Portfolio Management Under Epistemic Uncertainty Using Stochastic Dominance and Information-Gap Theory

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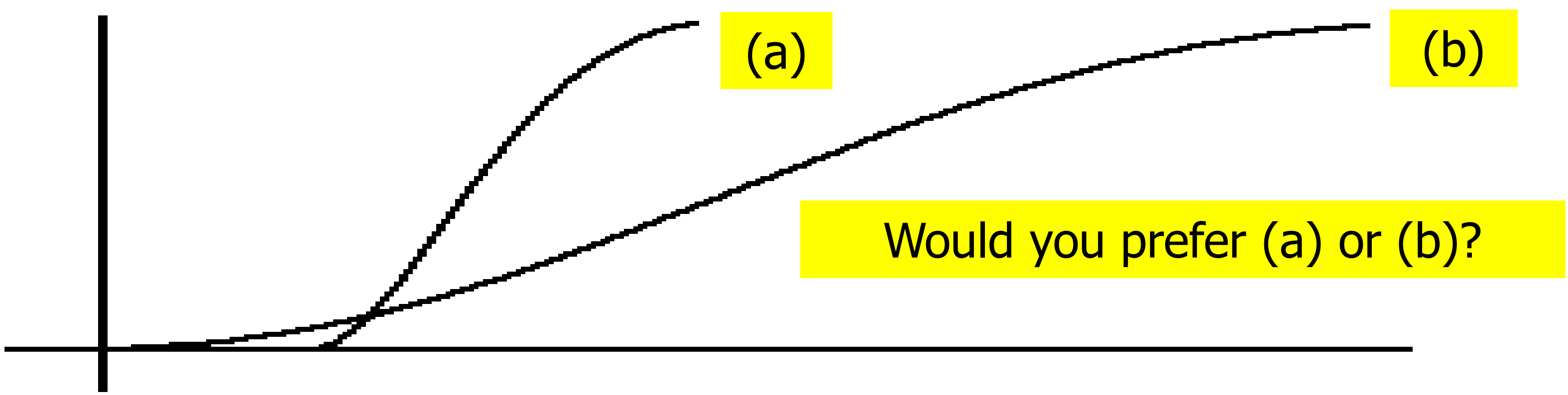
What is Portfolio Management (and why REC ’08)?

- A portfolio in finance is
  - A collection of assets
    - Stocks, bonds, electric energy futures, etc.
  - A big part of portfolio management is…
    - Figuring out what % of each asset to hold
- Financial Engineering
  - Using mathematics and computers to make investment decisions

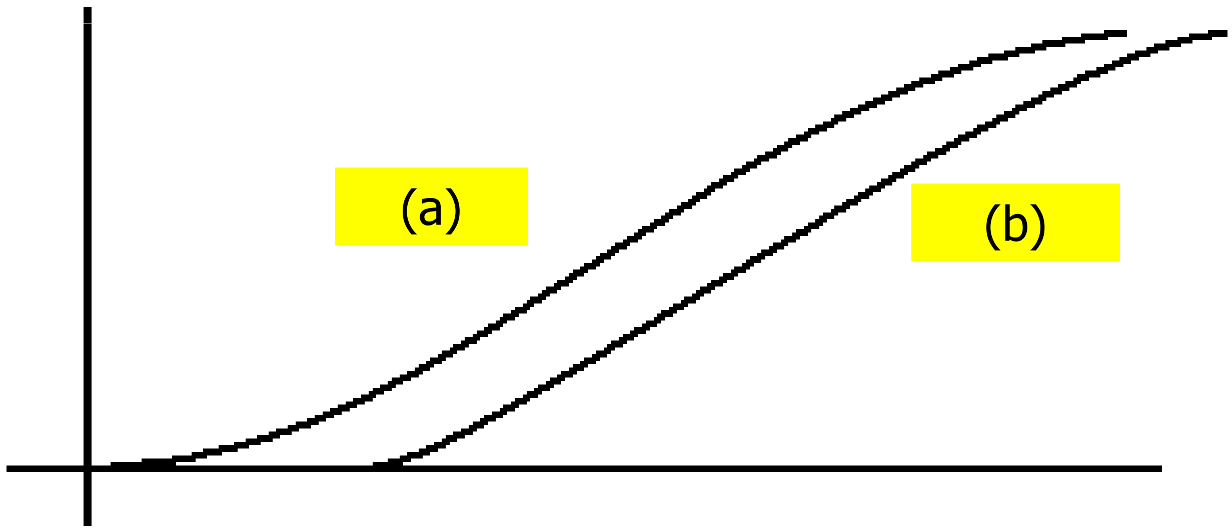
Portfolio Basics

- Let projected return be a distribution
- The usual goal is to balance
  - High expected return (mean, μ)
  - Low risk (variance, σ)
- The problem is these often conflict:



First Order Stochastic Dominance
…and portfolio selection

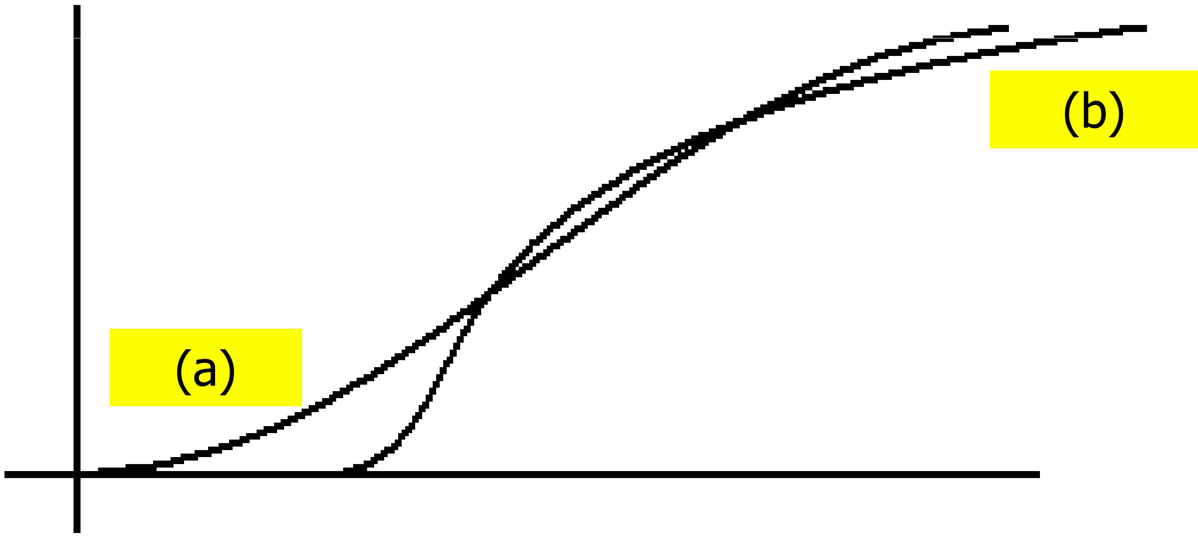
- (b) has **FSD** over (a)
  - Which would you choose now?



