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Structural Design under Fuzzy Randomness

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Program

Introduction (1)

the engineer‘s endeavor

- **realistic structural analysis and safety assessment**

prerequisites:

- **suitably matched computational models**
  ➡ **geometrically and physically nonlinear algorithms for numerical simulation of structural behavior**
- **appropriate description of structural parameters**
  ➡ **uncertainty has to be accounted for in its natural form**

Introduction (2)

modeling structural parameters

geometry ?

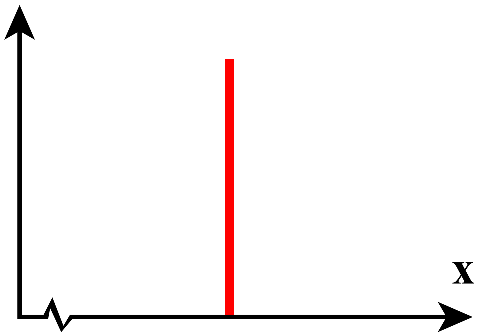
material ?

loading ?

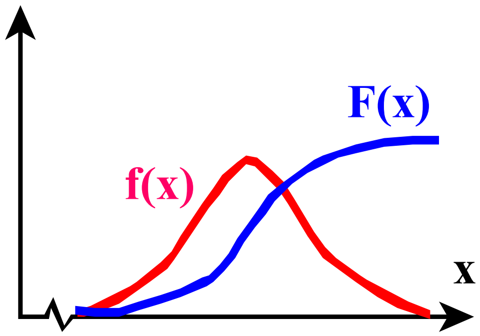
foundation ?

Introduction (3)

- **deterministic variable**



- **random variable**



! very few information

! changing reproduction conditions

! uncertain measured values

! linguistic assessments

! experience, expert knowledge

alternative uncertainty models

- **fuzzy variable**
- **fuzzy random variable**

Fuzzy sets

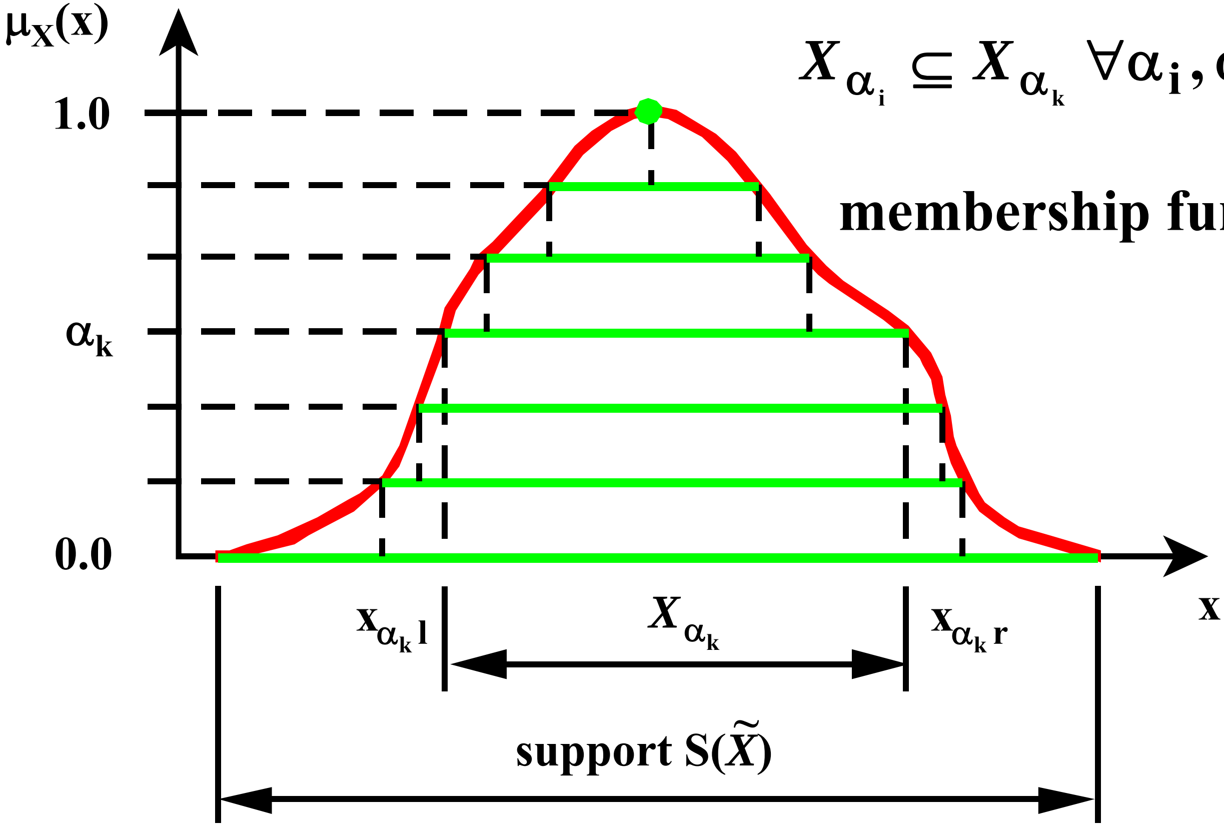
α-level set

$$X_\alpha = \{\, x \in \mathbf{X} \mid \mu_X(x) \ge \alpha \,\}$$

α-discretization

$$\tilde{X} = \{(X_\alpha ; \mu(X_\alpha))\}$$

$$X_{\alpha_i} \subseteq X_{\alpha_k} \;\; \forall \alpha_i, \alpha_k \;\left|\; \begin{array}{l} \alpha_i \ge \alpha_k \\ \alpha_i, \alpha_k \in (0;1] \end{array}\right.$$



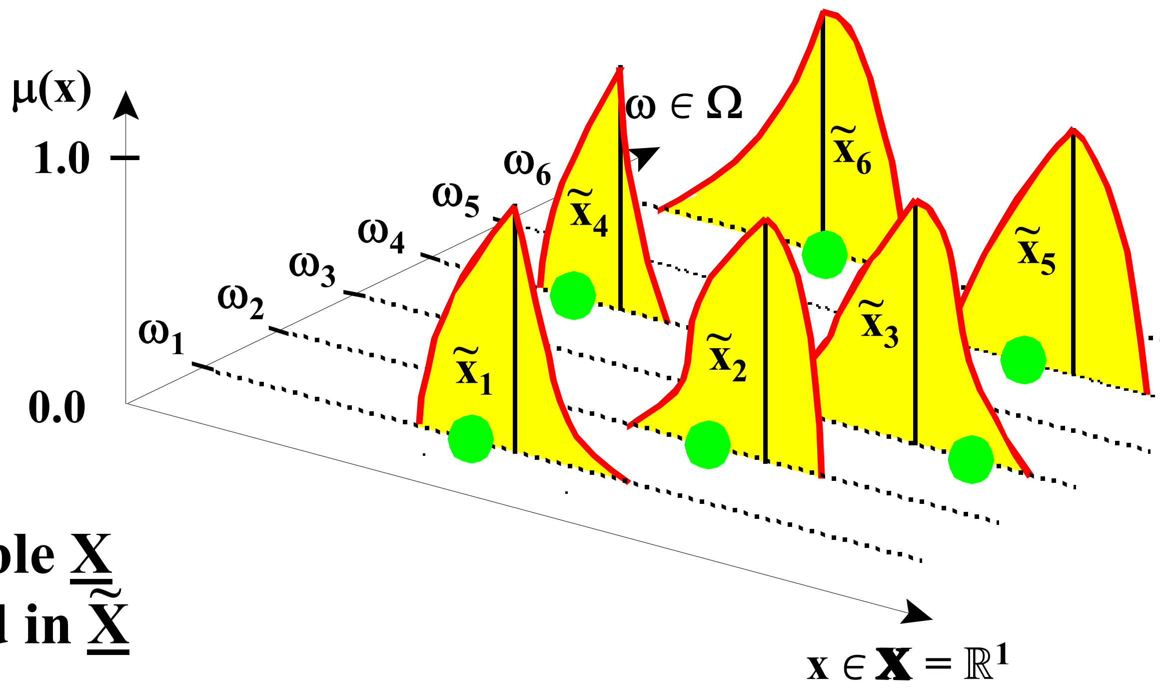
Fuzzy random variables (1)

fuzzy random variable $\underline{\tilde{X}}$

- fuzzy result of the uncertain mapping $\Omega \rightarrow F(\mathbb{R}^n)$
- Ω ... space of the random elementary events ω
- $F(\mathbb{R}^n)$ ... set of all fuzzy numbers $\underline{\tilde{x}}$ in $\mathbb{R}^n$

