# A Computational Approach to Existence Verification and Construction of Robust QFT Controllers

*P.S.V. Nataraj and Sachin Tharewal*

*Systems & Control Engineering,*

*IIT Bombay, India*

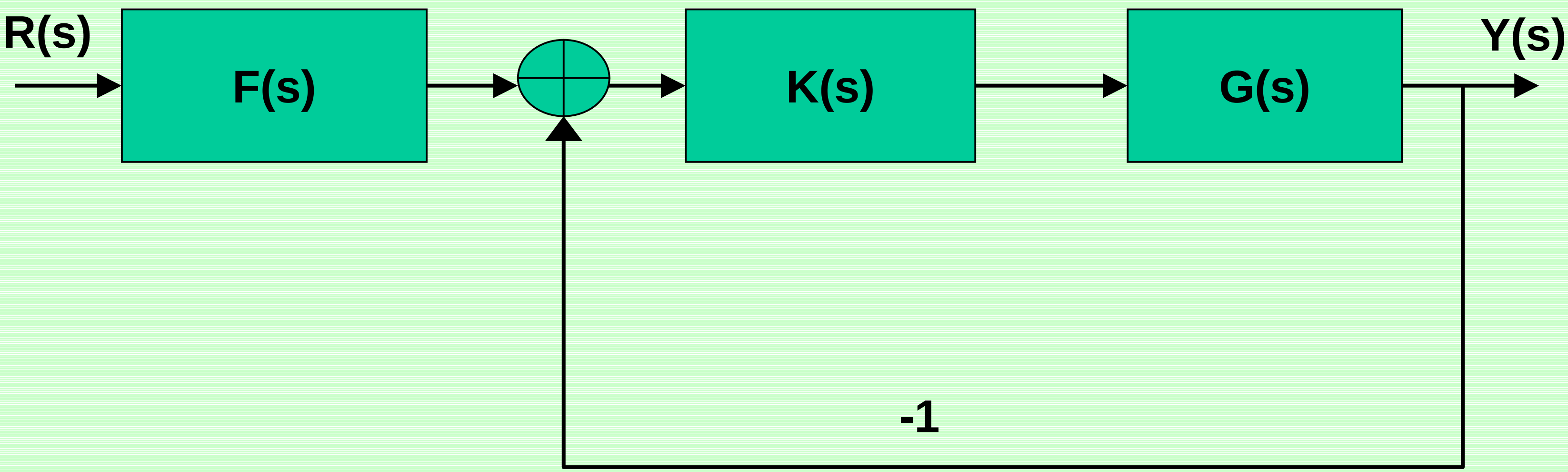
# Outline

- *Contribution*
- *Introduction to QFT*
- *Problem definition*
- *Proposed Algorithm*
- *Example*
- *Conclusions*

# Contribution

- ***Existence verification***
- ***Constructive approach –***
  - Gauranteed existence verification
  - Give all solutions, if any solution exist.

# Introduction




2-DOF Structure for QFT formulation

# Introduction …

## QFT Objective :

**Synthesize K(s) and F(s) for the following specifications:**

- **Robust Stability margin**
- **Tracking performance**
- **Disturbance Attenuation**

# Introduction …

## QFT Procedure :

1. **Generate the plant template at the given design frequencies $\omega_i$.**

2. **Generate the bounds in terms of nominal plant, at each design frequency, on the Nichols chart.**

# Introduction …

## QFT Procedure …

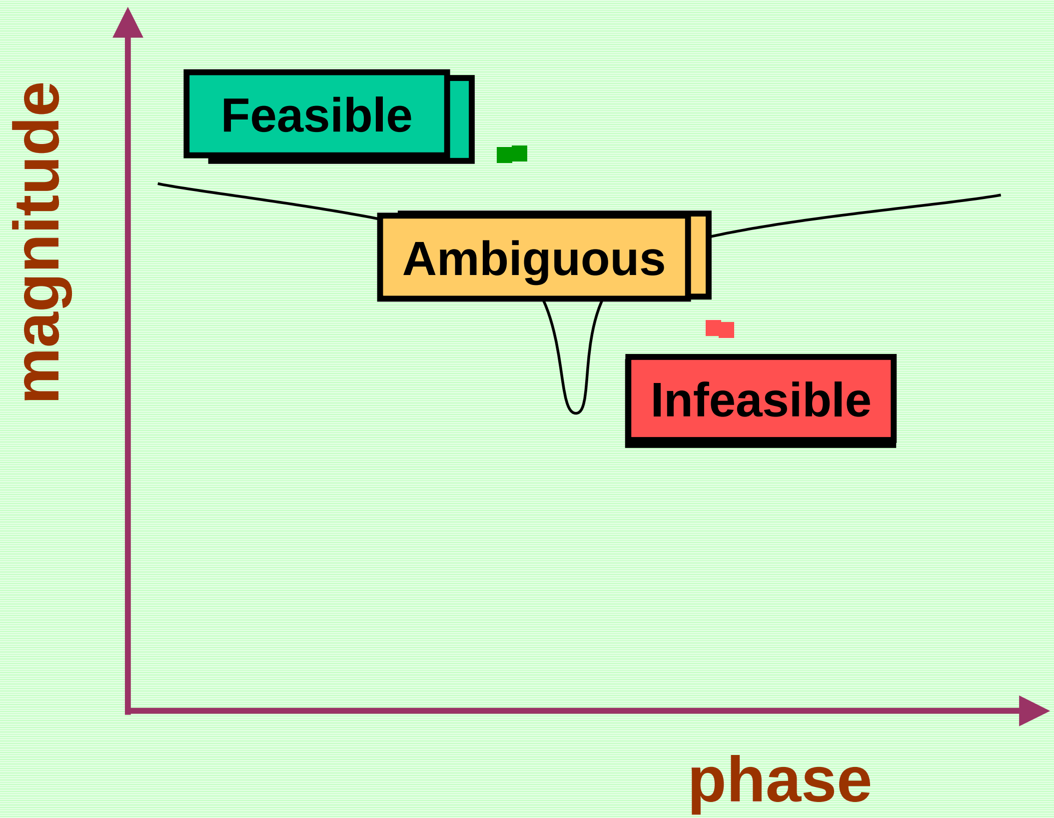
3. **Synthesize a controller K(s) such that**
    1. The open loop response satisfies the given performance bounds,
    2. And gives a nominal closed loop stable system.

