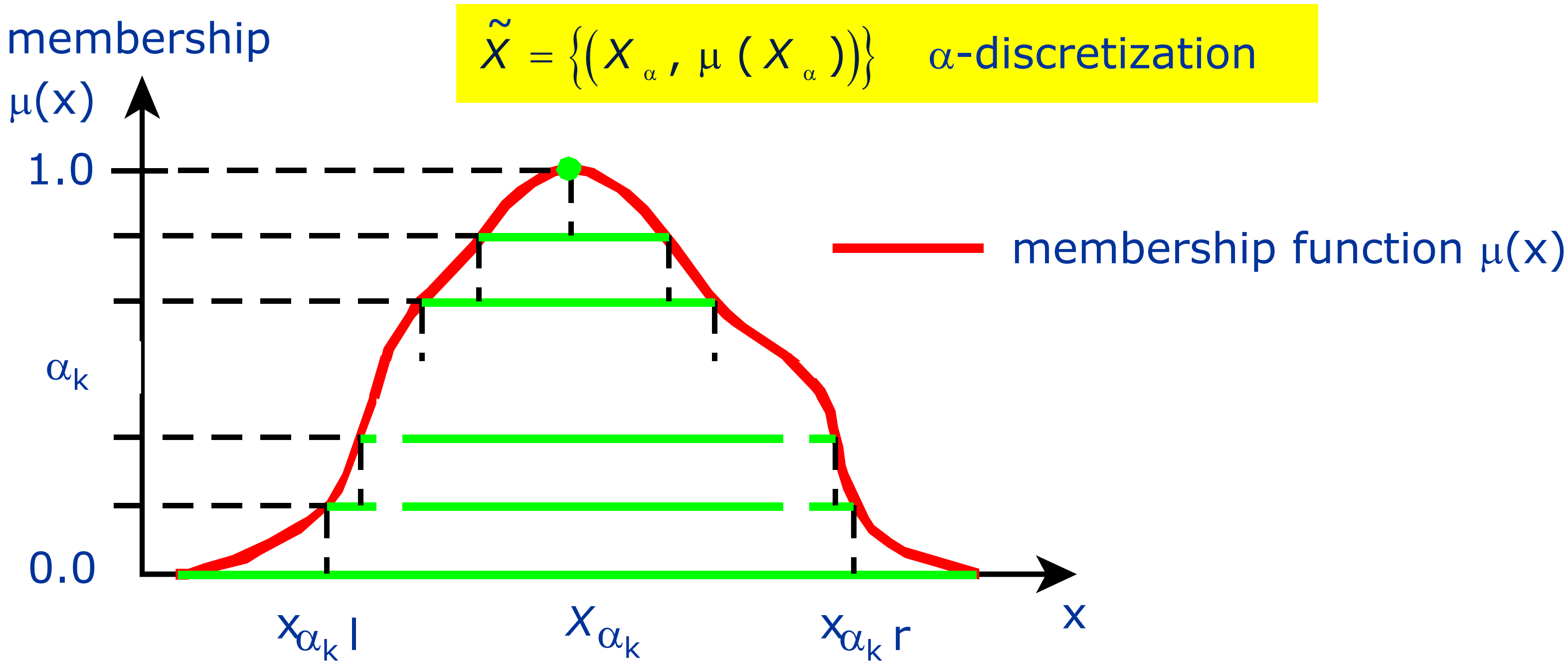
Single imprecise value

Fuzzy set

- $\tilde{X} = \{(x, \mu(x)) \mid x \in \mathbf{X}\}, \; \mu(x) \geq 0 \quad \forall x \in \mathbf{X}$

Numerical representation

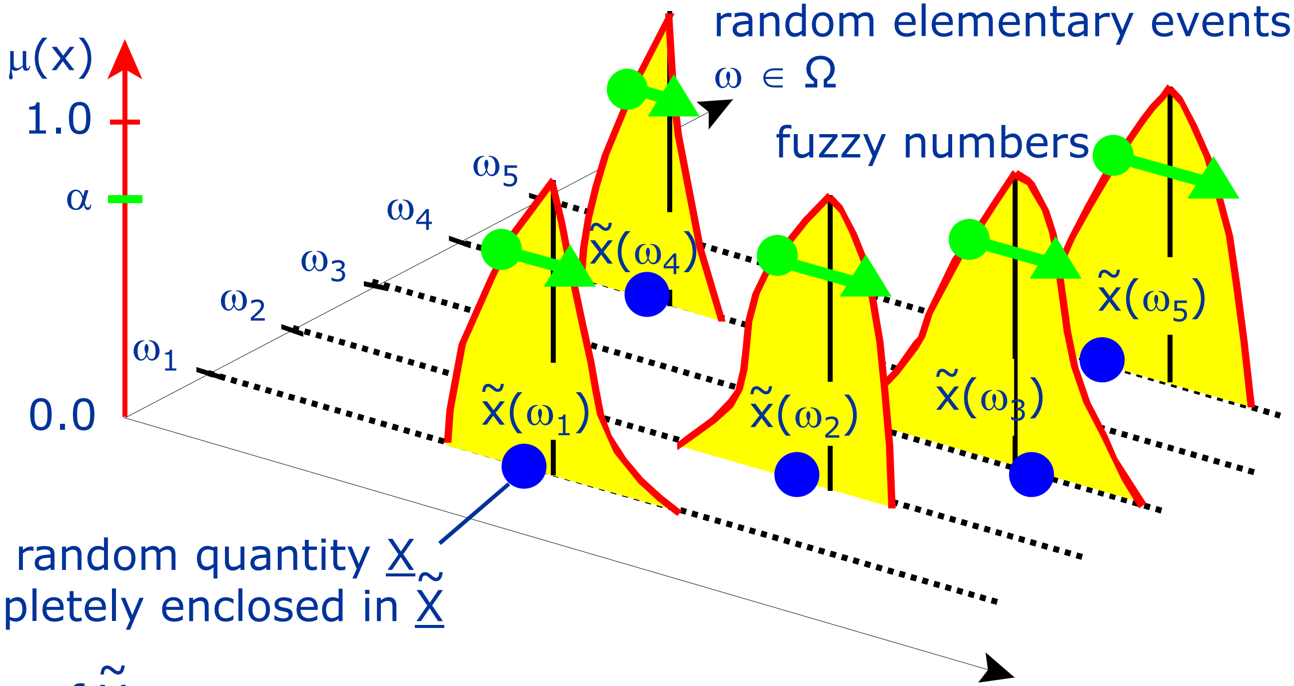
- α-level set $X_\alpha = \{ x \in \mathbf{X} \mid \mu(x) \geq \alpha \}$