$\underline{\tilde{X}}$ is the fuzzy set of all possible originals $\underline{X}_j$

original $\underline{X}_j$

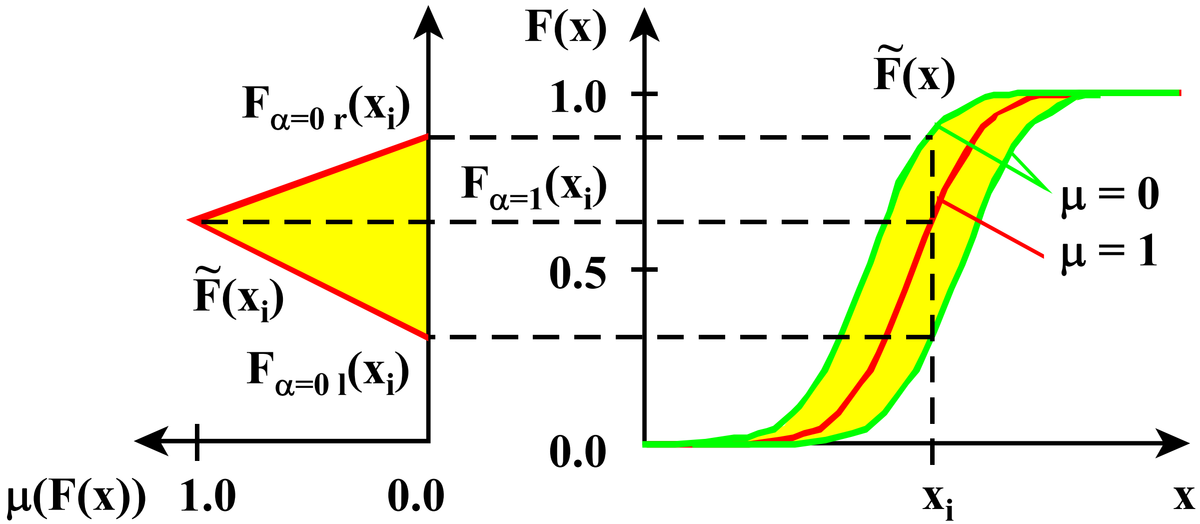


- real valued random variable $\underline{X}$ that is completely enclosed in $\underline{\tilde{X}}$

Fuzzy random variables (2)

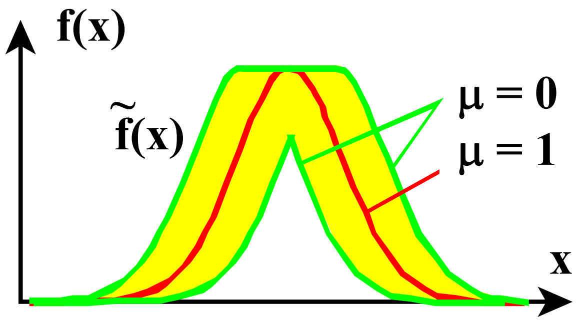
fuzzy probability distribution function $\tilde{F}(\underline{x})$

$\tilde{F}(\underline{x})$ = bunch of the $F_j(\underline{x})$ of the originals $\underline{X}_j$ of $\underline{\tilde{X}}$



- $\tilde{F}(\underline{x}) = \{(F_\alpha(\underline{x}); \mu(F_\alpha(\underline{x})))\}$

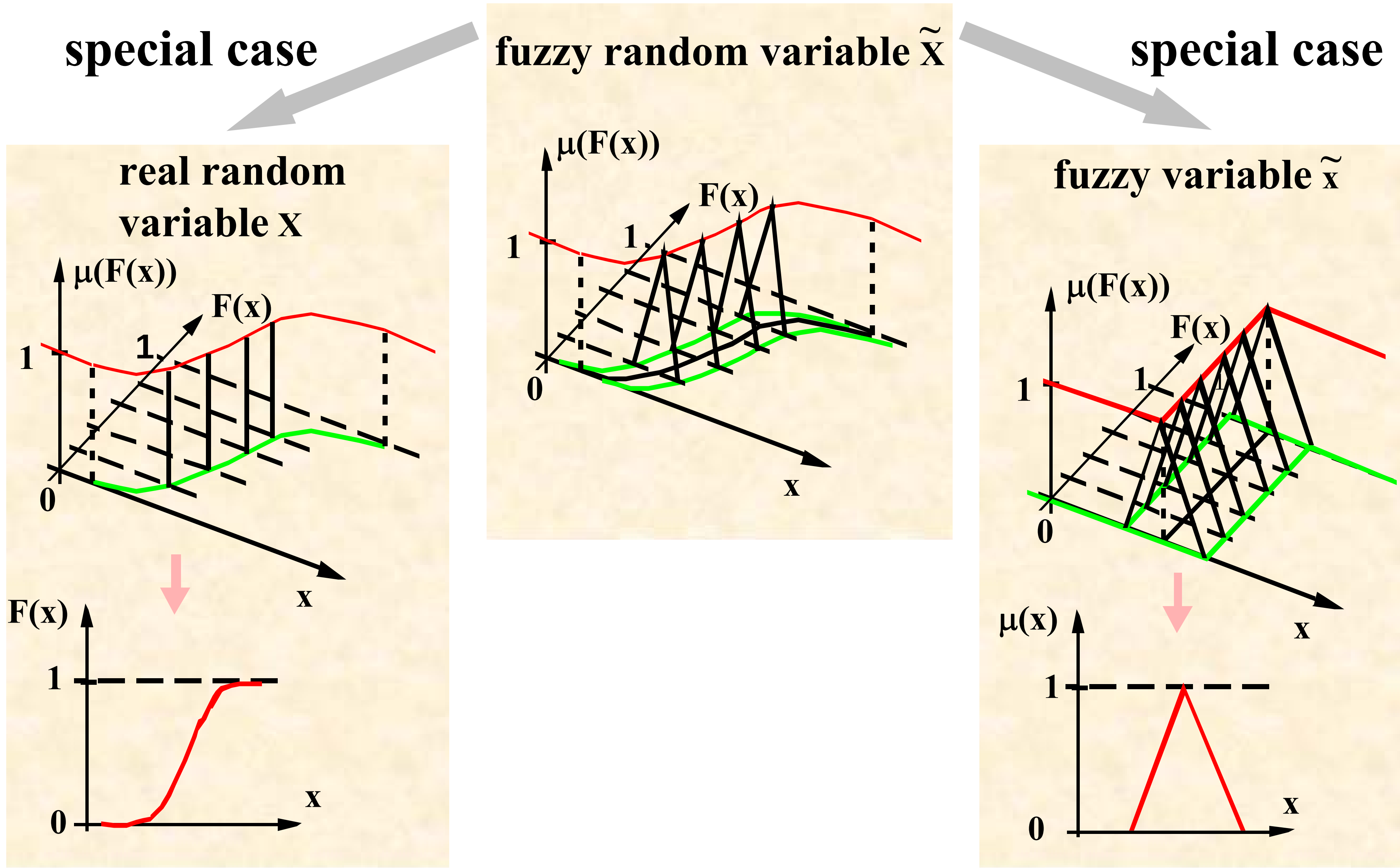
- $F_\alpha(\underline{x}) = [F_{\alpha\,l}(\underline{x}); F_{\alpha\,r}(\underline{x})]$;
  $\mu(F_\alpha(\underline{x})) = \alpha \;\forall\; \alpha \in (0;1]$