4. **Synthesize a prefilter F(s) which satisfies the closed loop specifications.**

# Problem Definition

- ***Given an uncertain plant and time domain or frequency domain specification, find out if a controller of specified (transfer function) structure exist.***

# Feasibility Test



# Proposed Algorithm

**Inputs:**

1. Numerical bound set,
2. The discrete design frequency set,
3. Inclusion function for magnitude and angle,
4. The initial search space box $z^0$.

**Output:**

Set of Controller parameters, **OR**

The message “No solution Exist”.

# Proposed Algorithm…

**BEGIN Algorithm**

1. Check the feasibility of initial search box $\mathbf{z}^0$.
   - Feasible ⇔ Complete $\mathbf{z}^0$ is feasible solution.
   - Infeasible ⇔ No solution exist in $\mathbf{z}^0$
   - Ambiguous ⇔ Further processing required : Initialize stack list $L^{stack}$ and solution list $L^{sol}$.

# Proposed Algorithm …

2. Choose the first box from the stack list $\boldsymbol{L^{stack}}$ as the current box **z**, and delete its entry form the stack list.

3. Split **z** along the maximum width direction into two subboxes.

# Proposed Algorithm …

4. Find the feasibility of each new subbox:
   - Feasible ⇔ Add to solution list $L^{sol}$.
   - Infeasible ⇔ Discard.
   - Ambiguous ⇔ Further processing required : Add to stack list $L^{stack}$.

# Proposed Algorithm

5. IF $\boldsymbol{L}^{\boldsymbol{stack}}$ = {empty} THEN (terminate)
   - $\boldsymbol{L}^{\boldsymbol{sol}}$ = {empty} ⇔ "No solution exist", **Exit**.
   - $\boldsymbol{L}^{\boldsymbol{sol}} \neq$ {empty} ⇔ "Solution set = " $\boldsymbol{L}^{\boldsymbol{sol}}$, **Exit**.