FSD is decisive for the rational investor

Second Order Stochastic Dominance
…and portfolio selection

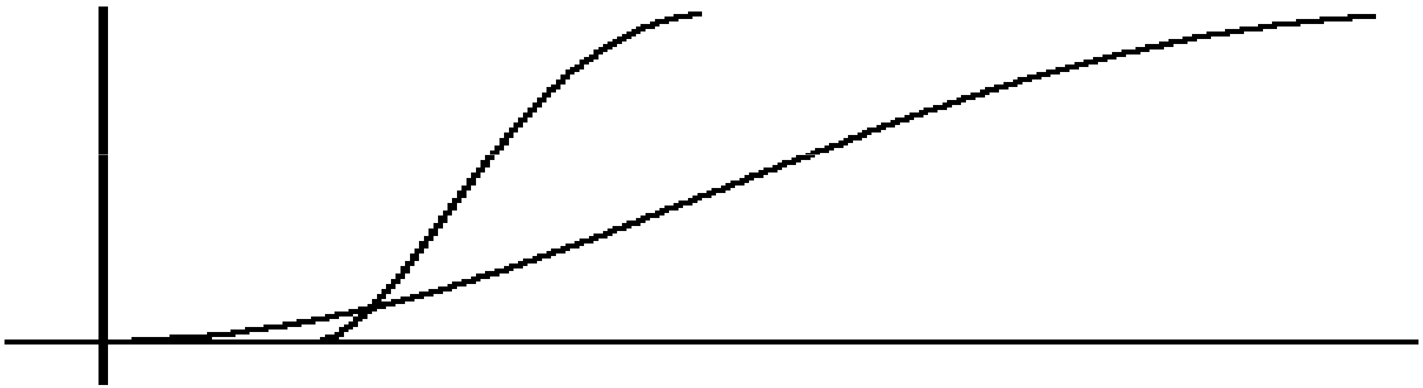
- **SSD**: integrals do not cross
- (b) has **SSD** over (a) … which is better?



SSD is decisive for the risk-averse investor

Choosing is not always easy

- Recall:



- If each curve models a different asset
  - A portfolio can be a weighted average
  - N assets, pick some of each

Portfolio Selection Problem

- r is the portfolio return distribution
- $\tilde{R}$ is a “reference” return distribution

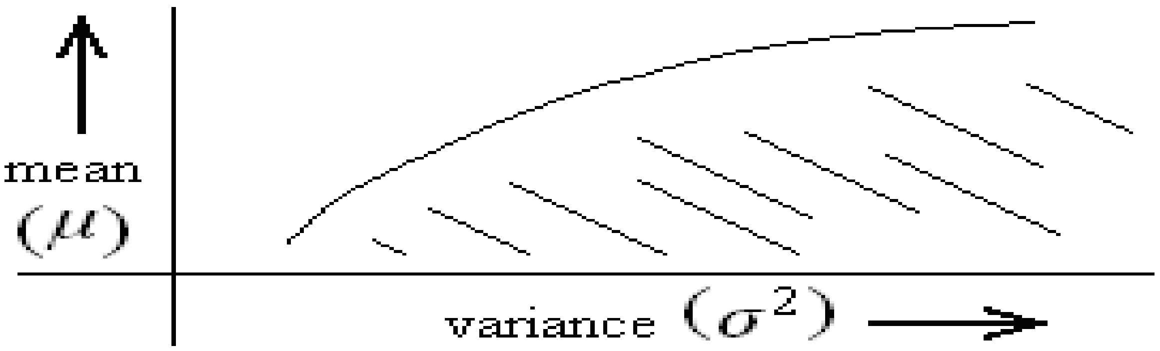
$$r = \sum_{i=1}^{s} w_i r_i \succ_2 \tilde{R}$$

$$\sum_{i=1}^{s} w_i = 1$$

The SSD constraint is not typical of current practice

Portfolio Selection II

- The *efficient frontier*
  - Contains the set of candidate portfolios
  - Due to Markowitz (1952)



$$\text{desirability} = f\,(z, r) = mean\,(r) - z * risk\,(r)$$

z is risk attitude (we assume risk aversion … $z > 0$)