- $$F_{\alpha\,l}(\underline{x}) = 1 - \max_j P\left(\underline{X}_{j,\alpha} = \underline{t} \;\middle|\; \begin{array}{l} \underline{x}, \underline{t} \in \underline{\mathbf{X}} = \mathbb{R}^n \\ \exists\, t_k \geq x_k ; 1 \leq k \leq n \end{array}\right)$$

- $$F_{\alpha\,r}(\underline{x}) = \max_j P\left(\underline{X}_{j,\alpha} = \underline{t} \;\middle|\; \begin{array}{l} \underline{x}, \underline{t} \in \underline{\mathbf{X}} = \mathbb{R}^n \\ t_k < x_k ; k = 1, \ldots, n \end{array}\right)$$

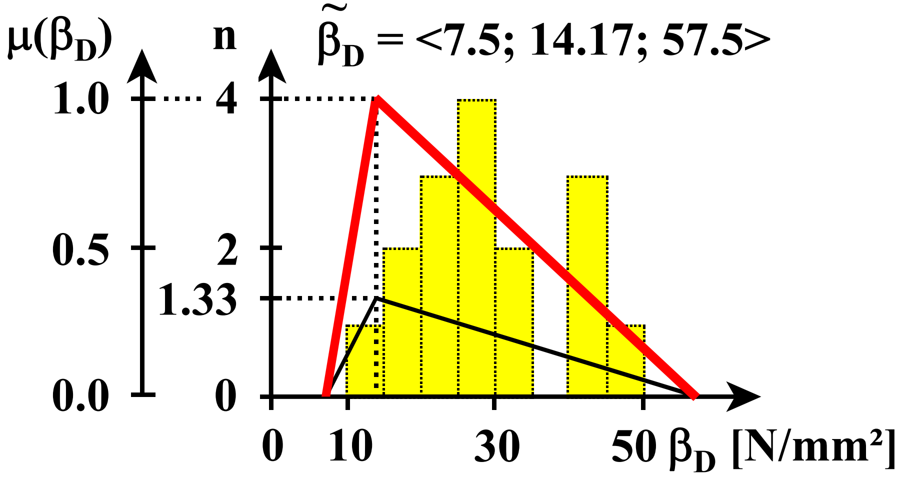
Fuzzy random variables (3)



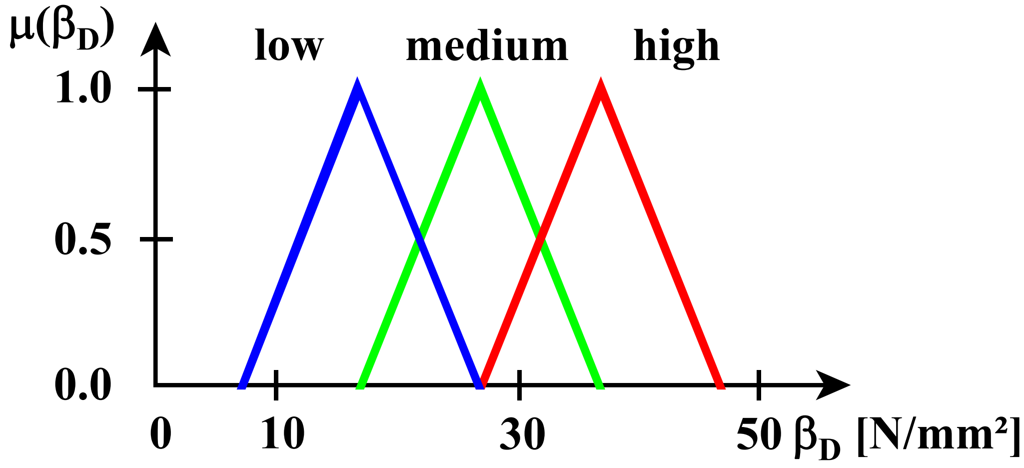
Quantification of fuzzy variables (fuzzification)