Sample of imprecise values

Fuzzy random quantity

- fuzzy result of the mapping $\Omega \to F(\mathbb{R}^n)$



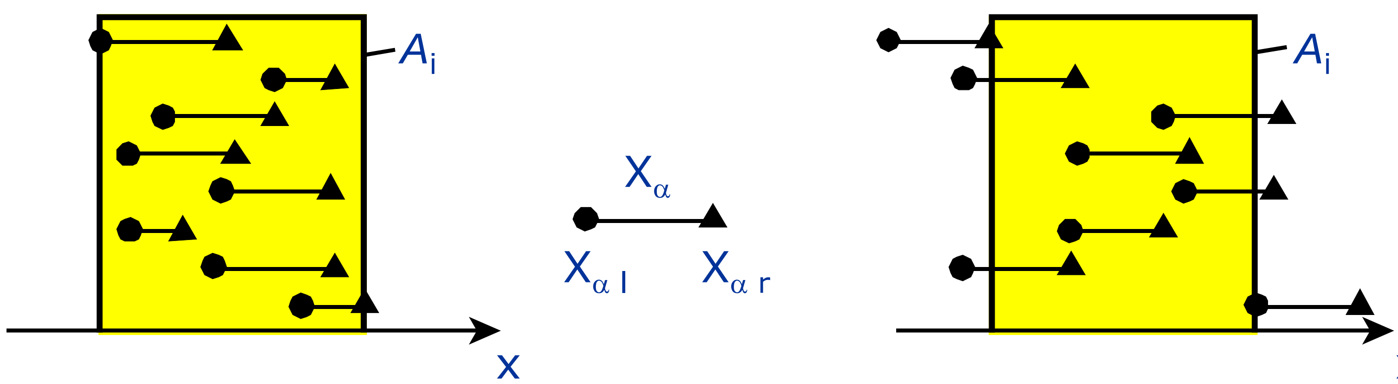
- original $\underline{X}_j$
  » real-valued random quantity $\underline{X}$ that is completely enclosed in $\underset{\sim}{\underline{X}}$
- representation of $\underset{\sim}{\underline{X}}$
  » fuzzy set of all possible originals $\underline{X}_j$
- α-discretization ➡ random α-level sets X_α

Sample of imprecise values

Fuzzy probability

- evaluation of all random α-level sets $\underline{X}_\alpha$
- $\tilde{P}(\underline{A}_i) = \left\{\left(P_\alpha(\underline{A}_i);\ \mu(P_\alpha(\underline{A}_i))\right)\right\}$
- $P_\alpha(\underline{A}_i) = \left[P_{\alpha\,l}(\underline{A}_i);\ P_{\alpha\,r}(\underline{A}_i)\right]\ ; \quad \mu(P_\alpha(\underline{A}_i)) = \alpha \ \forall\ \alpha \in (0;\ 1]$

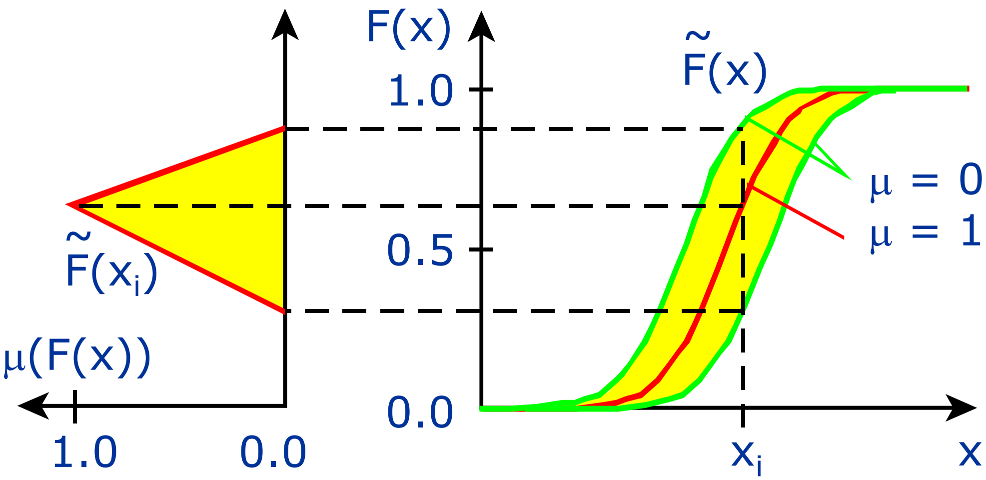
$P_{\alpha\,l}(\underline{A}_i) = P(\underline{X}_\alpha \subseteq \underline{A}_i)$ $\qquad$ $P_{\alpha\,r}(\underline{A}_i) = P(\underline{X}_\alpha \cap \underline{A}_i \neq \emptyset)$