Example

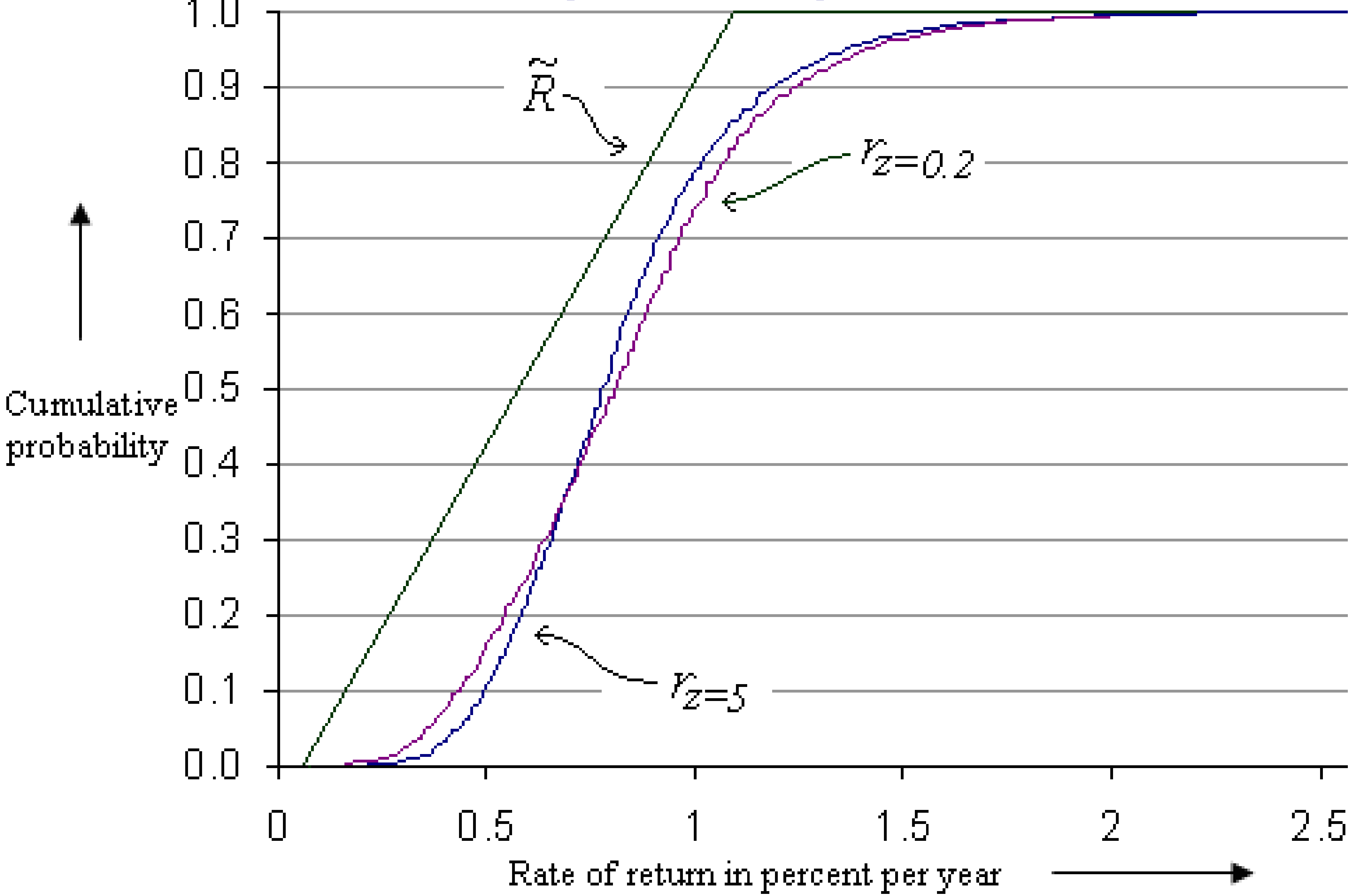
- 3 segment portfolio:

$$r_1 \sim Normal\ (1.1,\ 0.25) \qquad w_1 \in [0.2,\ 0.3]$$

$$r_2 \sim Exponent\ (1.0,\ 1.0) \qquad w_2 \in [0.4,\ 0.6]$$

$$r_3 \sim Uniform\ (1.2,\ 0.48) \qquad w_3 \in [0.2,\ 0.3]$$

Two optimal portfolios



We're not Done Yet

- If z was known, we would be happier
- But z is unknown
  - Therefore, more work is required
    - (if we are to find the right portfolio)
- By the way, $\sigma_y^2 = \sum_{i=1}^{s} \sum_{j=1}^{s} w_i w_j \sigma_{ij}$
  - So, in $mean(r) - z * risk(r)$
  - z is still a number.