objective and subjective information, expert knowledge

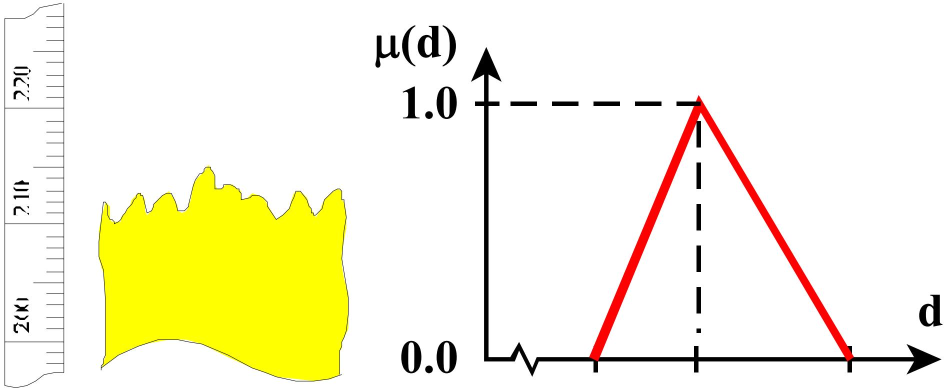
- **limited data**



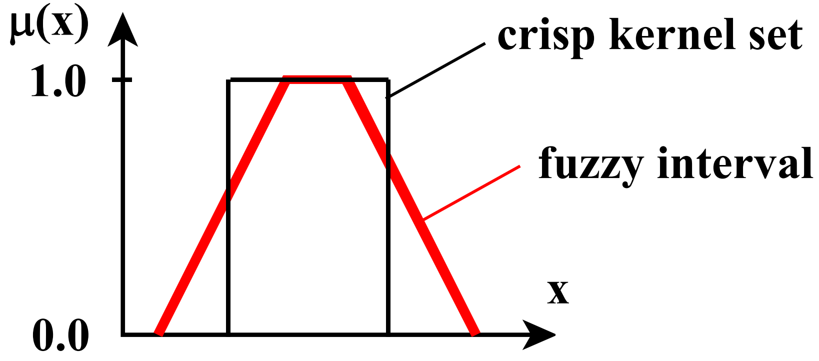
- **linguistic assessment**



- **single uncertain measurement**



- **expert experience**

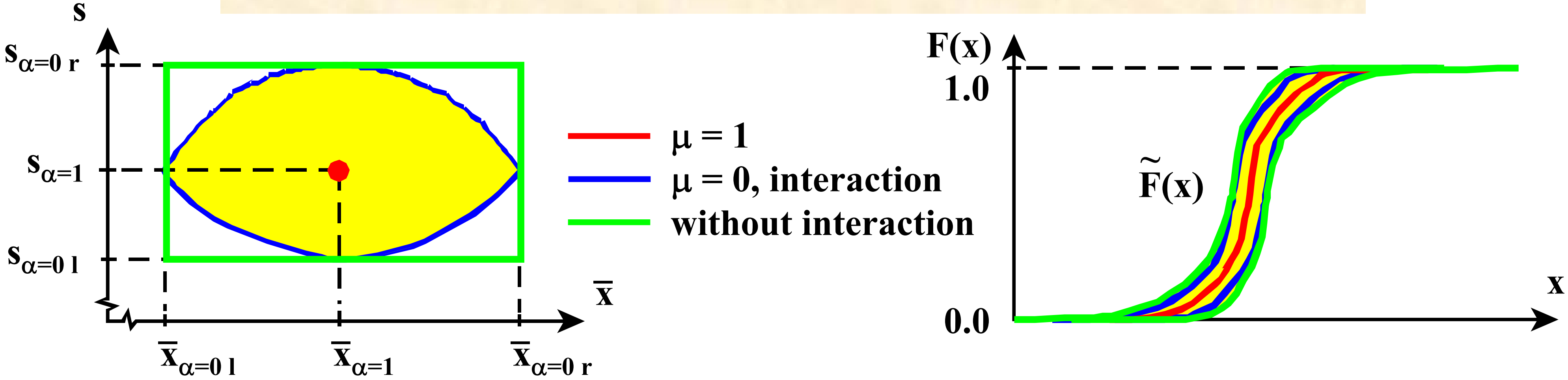


Quantification of fuzzy random variables

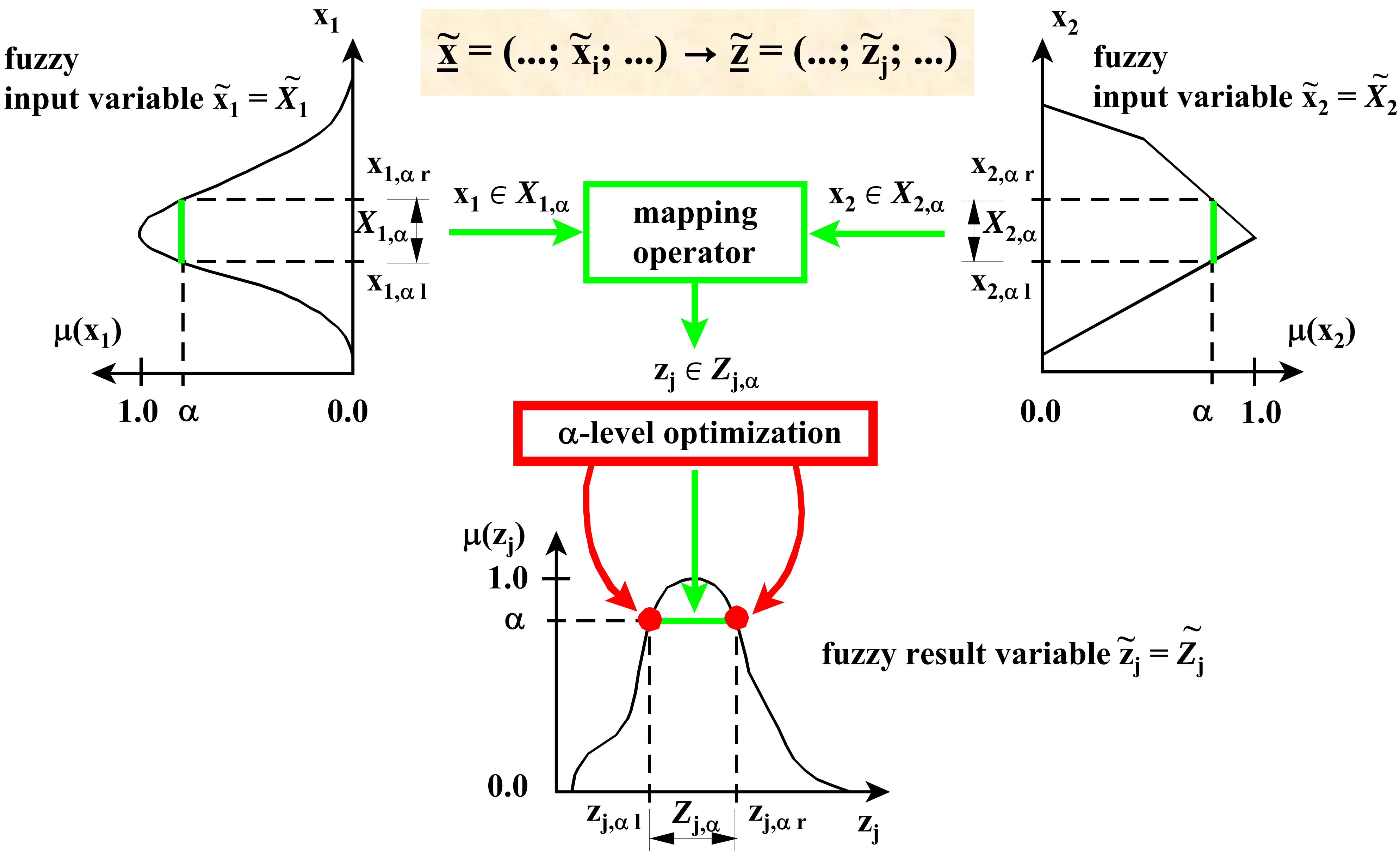
statistical information comprising partially non-stochastic uncertainty

- **small sample size**
  - ➡ **fuzzification of the uncertainty of statistical estimations and tests**
- **non-constant reproduction conditions that are known in detail**
  - ➡ **separation of fuzziness and randomness by constructing groups**
- **non-constant, unknown reproduction conditions; uncertain measure values**
  - ➡ **fuzzification of the sample elements followed by statistical evaluation**