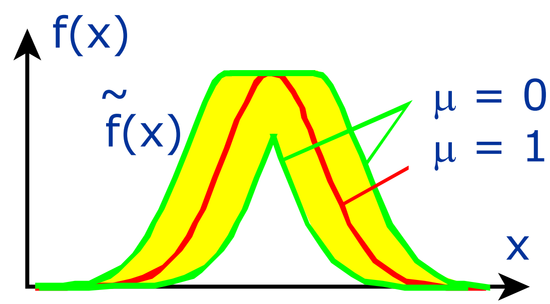


Sample of imprecise values

Fuzzy probability distribution function $\tilde{F}(\underline{x})$

- bunch of the $F_j(\underline{x})$ of the originals $\underline{X}_j$ of $\tilde{\underline{X}}$
  - » α-discretization
  - » $\tilde{F}(\underline{x}) = \left\{ \left(F_\alpha(\underline{x}),\ \mu(F_\alpha(\underline{x})) \right) \right\}$
  - » $F_\alpha(\underline{x}) = \left[F_{\alpha\,l}(\underline{x}),\ F_{\alpha\,r}(\underline{x}) \right]$, $\mu(F_\alpha(\underline{x})) = \alpha \ \forall\ \alpha \in (0,\, 1]$
  - » $F_{\alpha\,l}(\underline{x}) = 1 - \max_j \ P\left(\underline{X}_{j,\alpha} = \underline{t} \ \middle|\ \begin{array}{l} \underline{x},\, \underline{t} \in \mathbf{\underline{X}} = \mathbb{R}^n \\ \exists\ t_k \geq x_k,\ 1 \leq k \leq n \end{array} \right)$
  - » $F_{\alpha\,r}(\underline{x}) = \max_j \ P\left(\underline{X}_{j,\alpha} = \underline{t} \ \middle|\ \begin{array}{l} \underline{x},\, \underline{t} \in \mathbf{\underline{X}} = \mathbb{R}^n \\ t_k < x_k,\ k = 1,\, \ldots,\, n \end{array} \right)$





$\tilde{F}(\underline{x})$ with fuzzy parameters and fuzzy functional type

Quantification of uncertainty

General concept

- exploitation of statistical information
- realistic consideration of imprecision
- no mixing between statistical information and imprecision

Typical cases in engineering

- small sample size, expert knowledge
  » weak statistical information from estimations and tests
  ➡ utilization of statistical imprecision
  in the specification of fuzzy parameters and fuzzy distribution types
- imprecise sample elements
  » statistics with fuzzy quantities
  ➡ utilization of fuzzy arithmetic in statistical estimations and tests
- inconsistent environmental conditions, expert knowledge
  » critical conditions for statistical estimations and tests
  ➡ separation of fuzziness and randomness by constructing groups

Example I

Small sample size, expert knowledge

- measurement of the compressive strength of concrete
  - » 20 sample elements for $x = f_c$ [N/mm²]

    28.3, 26.8, 31.5, 35.3, 35.2, 26.3, 29.8, 23.1, 27.6, 20.2
    30.7, 29.2, 25.2, 25.7, 34.6, 34.2, 28.9, 24.8, 19.2, 22.8

- expert knowledge
  - » distribution type
    - normal distribution
  - » choice of estimator
    - sample mean for m_X
    - sample variance for σ_X^2
  - » construction of confidence intervals (type and level)
    - both-sided
    - levels: $\gamma = 0.50, 0.75, 0.90, 0.99$
  - » assignment of membership degrees to confidence levels
    - point estimation – $\mu = 1.0$
    - $\gamma = 0.50 - \mu = 0.75$, $\gamma = 0.75 - \mu = 0.50$
      $\gamma = 0.75 - \mu = 0.25$, $\gamma = 0.99 - \mu = 0.00$
  - » subsequent modification of the initial draft of the membership functions