Alternative Optimality Criteria

<table>
<tr><td colspan="2"></td><td colspan="2">Quality Metric</td></tr>
<tr><td colspan="2"></td><td>|SSD|</td><td>α (alpha)</td></tr>
<tr><td rowspan="4">Objective</td><td>Maximize Robustness (to achieve secure performance)</td><td>1. Find the portfolio(s) with the highest |SSD| over the reference curve, i.e., move to the right until further movement would disqualify every portfolio.</td><td>3. Find a portfolio with SSD over the reference curve, and has the highest possible α.</td></tr>
<tr><td rowspan="3">Maximize μ (to achieve best performance within the risk limit)</td><td rowspan="3">2. Find a portfolio y with return distribution r_y and SSD over the reference curve, choosing one with the highest possible mean return μ.</td><td>4a. Find a portfolio with the highest mean return μ from among those with SSD over the reference curve for any dependency relationships among segments.</td></tr>
<tr><td>4b. Generalize 4a by requiring SSD for only some dependencies. The precise meaning of “some” is determined by the value of α.</td></tr>
<tr><td>4c. Find the demand value of information about α in order to choose what value of α to use in 4b.</td></tr>
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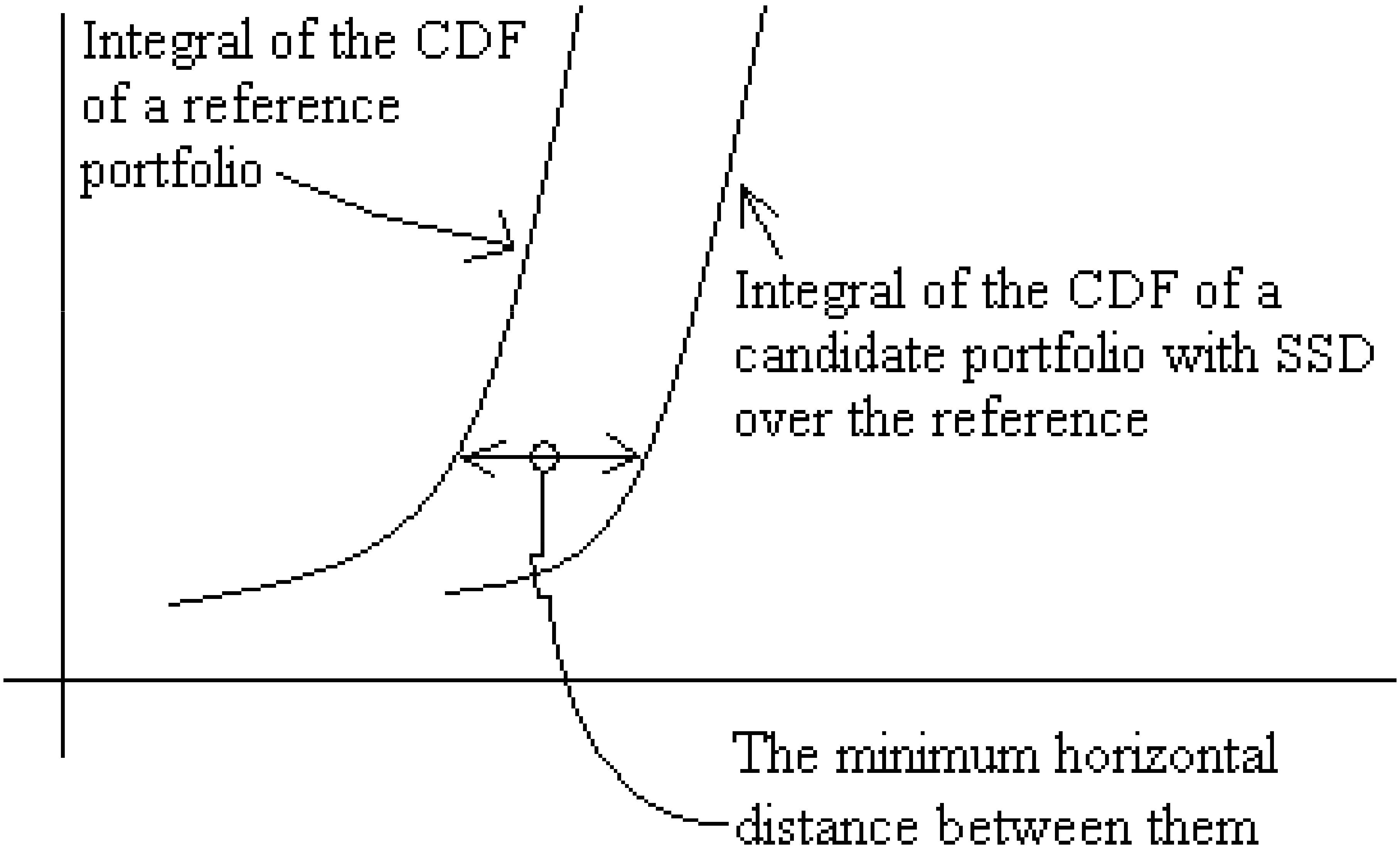
1. Maximize robustness Measure with |SSD|

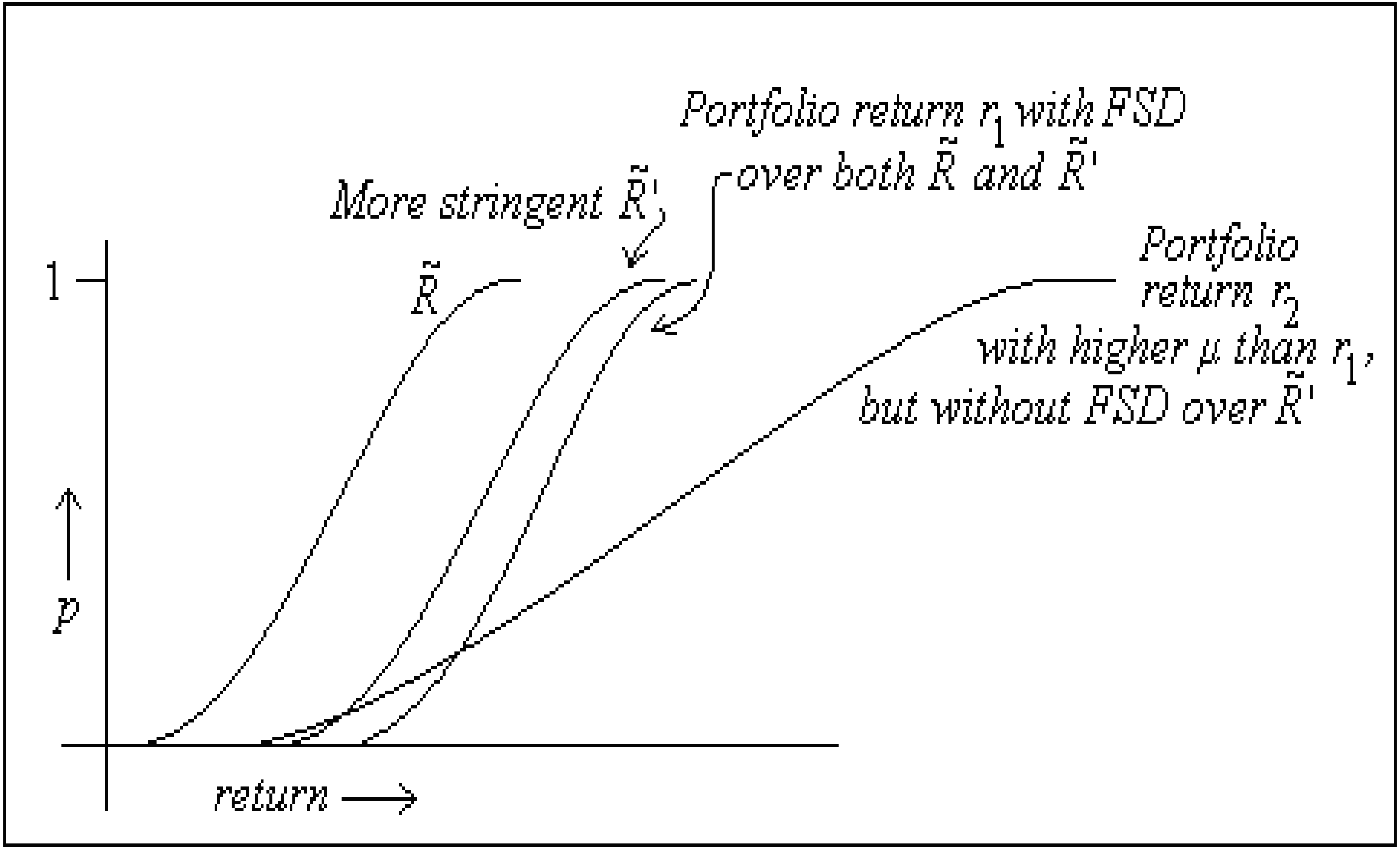


Results

- Maximizes chances that SSD indeed holds
- Does <u>not</u> (necessarily) maximize mean μ