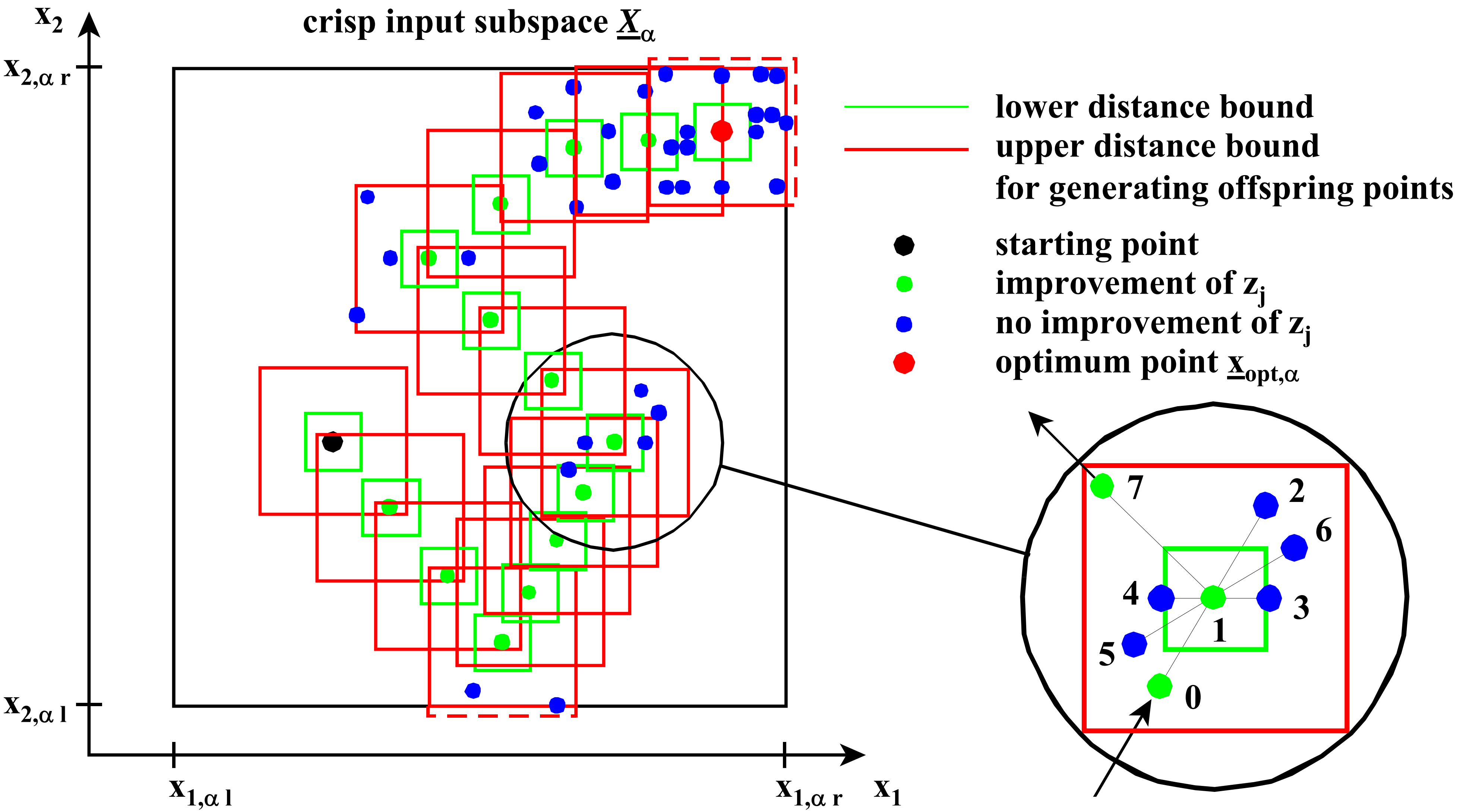
$\tilde{F}(\underline{x})$ with fuzzy parameters and fuzzy functional type