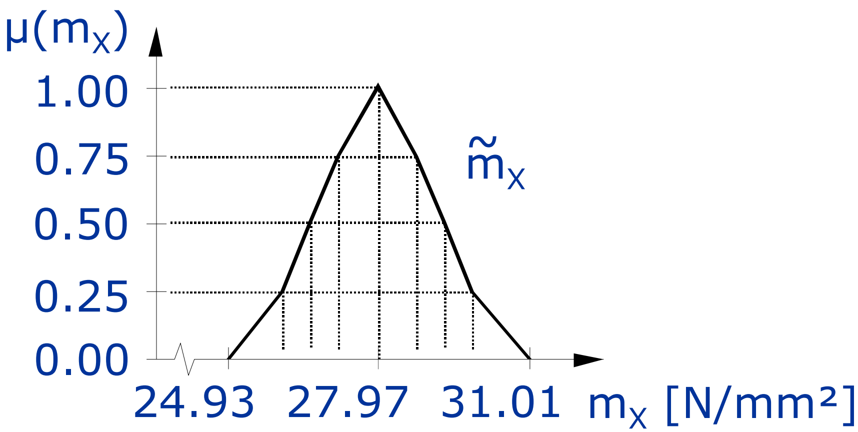
Example I

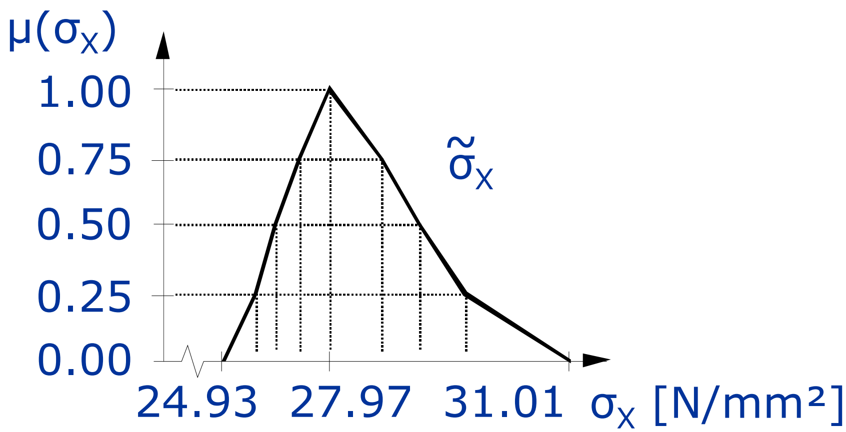
Small sample size, expert knowledge

- statistical estimation

	confidence level γ	expected value m_X [N/mm²]	standard deviation σ_X [N/mm²]
point estimation	–	27.97	4.75
interval estimation	0.50	[27.24, 28.70]	[4.35, 5.43]
	0.75	[26.71, 29.23]	[4.05, 5.92]
	0.90	[26.13, 29.81]	[3.77, 6.52]
	0.99	[24.93, 31.01]	[3.34, 7.92]

- construction of membership functions





Example II

Imprecise sample elements

- measurement of the compressive strength of concrete
  - » 20 sample elements for $x = f_c$ [N/mm²]
    - imprecision due to individual care and readings in the tests
    - measurements modeled with fuzzy triangular numbers

<26.3, 28.3, 30.3>, <24.8, 26.8, 28.8>, <29.5, 31.5, 33.5>,
<33.3, 35.3, 37.3>, <33.2, 35.2, 37.2>, <24.3, 26.3, 28.3>,
<27.8, 29.8, 31.8>, <21.1, 23.1, 25.1>, <25.6, 27.6, 29.6>,
<18.2, 20.2, 22.2>, <28.7, 30.7, 32.7>, <27.2, 29.2, 31.2>,
<23.2, 25.2, 27.2>, <23.7, 25.7, 27.7>, <32.6, 34.6, 36.6>,
<32.2, 34.2, 36.2>, <26.9, 28.9, 30.9>, <22.8, 24.8, 26.8>,
<17.2, 19.2, 21.2>, <20.8, 22.8, 24.8>

- statistical evaluation
  - » distribution type: normal distribution (expert knowledge)
  - » application of estimators to fuzzy sample elements

$$\tilde{\tilde{x}} = \frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i \qquad \tilde{s}_x^2 = \frac{1}{n-1}\left[\sum_{i=1}^{n}\left(\tilde{x}_i\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n}\tilde{x}_i\right)^2\right]$$

$$\underline{x} = \left(x_1, \ldots, x_n\right) \in \underline{\tilde{x}} = \left(\tilde{x}_1, \ldots, \tilde{x}_n\right) \Rightarrow \left(x, s_x\right) \in \left(\tilde{\tilde{x}}, \tilde{s}_x\right)$$

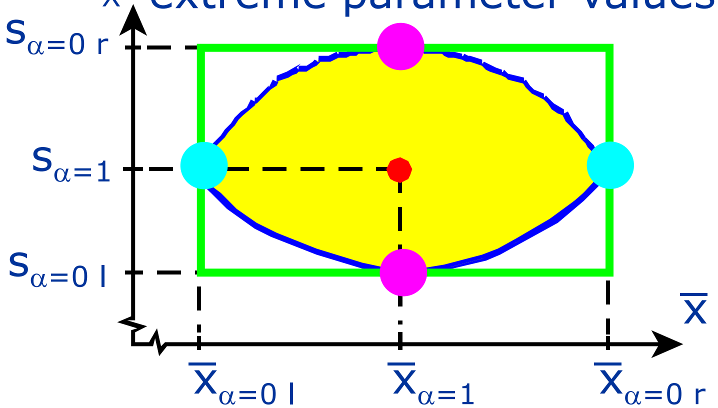
interaction between $\tilde{\tilde{x}}$ and $\tilde{s}_x$!