z	|SSD|	μ
0.2	0.2662	1.0800
1	0.2662	1.0800
2	0.2841	1.0740
3	0.2872	1.0717
4	**0.2886**	1.0705
5	0.2864	1.0700

2. Maximize mean μ Demand FSD/SSD over $\tilde{R}$



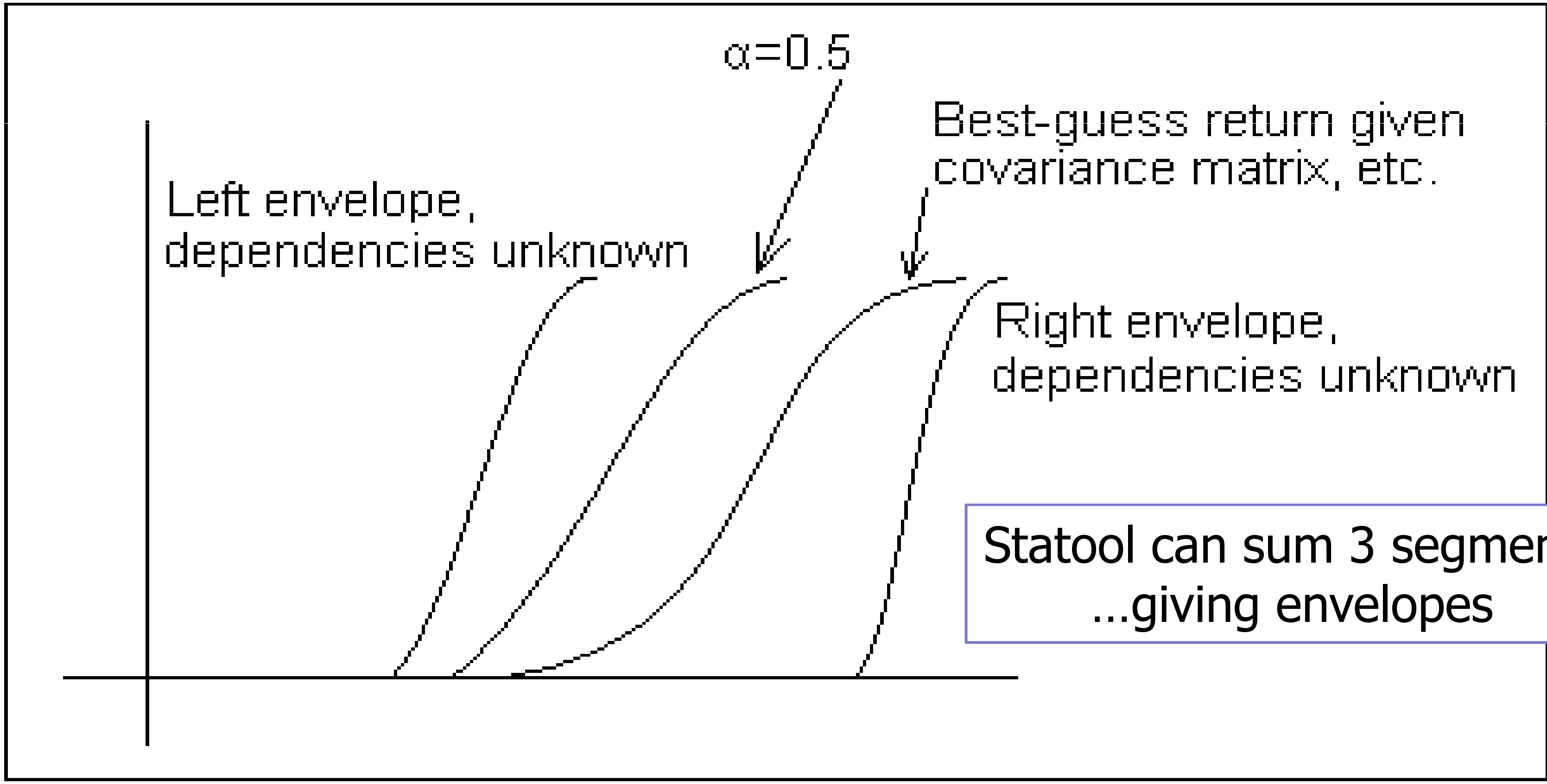
Results

- Maximizes expected return
- Can respond paradoxically to different $\tilde{R}$

z	|SSD|	μ
0.2	0.2662	1.0800
1	0.2662	1.0800
2	0.2841	1.0740
3	0.2872	1.0717
4	**0.2886**	1.0705
5	0.2864	1.0700

