Interval Finite Elements as a Basis for Generalized Models of Uncertainty in Engineering Mechanics

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Latest scientific and engineering advances have started to recognize the need of defining multiple types of uncertainty. Probabilistic modeling cannot handle situations with incomplete or little information on which to evaluate a probability, or when that information is nonspecific, ambiguous, or conflicting [1]. Many generalized models of uncertainty have been developed to treat such situations. Among them, there are five major frameworks that use interval-based representation of uncertainty, namely: imprecise probabilities, possibility theory, Dempster-Shafer theory of evidence, Fuzzy set theory, and convex set modeling. Regardless what model is adopted, the proper interval solution will represents the first requirement for any further rigorous formulation.

In this work an interval technique is applied to Finite Element Methods. Finite Element Methods (FEM) are an essential and frequently indispensable part of engineering analysis and design.

An Interval Finite Element Method (IFEM) is presented that handles stiffness and load uncertainty in the linear static problems of mechanics. Uncertain parameters are introduced in the form of unknown but bounded quantities (intervals). To avoid overestimation, the new formulation is based on an element-by-element (EBE) technique. Element matrices are formulated, based on the physics of materials, and the Lagrange multiplier method is applied to impose the necessary constraints for compatibility and equilibrium. Earlier EBE formulation provided sharp bounds only on displacements [2]. Based on the developed formulation, the bounds on the system's displacement and stresses/forces are obtained simultaneously and have the same level of accuracy. Very sharp enclosures for the exact system responses are obtained. A number of numerical examples are introduced and scalability is illustrated.

References:

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