How to Estimate, Take Into Account, and Improve Travel Time Reliability in Transportation Networks

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1. Decreasing Traffic Congestion: Formulation of the Problem

- *Practical problem:* decreasing traffic congestion.
- *Important difficulty:* a new road can worsen traffic congestion.
- *Conclusion:* importance of the preliminary analysis of the results of road expansion.
- *Traditional approach* assumes that we know:
  - the exact amount of traffic going from zone A to zone B (*OD-matrix*), and
  - the exact capacity of each road segment.
- *Limitations:* in practice, we only know all this with uncertainty.
- *What we do:* we show how to take this uncertainty into account in traffic simulations.

2. Traffic Assignment: Brief Reminder

- *Traffic demand:* # of drivers d_{ij} who need to go from zone i to zone j – *origin-to-destination* (O-D) matrix.
- *Capacity* of a road link – the number c of cars per hour which can pass through this link.
- *Travel time along a link*: $t = t^f \cdot \left[1 + a \cdot \left(\frac{v}{c}\right)^\beta\right]$, where:
  - $t^f = L/s$ is a *free-flow* time (s is the speed limit),
  - $a \approx 0.15$ and $\beta \approx 4$ are empirical constants.
- *Equilibrium:* when
  - the travel time along all used alternative routes is exactly the same, and
  - the travel times along other un-used routes is higher.
- *Algorithms:* there exist efficient algorithms for finding the equilibrium.

3. How We Can Use the Existing Traffic Assignment Algorithms to Solve Our Problem: Analysis

- *Main objective:* predict how different road project affect future traffic congestion.
- *Future traffic demands – what is known:* there exist techniques for predicting daily O-D matrices.
- *What is lacking:* we need to "decompose" the daily O-D matrix into 1 hour (or 15 minute) intervals.
- *1st approximation:* assume that the proportion of drivers starting at, say 6 to 7 am is the same as now.
- *Need for a more accurate approximation:*
  - drivers may start early because of congestion;
  - if a new road is built, they will start later;
  - the % of those who start 6–7 am will decrease.
- *We cover:* both approximations.

4. Towards a More Accurate Approximation to O-D Matrices

- *Describing preferences:* empirical utility formula

  $$u_i = -0.1051 \cdot E(T) - 0.0931 \cdot E(SDE) - 0.1299 \cdot E(SDL) -$$
  $$1.3466 \cdot P_L - 0.3463 \cdot \frac{S}{E(T)},$$

  where $E(X)$ means expected value,

  - T is the travel time T,
  - SDE is the wait time when arriving early,
  - SDL is the delay when arriving late,
  - P_L is the probability of arriving late, and
  - S is the variance of the travel time.

- *Logit model:* the probability P_i that a driver will choose the i-th time interval is proportional to $\exp(u_i)$:

  $$P_i = \frac{\exp(u_i)}{\exp(u_1) + \ldots + \exp(u_n)}.$$

5. A Seemingly Natural Idea and Its Limitations

- *Seemingly natural idea:*
  - start with the 1st approximation O-D matrices M_1;
  - based on M_1, we find travel times, and use them to find the new O-D matrices $M_2 \stackrel{\rm def}{=} F(M_1)$;
  - based on M_2, we find travel times, and use them to find the new O-D matrices $M_3 \stackrel{\rm def}{=} F(M_2)$;
  - repeat until converges.
- *Toy example illustrating a problem:*
  - now: no congestion, all start at 7:30, work at 8 am;
  - M_1: full O-D matrix for 7:30 am, 0 for 7:15 am;
  - based on this M_1, we get huge delays;
  - M_2: everyone leaves for work early at 7:15 am;
  - at 7:30, roads are freer, so in M_3, all start at 7:30;
  - no convergence: $M_1 = M_3 = \ldots \neq M_2 = M_4 \ldots$

6. A More Realistic Approach

- *Above idea:* drivers make decisions based only on *previous* day traffic.
- *More accurate idea:* drivers make decisions based on the *average* traffic over a few past days.
- *Resulting process:*
  - start with the 1st approximation O-D matrices M_1;
  - for $i = 2, 3, \ldots$:
    - compute the average $E_i = \dfrac{M_1 + \ldots + M_i}{i}$,
    - find traffic times based on E_i;
    - use these traffic times to compute a new O-D matrix $M_{i+1} = F(E_i)$;
    - repeat until converges.
- *Process converges:* on toy examples, on El Paso network, etc.

7. Algorithm Simplified

- *Main idea:* once we know the previous average E_i, we can compute

$$E_{i+1} = \frac{(M_1 + \ldots + M_i) + M_{i+1}}{i+1} = \frac{i \cdot E_i + M_{i+1}}{i+1} =$$

$$E_i \cdot \left(1 - \frac{1}{i+1}\right) + M_{i+1} \cdot \frac{1}{i+1}.$$

- *We know:* that $M_{i+1} = F(E_i)$.
- *Resulting algorithm:*
  - start with the 1st approximation O-D matrices

$$E_1 = M_1;$$

  - compute $E_{i+1} = E_i \cdot \left(1 - \frac{1}{i+1}\right) + F(E_i) \cdot \frac{1}{i+1};$
  - repeat until converges.