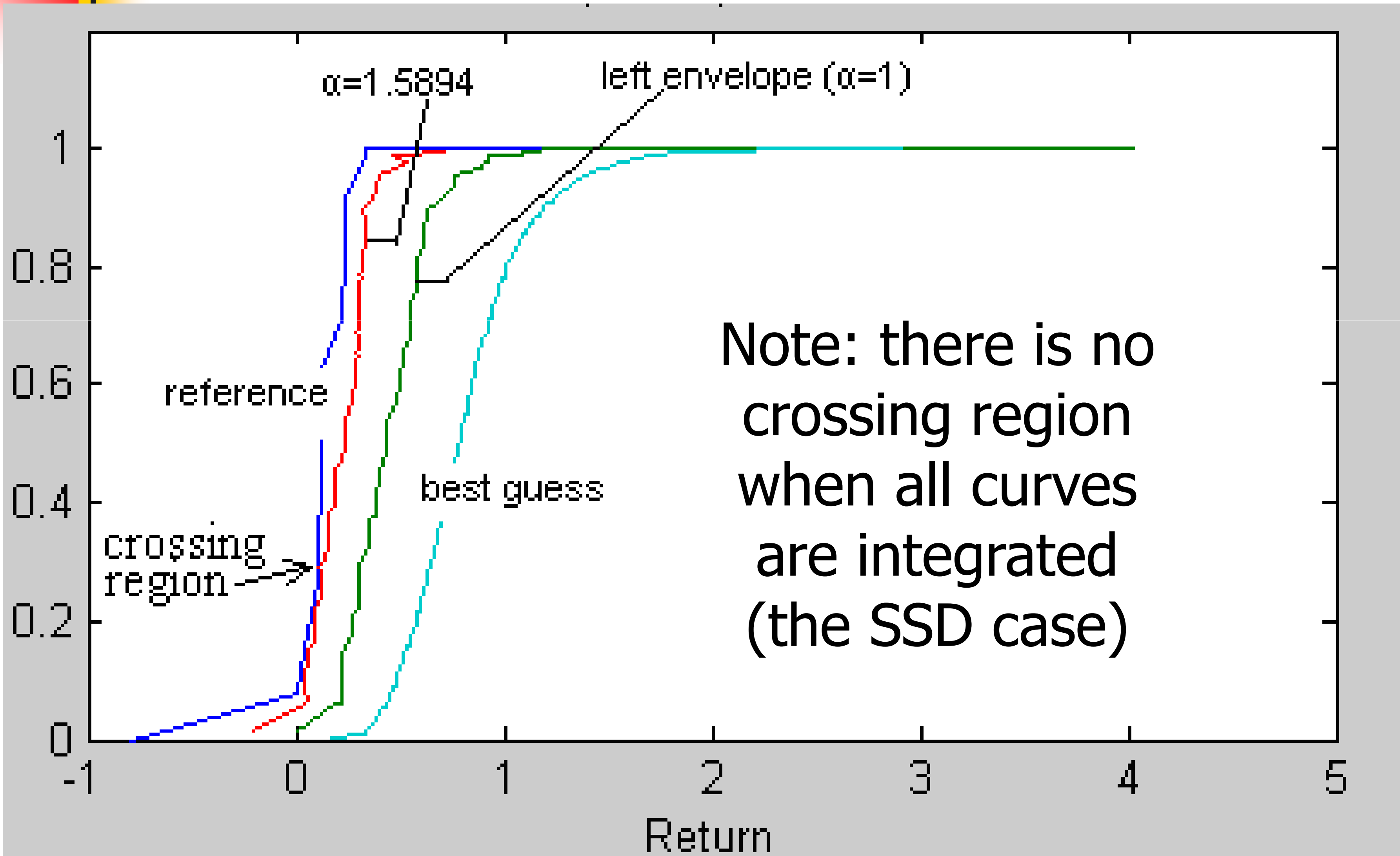
3. Maximize robustness Measure with α (alpha)