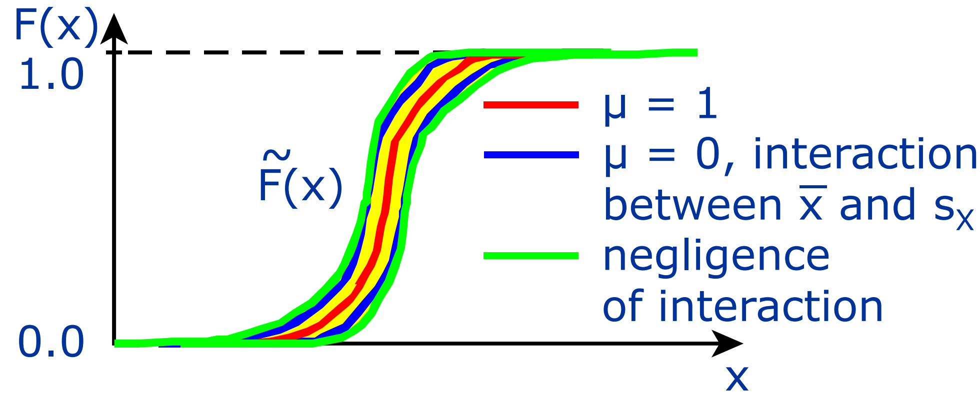
Example II

Imprecise sample elements

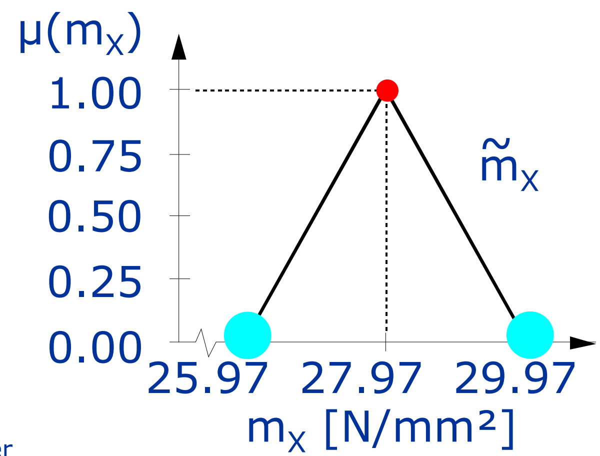
- numerical evaluation of statistical estimations