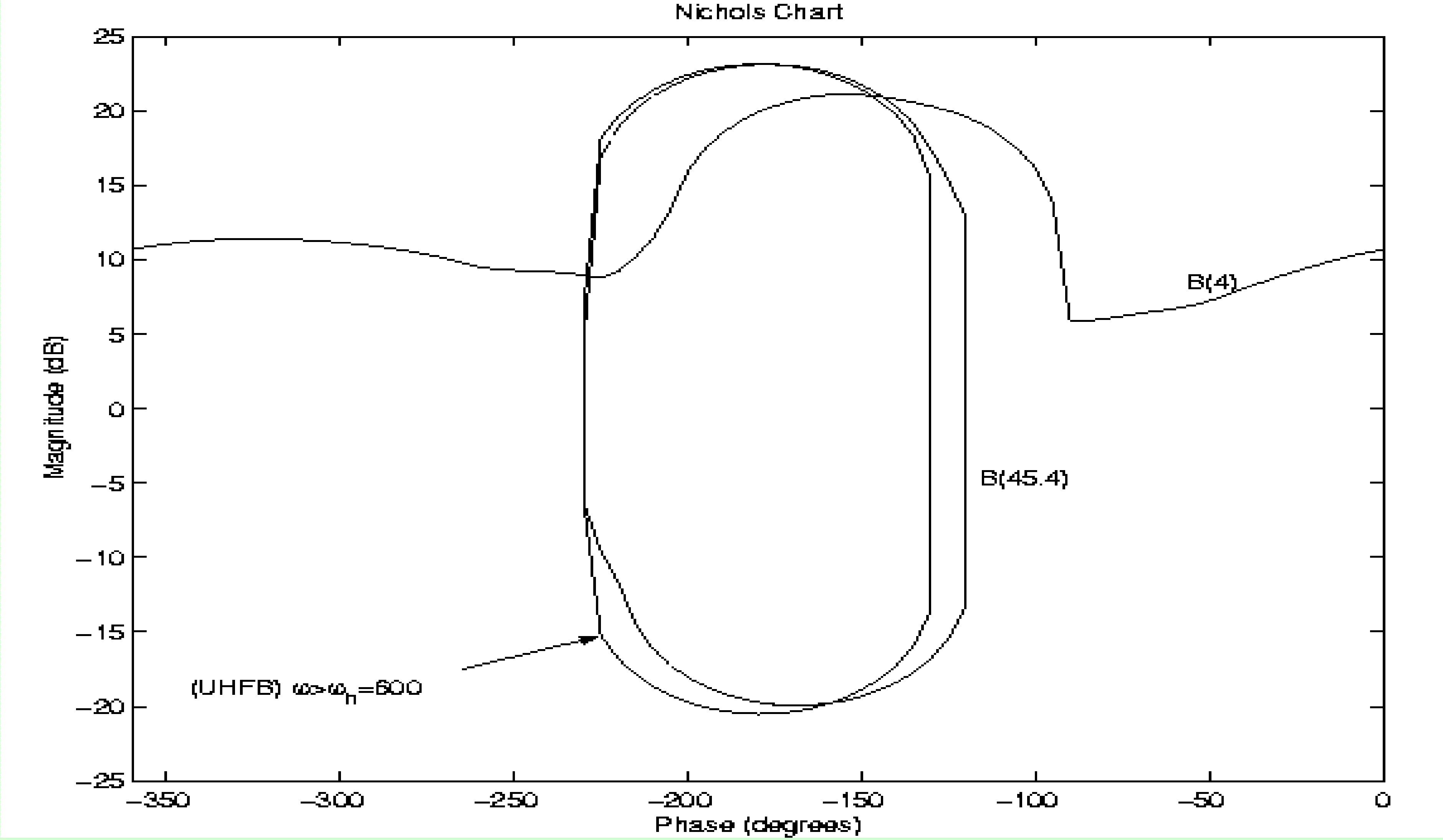
6. Go to step 2 (iterate).

**End Algorithm**

# Example

- Uncertain plant : $$G(s) = \frac{k(1-\tau s)}{s(1+\beta s)}$$
- Uncertainty : $k \in [1,3], \beta \in [0.3,1], \tau \in [0.05,0.1]$
- Robust stability spec: $\omega_s = 1.3032$
- Tracking spec: $|T_U(j4)| = 0.5 \quad and \quad |T_L(j4)| = -3.5$

# QFT Bounds


Nichols Chart
Magnitude (dB)
Phase (degrees)
25
20
15
10
5
0
−5
−10
−15
−20
−25
−350
−300
−250
−200
−150
−100
−50
0
B(4)
B(45.4)
(UHFB) ω>ω$_h$=600

# Example …

- **For first order controller**:
  - Parameter vector $\boldsymbol{z} = \{\boldsymbol{k}, \breve{\boldsymbol{z}}_1, \boldsymbol{p}_1\}$
  - Initial search box $\boldsymbol{z}^0 = (0,10^8], (0,10^4], (0,10^4]$
- **For second order controller**:
  - Parameter vector $\boldsymbol{z} = \{\boldsymbol{k}, \breve{\boldsymbol{z}}_1, \breve{\boldsymbol{z}}_2, \boldsymbol{p}_1, \boldsymbol{p}_2\}$
  - Initial search box $\boldsymbol{z}^0 = (0,10^8], (0,10^4], (0,10^4], (0,10^4], (0,10^4]$

# Example …

- **For third order controller**:
  - Parameter vector $z = \{k, \breve{z}_1, \breve{z}_2, \breve{z}_3, p_1, p_2, p_3\}$
  - Initial search box

$$z^0 = (0,10^8], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4]$$

- **For fourth order controller**:
  - Parameter vector $z = \{k, \breve{z}_1, \breve{z}_2, \breve{z}_3, \breve{z}_4, p_1, p_2, p_3, p_4\}$
  - Initial search box

$$z^0 = (0,10^8], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4], (0,10^4]$$

# Results

- For the aforementioned structures and the initial search domains, the proposed algorithm terminated with the message: "**No feasible solution exists in the given initial search domain**".
- This finding is in agreement with the analytically found '**non-existence**' of Horowitz.

# Conclusions

- An algorithm has been proposed to computationally verify the existence (or non-existence) of a QFT controller solution.
- Proposed algorithm has been tested successfully on a QFT benchmark example.
- The proposed algorithm is based on interval analysis, and hence provides the most reliable technique for existence verification.

# Thank you !