- α is an Info Gap Theory parameter
  - it describes amount of uncertainty



Statool can sum 3 segments
...giving envelopes

Example

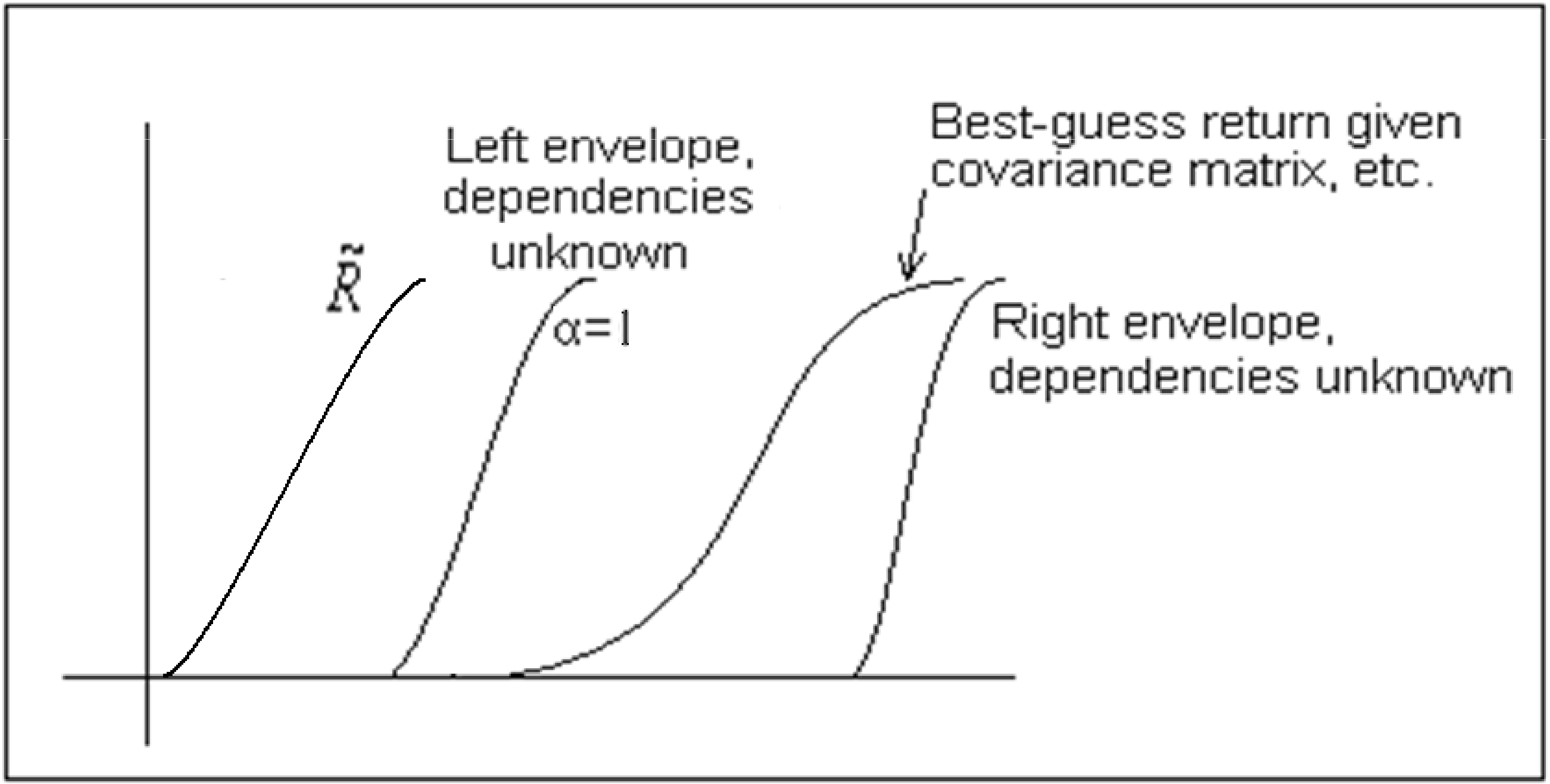

α=1.5894
left envelope (α=1)
reference
best guess
crossing region
Note: there is no crossing region when all curves are integrated (the SSD case)
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
Return

Results

z	**max** α	μ
0.20	1.33	1.0800
1.00	1.33	1.0800
2.00	1.3380	1.0740
3.00	1.3410	1.0718
3.90	1.3408	1.0717
3.96	1.371	1.0706
3.97	**1.373**	1.0706
3.98	**1.373**	1.0706
3.99	1.371	1.0706
4.00	1.3700	1.0715
4.01	1.3680	1.0705
4.10	1.362	1.0705
5.00	1.36	1.0700

4a. Maximize mean μ Require α=1 (and FSD/SSD)

- Makes no assumption about dependency



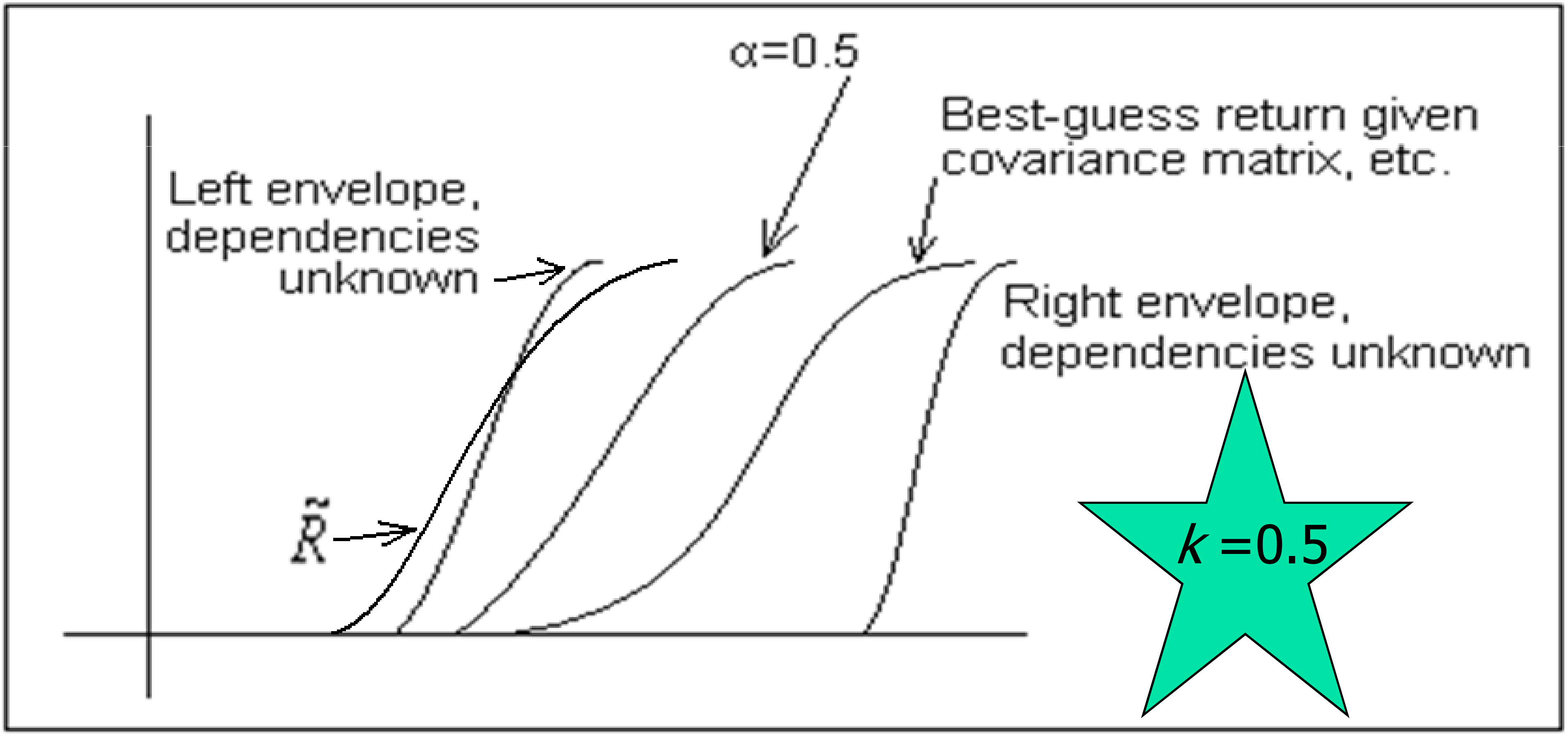
Results

z	**maximum** α	μ
$z = 0.2$	1.3500	**1.0800**
$z = 1$	1.3500	**1.0800**
$z = 2$	1.5300	1.0740
$z = 3$	1.5894	1.0717
$z = 4$	1.5500	1.0705
$z = 5$	1.4000	1.0700

All portfolios shown qualify (α >=1)

4b. Maximize mean μ Require $\alpha = k$ (and FSD/SSD)