8. Taking Uncertainty into Account

- *Deterministic model:* $t = t^f \cdot \left[1 + a \cdot \left(\frac{v}{c}\right)^\beta\right]$.
- *Traffic assignment:* a driver minimizes the travel time $t = t_1 + \ldots + t_n$.
- *In practice:* travel times vary.
- *Decision theory:* maximize expected utility $E[u]$.
- *How utility depends on travel time:* $u(t) = -U(t)$, where $U(t) = \exp(\alpha \cdot t)$.
- *Conclusion:* the driver minimizes

$$E[U(t)] = E[\exp(\alpha \cdot t)] = E[\exp(\alpha \cdot (t_1 + \ldots + t_n)] =$$

$$E[\exp(\alpha \cdot t_1) \cdot \ldots \cdot \exp(\alpha \cdot t_n)].$$

- Deviations on different links are independent, so

$$E[U(t)] = E[\exp(\alpha \cdot t_1)] \cdot \ldots \cdot E[\exp(\alpha \cdot t_n)].$$

9. Taking Uncertainty into Account (cont-d)

- Minimizing $E[U(t)] = E[\exp(\alpha \cdot t_1)] \cdot \ldots \cdot E[\exp(\alpha \cdot t_n)]$ $\Leftrightarrow$ minimizing $\sum\limits_{i=1}^{n} \widetilde{t}_i$, where $\widetilde{t}_i \stackrel{\text{def}}{=} \ln(E[\exp(\alpha \cdot t_i)])$.
- $\widetilde{t}$ depends on t^f and $r \stackrel{\text{def}}{=} \dfrac{\overline{t} - t^f}{t}$: $\widetilde{t} = F(t^f, r)$.
- If we divide a link into sublinks, we conclude that $F(t_1^f + t_2^f, r) = F(t_1^f, r) + F(t_2^f, r)$, hence $\widetilde{t} = t^f \cdot k(r)$.
- For no-congestion case $r = 0$, we have $\widetilde{t} = t^f$, so $k(0) = 1$ and $k(r) = 1 + a_0 \cdot r + a_2 \cdot r_2 + \ldots$
- *Empirical analysis:* $a_1 \approx 1.4$, $b \approx 0$, so

$$\widetilde{t} = t^f \cdot \left[1 + a \cdot a_1 \cdot \left(\frac{v}{c}\right)^{\beta}\right].$$

- *Solution:* use the standard travel time formula with $a \cdot a_1 \approx 0.21$ instead of $a \approx 0.14$.

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11. Logit Discrete Choice Model: A New Justification

- *Reasonable assumption:* if we add same incentive to all routes, probabilities will not change.
- *For 2 routes:* $P_1 = F(\Delta V)$, where $\Delta V \stackrel{\text{def}}{=} V_1 - V_2$.
- *Bayes theorem:*

$$P(H_i\,|\,E) = \frac{P(E\,|\,H_i)\cdot P_0(H_i)}{P(E\,|\,H_1)\cdot P_0(H_1) + \ldots + P(E\,|\,H_n)\cdot P_0(H_n)}.$$

- *Idea:* if we add an incentive v_0 to one of the routes, this changes the probability of selecting this route:

$$F(\Delta V + v_0) = \frac{A(v_0)\cdot F(\Delta V)}{A(v_0)\cdot F(\Delta V) + B(v_0)\cdot (1 - F(\Delta V))}.$$

- *Conclusion:* $F(\Delta V) = \dfrac{1}{1 + e^{-\beta\cdot\Delta V}}$, so

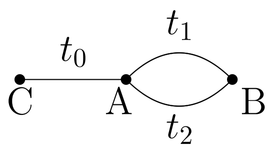
$$p_1 = F(V_1 - V_2) = \frac{e^{\beta\cdot V_1}}{e^{\beta\cdot V_1} + e^{\beta\cdot V_2}}.$$

12. Towards an Optimal Algorithm for Computing Fixed Points

- *Idea:* when iterations $x_{k+1} = f(x_k)$ do not converge, $x_{k+1} = x_k + \alpha \cdot (f(x_k) - x_k) = (1 - \alpha_k) \cdot x_k + \alpha_k \cdot f(x_k)$.
- *Question:* which choice of α_k is best?
- *Idea:* this is a discrete approximation to a continuous-time system $\dfrac{dx}{dt} = \alpha(t) \cdot (f(x) - x)$.
- *Scale invariance:* the system should not change if we use a different discretization, i.e., re-scale t to $t' = t/\lambda$:
$$\frac{dx}{dt'} = (\lambda \cdot \alpha(\lambda \cdot t')) \cdot (f(x) - x).$$
- *Conclusion:* $\lambda \cdot \alpha(\lambda \cdot t') = a(t')$, so for $\lambda = 1/t'$, we get $\alpha(t') = \dfrac{c}{t'}$ for some c.
- *Fact:* this is exactly what we used: $\alpha_k = 1/k$.

13. Exponential Disutility Functions in Transportation Modeling: Justification

- *Situation:*



- *Reasonable assumption:* the driver starting at C will choose the same road as the driver starting at A.
- *Formally:* if $E[u(t_1)] < E[u(t_2)]$ then

$$E[u(t_1 + t_0)] < E[u(t_2 + t_0)].$$

- *Result:* $u(t) = t$, $u(t) = \exp(c \cdot t)$, or

$$u(t) = -\exp(-c \cdot t).$$

- *Fact:* this is exactly the empirically justified formula used in transportation.