

# An Interval Analysis Algorithm for Automated Controller Synthesis in QFT Designs

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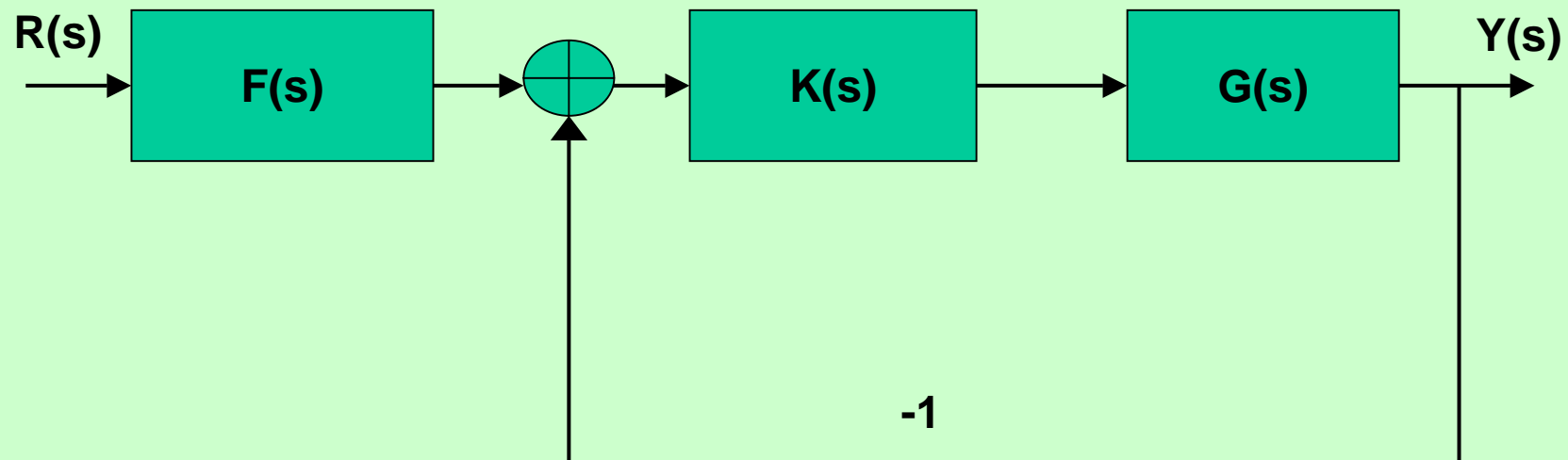
# Outline

- ◆ ***Salient Features***
- ◆ ***Introduction to QFT***
- ◆ ***Problem definition***
- ◆ ***Examples***
- ◆ ***Conclusions***

# Salient Features

- ◆ Design is fully Automatic.
- ◆ Enables the designer to pre-specify the controller structure.
- ◆ Deals directly with the numerical values of the possibly nonconvex, nonlinear QFT bounds.
- ◆ Guarantees the globally optimal solution, if the solution exists.

# Introduction



2-DOF Structure for QFT formulation

# Introduction ...

## QFT Objective :

Synthesize  $K(s)$  and  $F(s)$  for the following specifications:

- Robust Stability margin
- Tracking performance
- Disturbance Attenuation

# Introduction ...

## QFT Procedure :

1. **Generate the plant template at the given design frequencies  $\omega_i$ .**
2. **Generate the bounds in terms of nominal plant, at each design frequency, on the Nichols chart.**

# Introduction ...

## QFT Procedure ...

3. **Synthesize a controller  $K(s)$  such that**
  1. The open loop response satisfies the given performance bounds,
  2. And gives a nominal closed loop stable system.
  
4. **Synthesize a prefilter  $F(s)$  which satisfies the closed loop specifications.**

# Problem Definition

- ◆ ***Given an uncertain plant and time domain or frequency domain specification, automatically synthesize an optimal QFT controller of a pre-specified structure.***



# Example 1

- ◆ Application : **Control system design for DC Motor.**
- ◆ Compared with : **Genetic Algorithms\***.
  - \* *W. Chen et al. “Automatic Loop-shaping in QFT using genetic algorithms”, In Proceedings of 3<sup>rd</sup> Asia-pacific Conference on Control and Measurement, pages 63-67, 1998.*
- ◆ No. of optimization variables involved : **4**
- ◆ Reduction obtained with the proposed algorithm:
  - hf gain : **48.73 %**
  - Cutoff freq : **67.68 %**

## Example 2

- ◆ Application : **Control system design for DC Motor.**
- ◆ Compared with : **Non-iterative method based on SVD\***.
  - *A. Zolotas and G. Halikias, “Optimal design of PID controllers using the QFT method”, IEE Proc-Control Theory Appl., 146(6):585-589, Nov. 1999.*
- ◆ No. of optimization variables involved : **3**
- ◆ Reduction obtained with the proposed algorithm:
  - hf gain : **10.63 %**
  - Cutoff freq : **-21.87 %**

## Example 3

- ◆ Application : **Control system design for DC Motor.**
- ◆ Compared with : **LP solver NAG E04MBF\***.
  - *G. Bryant and G. Halikias, “Optimal loop-shaping for systems with large parameter uncertainty via linear programming”, Int. J. Control, 62(3):557-568, 1995.*
- ◆ No. of optimization variables involved : **5**
- ◆ Reduction obtained with the proposed algorithm:
  - hf gain : **23.80%**
  - Cutoff freq : **12.41 %**

## Example 4

- ◆ Application : **Control system design for DC Motor.**
- ◆ Compared with : **LP solver\***.
  - *Y. Chait et al., “Automatic Loop-shaping of QFT controllers via Linear Programming”, Trans. Of the ASME Journal of Dynamic Systems, Measurement and Control, 121:351-357, 1999*
- ◆ No. of optimization variables involved : **6**
- ◆ Reduction obtained with the proposed algorithm:
  - hf gain : **73 %**
  - Cutoff freq : **65 %**

## Example 5

- ◆ Application : **Control system design for Aircraft.**
- ◆ Compared with : **SQP solver IMSL DNCONG\***.
  - *D. F. Thompson, “ Optimal and Sub-optimal loop shaping in QFT”, PhD thesis, School of Mechanical Engineering, Purdue University, USA, 1990*
- ◆ No. of optimization variables involved : **5.**
- ◆ Reduction obtained with the proposed algorithm:
  - hf gain : **53.18 %**
  - Cutoff freq : **30 %**

## Example 6

- ◆ Application : **Control system design for Aircraft.**
- ◆ Compared with : **SQP solver IMSL DNCONG\***.
  - *D. F. Thompson and O. D. I. Nwokah, “Analytical loop shaping methods in QFT”, Trans. Of the ASME Journal of Dynamic Systems, Measurement and Control, 116:169-177, 1994.*
- ◆ No. of optimization variables involved : **7.**
- ◆ Reduction obtained with the proposed algorithm:
  - hf gain : **48.69 %**
  - Cutoff freq : **86.35 %**

# Conclusions

- ◆ An algorithm has been proposed to **automate** the controller synthesis step of QFT.
- ◆ Proposed algorithm is based on deterministic interval global optimization techniques that assures **convergence** and the **globalness** of the solution.
- ◆ The proposed algorithm uses the precise values of the numerical QFT bounds which **avoids** the problem associated with the **approximation** of the bounds.
- ◆ Overall, a reduction of **73%** in hf gain and **86%** reduction in cutoff frequency of the controller is obtained, over the existing methods for QFT controller synthesis.

**Thank you !**