- Parametrizes deviation from best guess

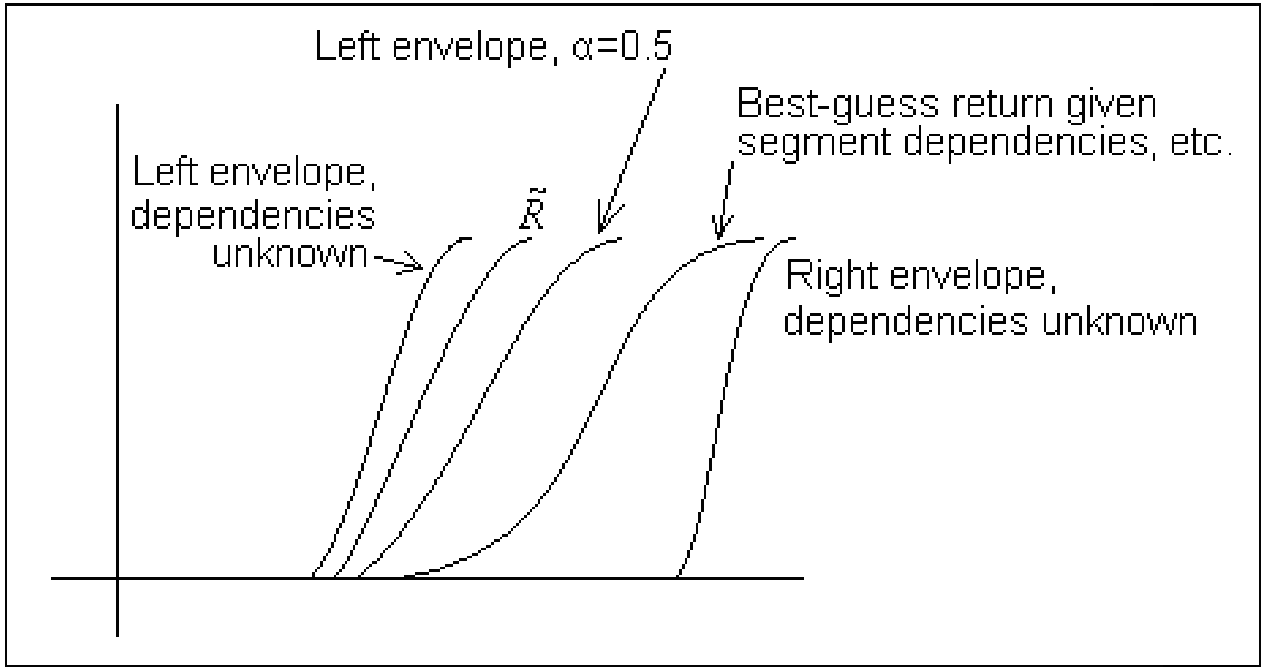


Results (for α = **1.50** not **0.50**)

α = **1.50**		μ
z	|SSD|	
0.2	Negative	1.08
1	Negative	1.08
2	**Positive**	**1.074**
3	**Positive**	1.0717
4	**Positive**	1.0705
5	Negative	1.07

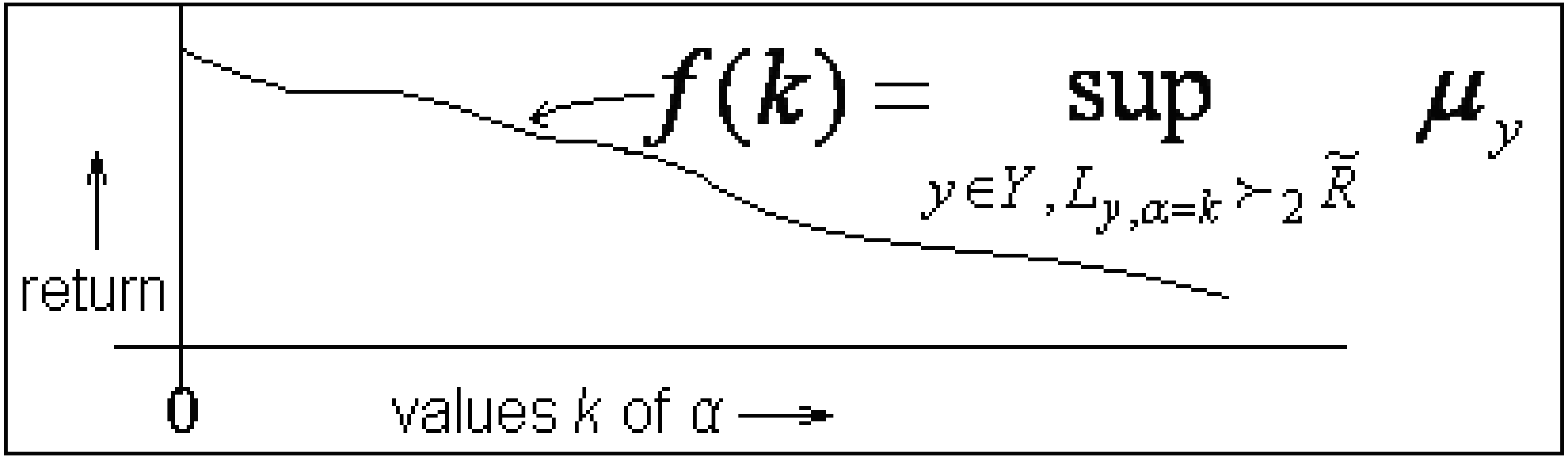
4c. Paying to reduce α

- Reducing α qualifies more portfolios
- More portfolios tends to raise maximum μ
- How much is it worth to reduce α?



Paying to reduce α (cont.)

- Information that reduces α...
  - ...from $k = \alpha_1$ to $k = \alpha_2$...
  - ...is worth paying $f(\alpha_1) - f(\alpha_2)$ for



Conclusion

- SSD and Info Gap Theory give different results
  - Of course – they generate different models
- But both apply when
  - Correlations are imperfectly known
  - Distribution shapes are imperfectly known
- In this domain as in others:
  - Severe uncertainty may be rationally addressed