Basic numerical solution procedure



Modified evolution strategy

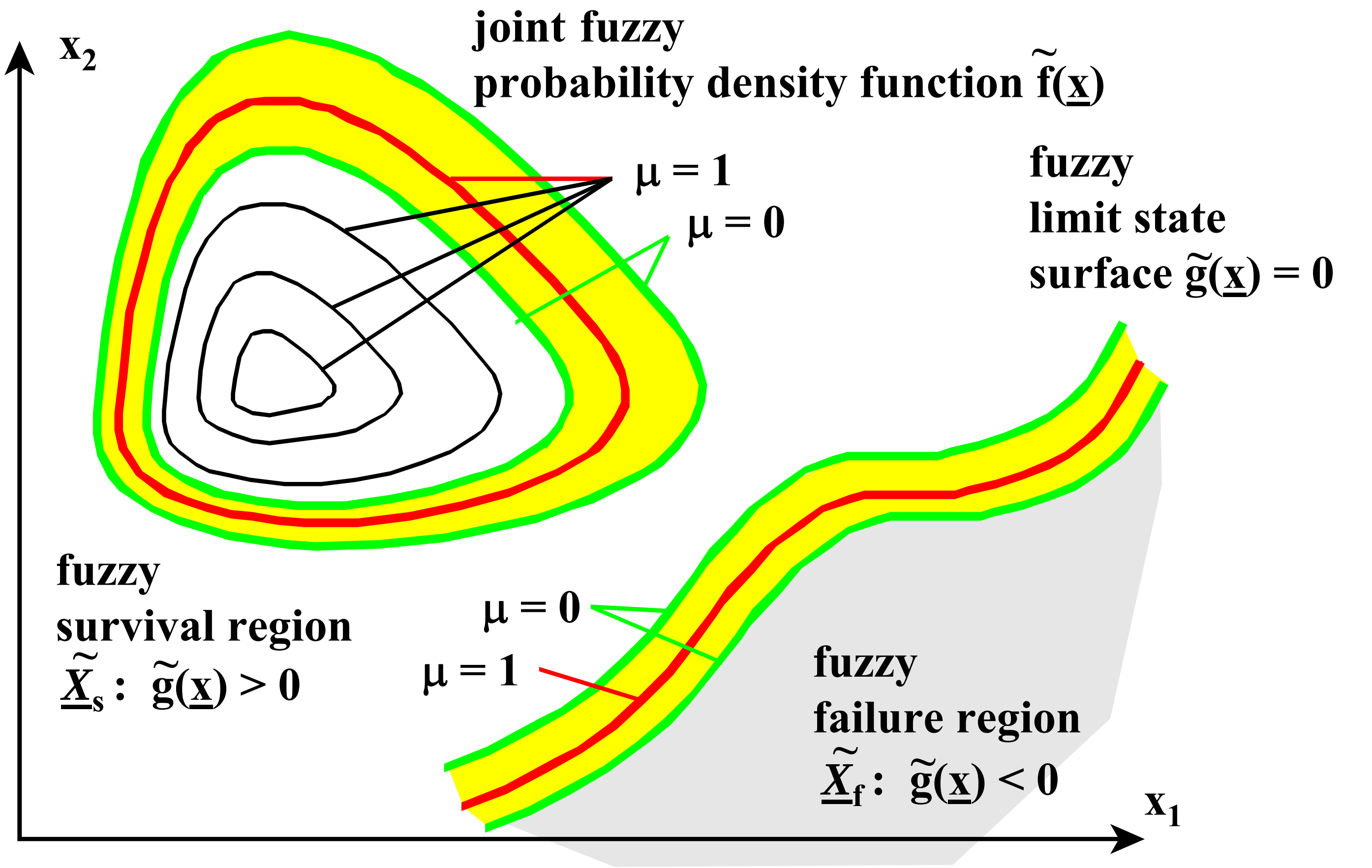


- post-computation ➡ recheck of all results for optimality

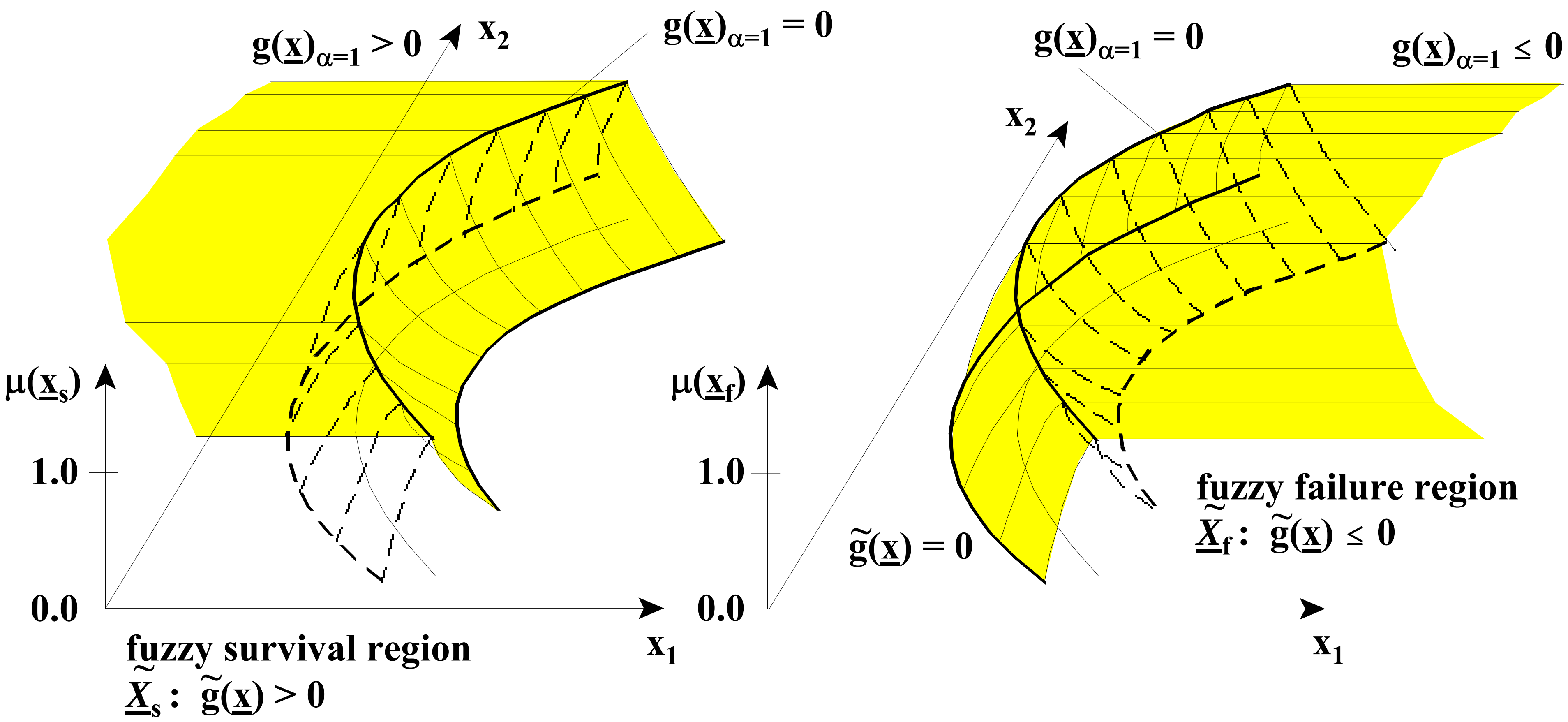
Fuzzy First Order Reliability Method (1)

original space of the basic variables

- **fuzzy randomness** $\Rightarrow \tilde{f}(\underline{x})$
- **fuzziness** $\Rightarrow \tilde{g}(\underline{x}) = 0$



Fuzzy First Order Reliability Method (1)



Fuzzy First Order Reliability Method (1)

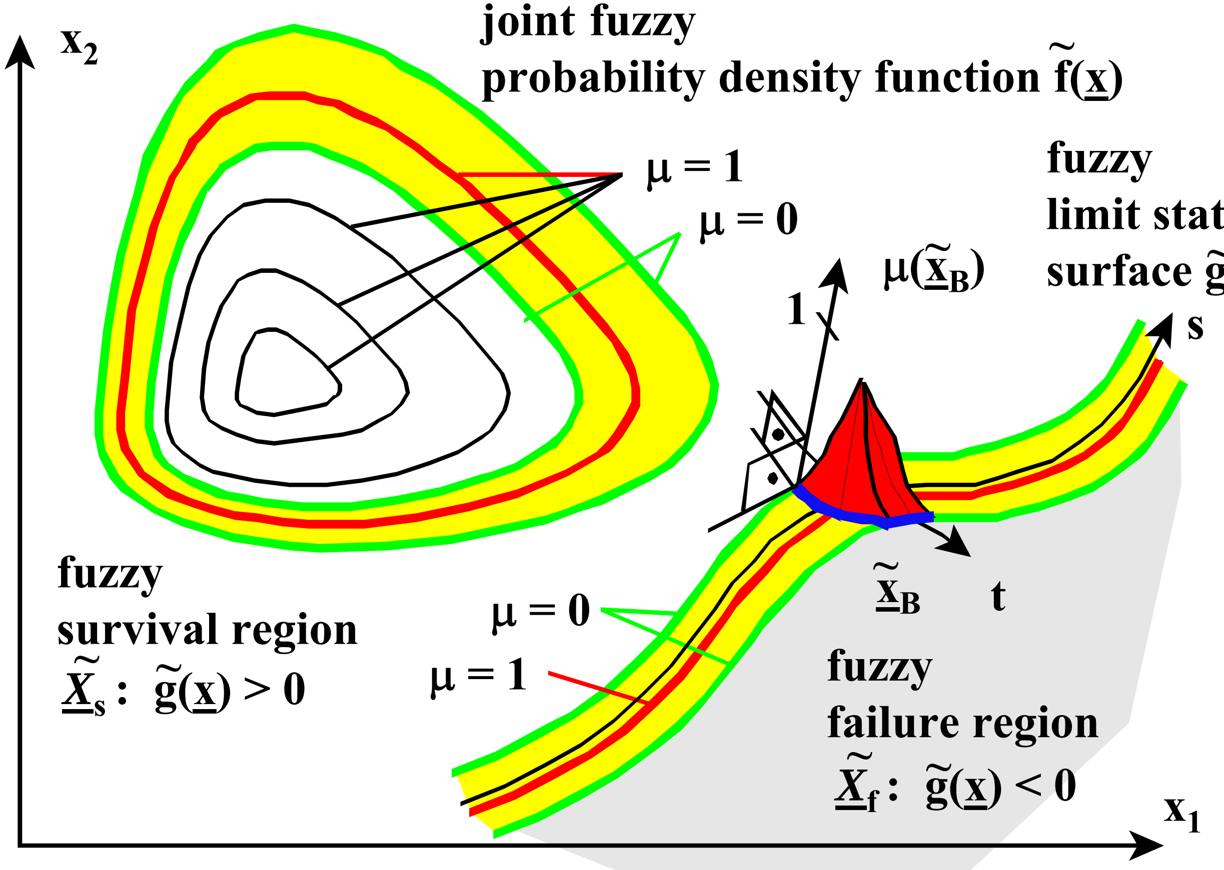
original space of the basic variables

- fuzzy randomness $\Rightarrow \tilde{f}(\underline{x})$
- fuzziness $\Rightarrow \tilde{g}(\underline{x}) = 0$

➡ • fuzzy design point $\tilde{\underline{x}}_B$

$$\tilde{P}_f = \int_{\underline{x} \in \tilde{X}_f;\, \underline{x} \notin \tilde{X}_s} \cdots \int \tilde{f}(\underline{x})\, d\underline{x}$$