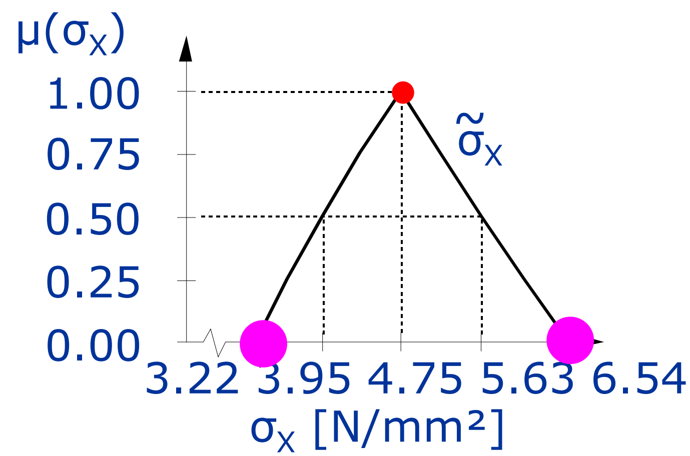
extreme parameter values



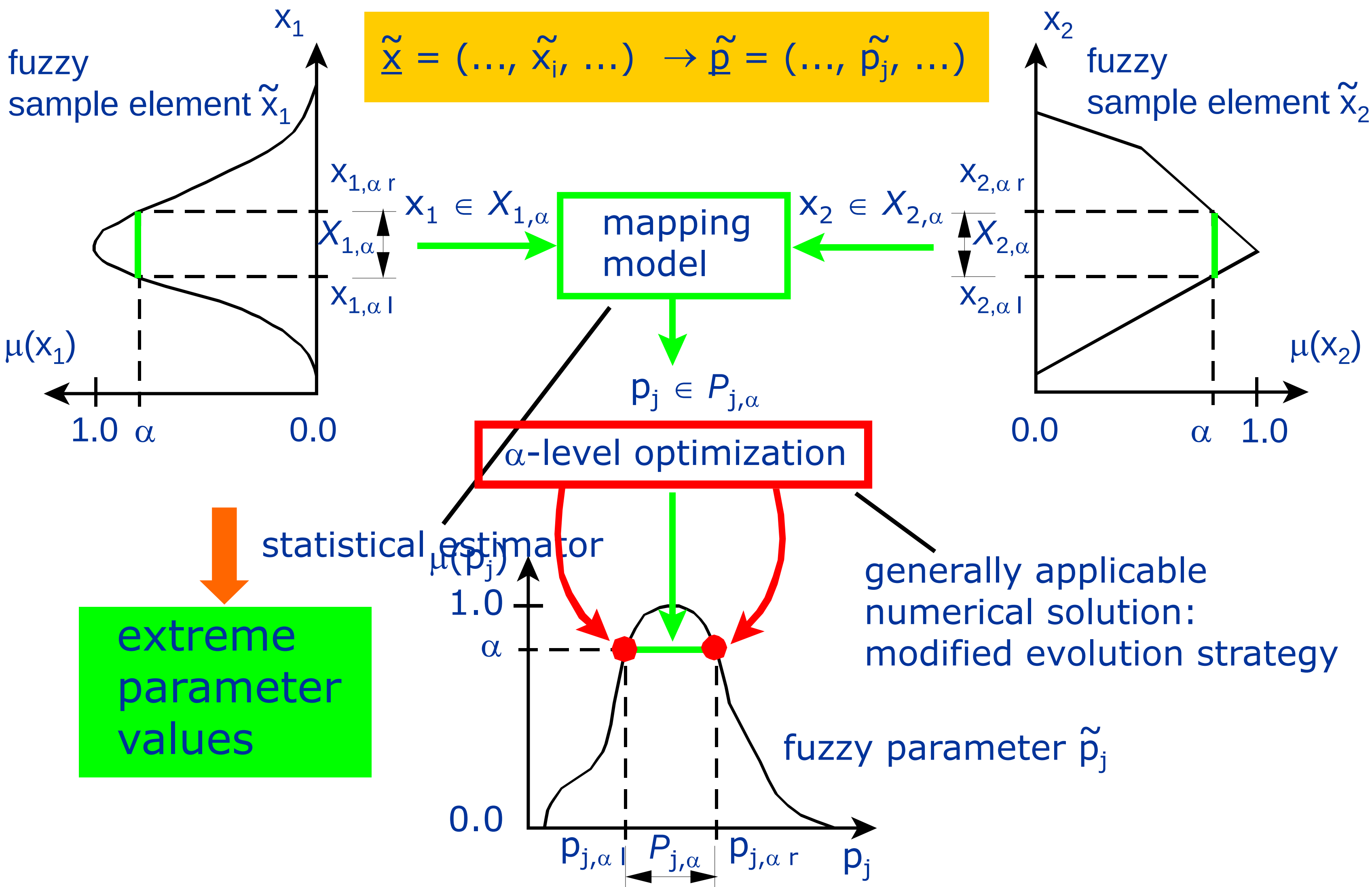


- construction of membership functions from extreme values for all α-levels





Fuzzy analysis



Example III

Inconsistent environmental conditions, expert knowledge

- measurement of the compressive strength of concrete
  - » 620 sample elements for $x = f_c$ [N/mm²]
  - » sample generation under varying environmental conditions
    - ▪ different manufacturers
    - ▪ different aggregates / additives (different suppliers)
    - ▪ different hardening conditions (temperature, humidity)
    - ▪ different motivation of personnel

- expert knowledge
  - » classify sample elements with respect to their attributes (conditions)
  - » determine groups of sample elements with same attributes

- quantification options
  - » parametric quantification
    - ▪ distribution assumption from expert knowledge
  - » non-parametric quantification
    - ▪ use of empirical distribution functions

Example III

Inconsistent environmental conditions, expert knowledge

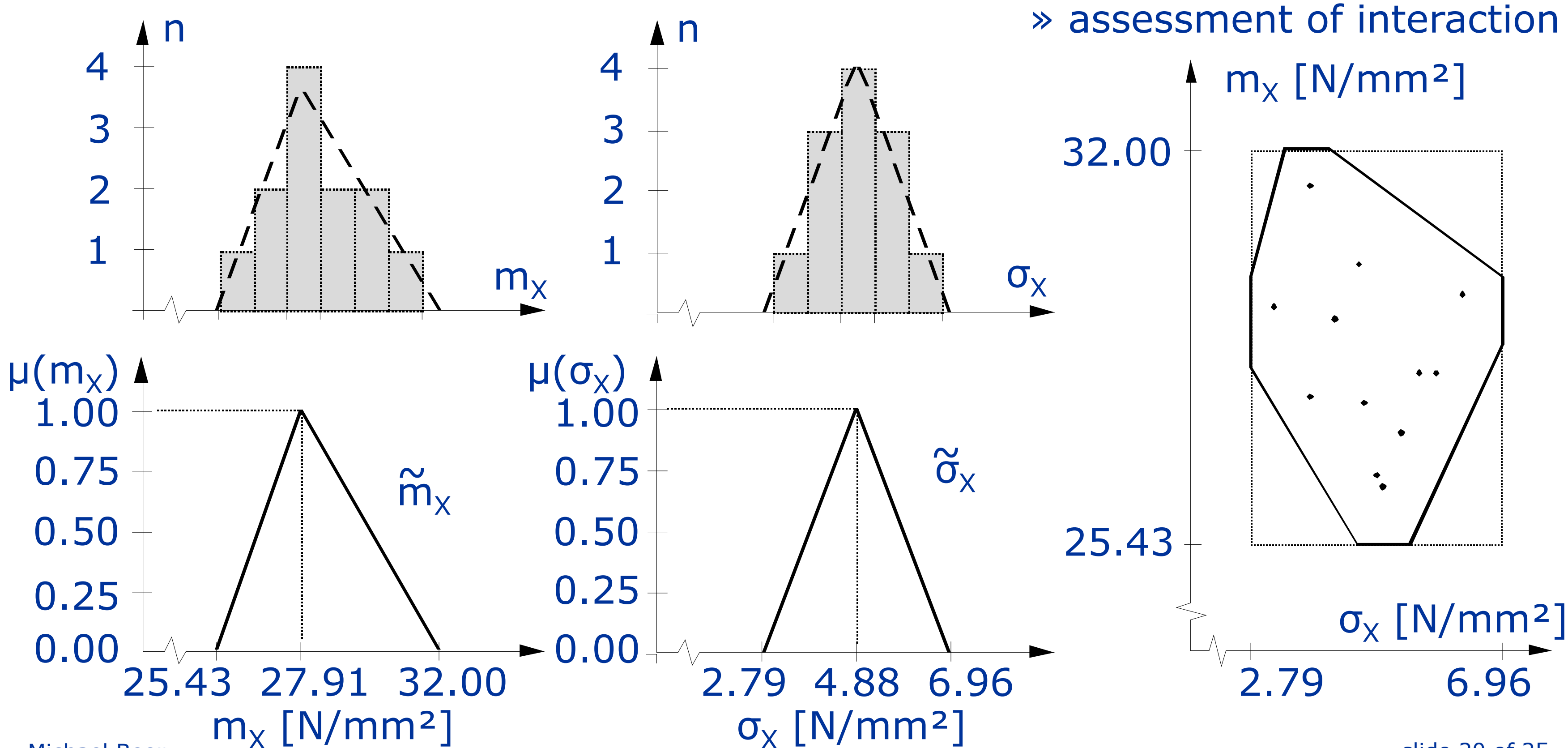
- option a) – parametric quantification
  - » distribution type for each group
    - normal distribution
  - » choice of estimator and point / interval estimation for each group
    - point estimation
    - sample mean for m_X
    - sample variance for σ_X^2