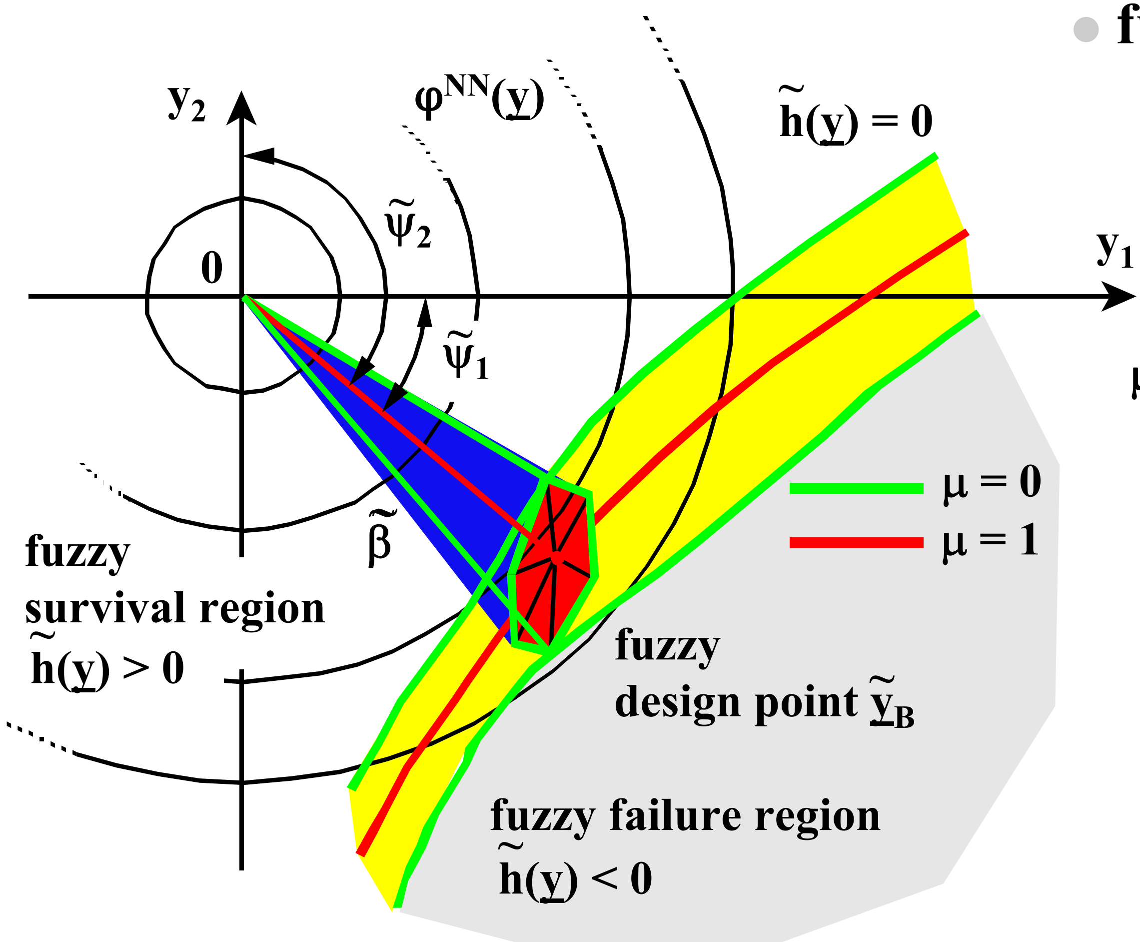
Fuzzy
First
Order
Reliability
Method



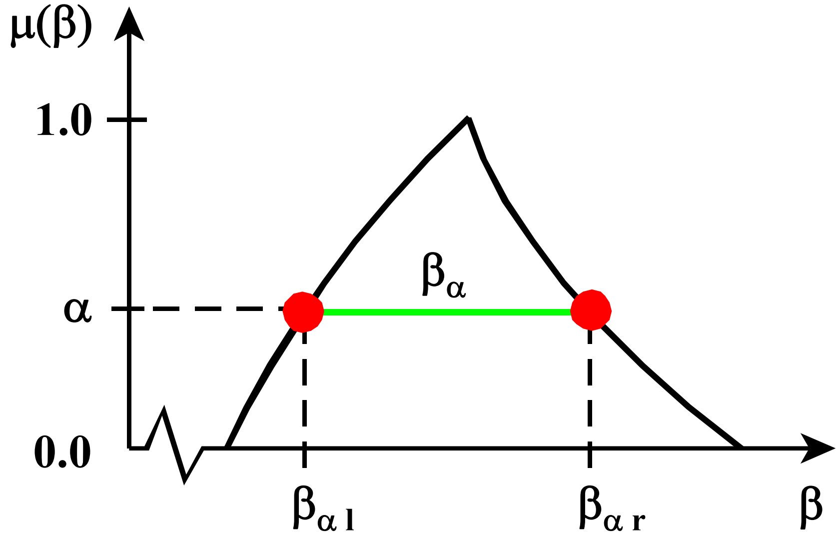
Fuzzy First Order Reliability Method (2)

standard normal space

- **complete fuzziness in $\tilde{h}(\underline{y}) = 0$** ➡
- **fuzzy design point $\tilde{\underline{y}}_B$**
- **fuzzy reliability index $\tilde{\beta}$**



$$\left\{\tilde{p}_t\left(\tilde{X}_i\right);\tilde{m}_j\right\} \rightarrow \tilde{\beta}$$



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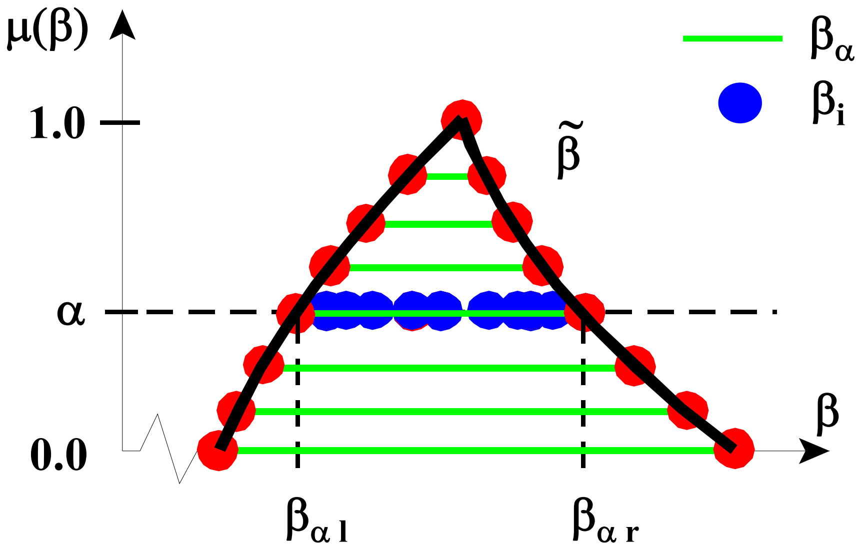
Fuzzy First Order Reliability Method (3)

numerical solution – α-level optimization

- selection of an α-level
- determination of an original of each basic variable
- determination of a single point in the space of the fuzzy structural parameters
- determination of one trajectory of $\tilde{f}(\underline{x})$
- determination of one element of $\tilde{g}(\underline{x}) = 0$
- computation of one element β_i of β_α
- comparison of β_i with $\beta_{\alpha\, l}$ and $\beta_{\alpha\, r}$
- number of α-levels sufficient ?