group number	sample size	m_X [N/mm²]	σ_X [N/mm²]	group number	sample size	m_X [N/mm²]	σ_X [N/mm²]
1	54	27.3	5.3	7	55	26.4	5.0
2	48	26.6	4.9	8	47	30.1	4.6
3	42	29.2	4.2	9	64	28.3	5.9
4	38	31.4	3.8	10	53	27.9	3.8
5	44	28.3	5.6	11	75	29.6	6.3
6	48	29.4	3.2	12	52	27.8	4.7

Example III

Inconsistent environmental conditions, expert knowledge

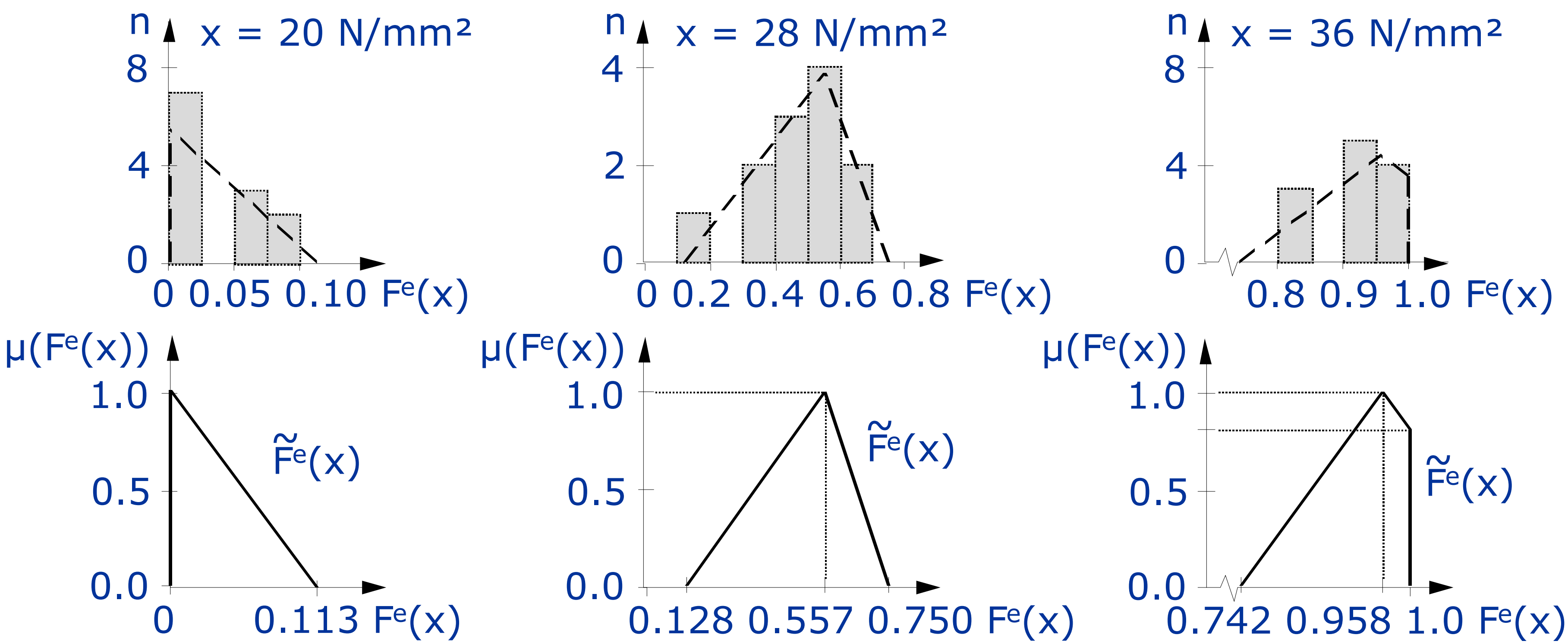
- option a) – parametric quantification
  - » histograms for parameters for all groups
  - » construction of membership functions for the parameters
  - » assessment of interaction



Example III

Inconsistent environmental conditions, expert knowledge

- option b) – non-parametric quantification
  - » construction of empirical distributions $F_i^e(x)$ for each group i
  - » histograms for $F^e(x)$ for selected values x for all groups
  - » construction of membership functions for the $F^e(x)$ for all selected x



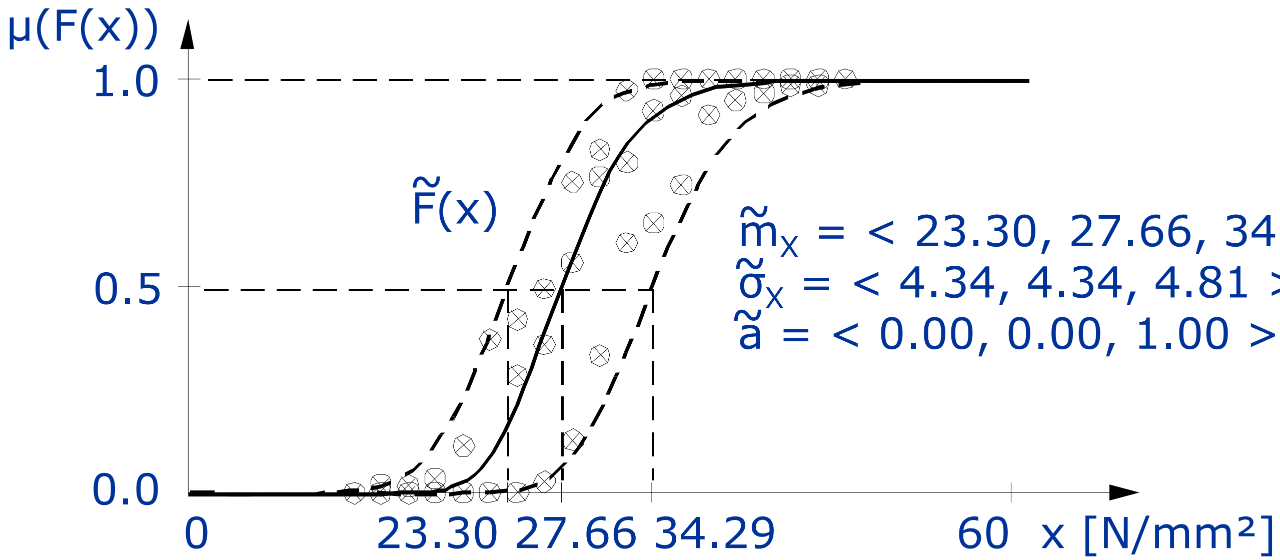
Example III

Inconsistent environmental conditions, expert knowledge

- option b) – non-parametric quantification
  - » determination of bounding distributions $F_{\alpha\, l}(x)$ and $F_{\alpha\, r}(x)$ for all α-levels
    - ▪ assumption of compound distribution – normal / logarithmic normal

$$\tilde{F}(x) = \tilde{a} \cdot \tilde{F}^{ND}(x) + (1 - \tilde{a}) \cdot \tilde{F}^{LND}(x)$$

    - ▪ least squares algorithm with bounding condition $F_{\alpha\, l}(x) \le F^{e}(x) \le F_{\alpha\, r}(x)$



$\tilde{m}_X$ = < 23.30, 27.66, 34.29 > N/mm²
$\tilde{\sigma}_X$ = < 4.34, 4.34, 4.81 > N/mm²
$\tilde{a}$ = < 0.00, 0.00, 1.00 >

Fuzzy stochastic structural analysis

$$\{\underline{x}(\underline{\theta}, t),\ \tilde{\underline{x}}(\underline{\theta}, t),\ \underline{X}(\underline{\theta}, t),\ \tilde{\underline{X}}(\underline{\theta}, t)\} \rightarrow \{\tilde{\underline{Z}}(\underline{\theta}, t),\ \tilde{P}_f(t)\}$$

deterministic parameters

fuzzy quantities

random quantities

fuzzy random quantities

fuzzy stochastic structural responses

fuzzy safety level

- depending on temporal and spatial coordinates

Numerical algorithm

coupling of ARBITRARY algorithms for

- fuzzy analysis
- deterministic structural analysis
- stochastic structural analysis or safety assessment

→ worst and best case results in terms of probability

Fuzzy stochastic structural analysis

Capabilities and performance features

- vague and imprecise statistical information, expert knowledge

- generally applicable,
  coupling of arbitrary algorithms for
  fuzzy analysis, deterministic structural analysis, and stochastic analysis
- simultaneous processing of
  random quantities, fuzzy quantities and fuzzy random quantities
- numerical effort $\leq$ cost(stochastic analysis) $\times$ cost(fuzzy analysis)
- applicable in combination with response surface approximations

- results reflect the uncertainty of distribution assumptions
- direct determination of worst and best case results in terms of probability
- qualitative information on sensitivities,
  in particular, with respect to the distribution assumptions

Resumé

Comprehensive evaluation of uncertainty

- high degree of flexibility in uncertainty quantification
  - » fuzzy random quantities / fuzzy probabilities
    - combination of traditional statistics with interval and fuzzy methods
    - appropriate uncertainty modeling in the particular situation
    - inclusion of subjective assessments
- high degree of generality in uncertainty processing
  - » processing of various uncertain quantities simultaneously
    - stochastic simulation
    - fuzzy analysis
    - fuzzy stochastic analysis

➡ adequate consideration of uncertainty in structural analysis, safety assessment and design

➡ complete reflection of the uncertainty in the computational results

➡ worst case analysis in terms of probability