Structural design based on clustering – concept

fuzzy structural analysis and fuzzy probabilistic safety assessment

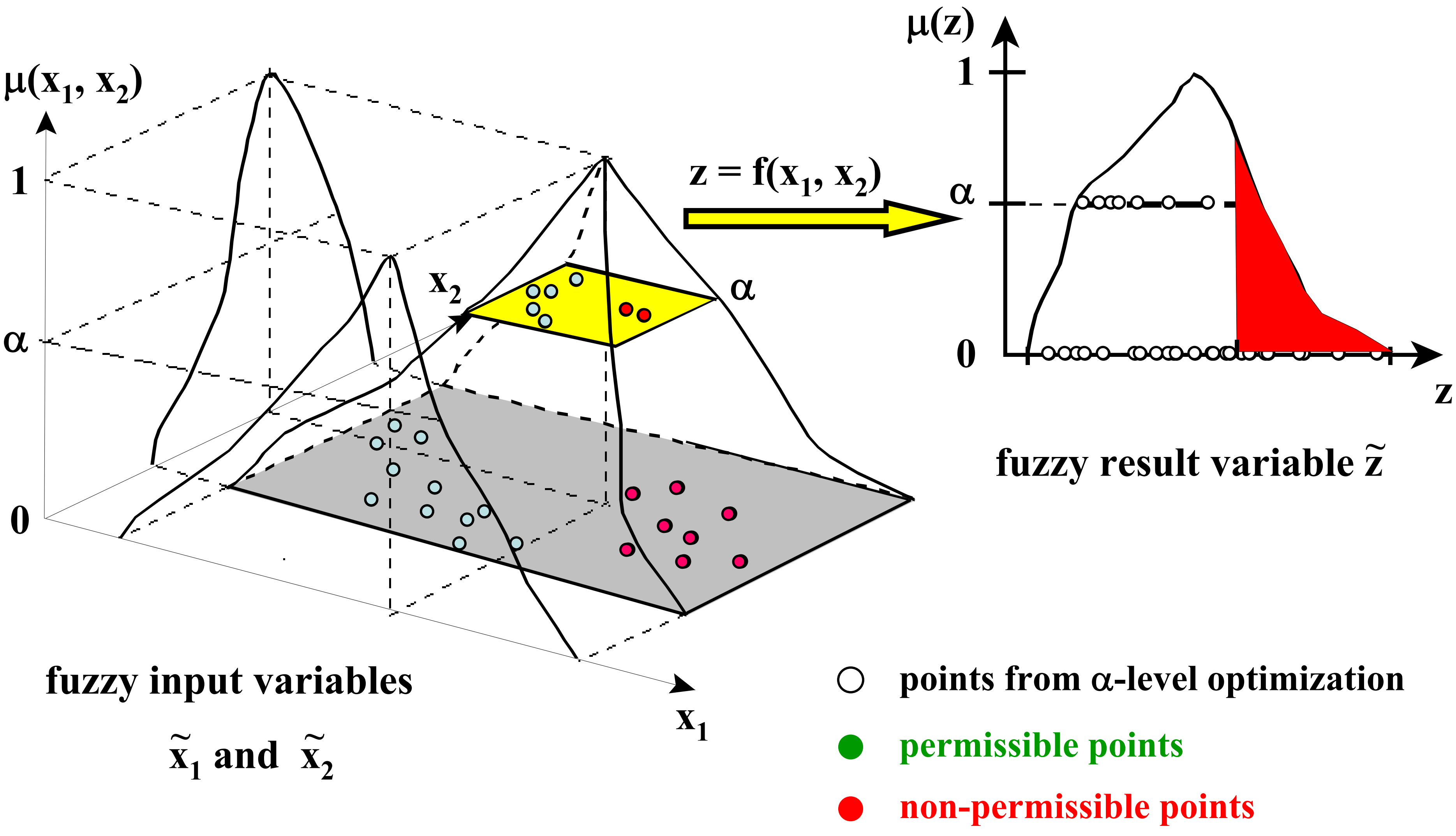
set M($\underline{x}$) of discrete points $\underline{x}_i$ in the input subspace,
set M($\underline{z}$) of the assigned discrete points $\underline{z}_i$ in the result subspace

- **design constraints CT_h**
  subdividing M($\underline{z}$) into permissible and non-permissible points
- **assigned permissible and non-permissible points within M($\underline{x}$)**
- **separate clustering of the permissible and non-permissible points from M($\underline{x}$)**
- **elimination of all non-permissible clusters**
  including intersections with permissible clusters

↓

permissible clusters of design parameters
representing alternative design variants

Design constraint

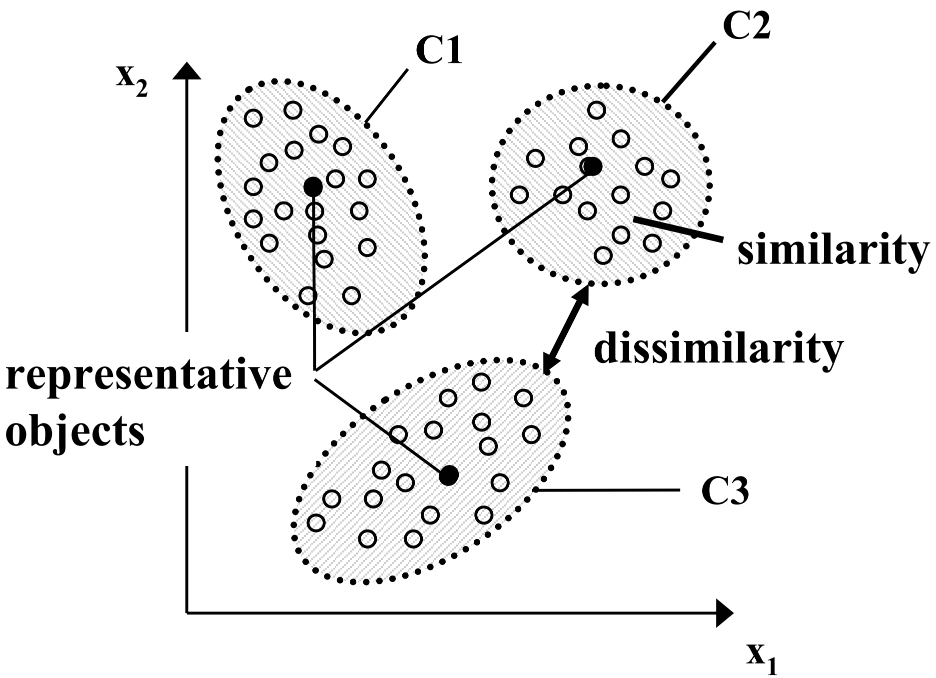


permissible structural design variants, assessment of alternatives

Cluster analysis

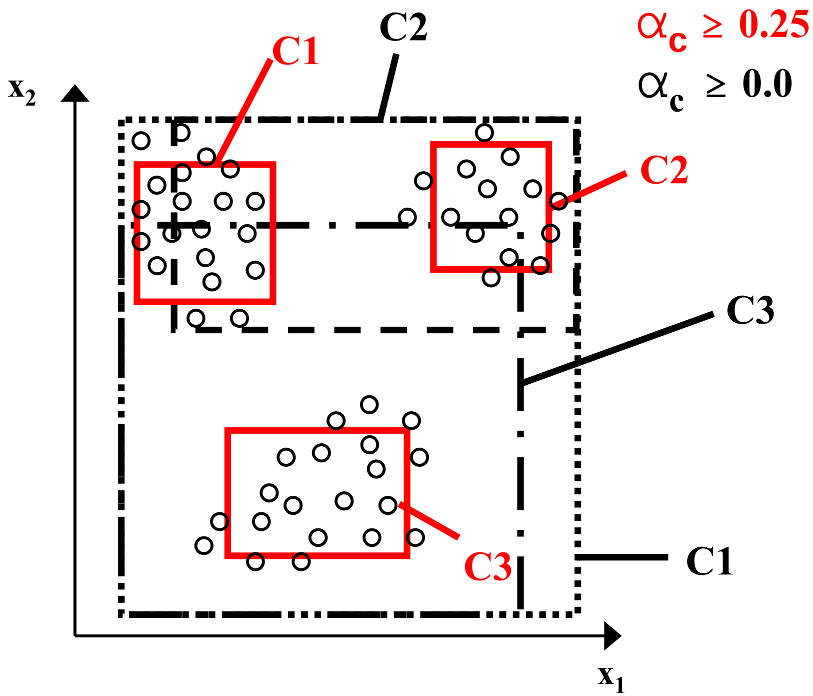
k-medoid method

crisp



- determination of representative objects
- assignment of remaining objects to the most similar representative objects

fuzzy cluster method



- minimization of the functional

$$c = \sum_{v=1}^{k} \frac{\sum_{i,j=1}^{n} \mu_{iv}^{r} \mu_{jv}^{r} d(i,j)}{2 \sum_{j=1}^{n} \mu_{jv}^{r}}$$

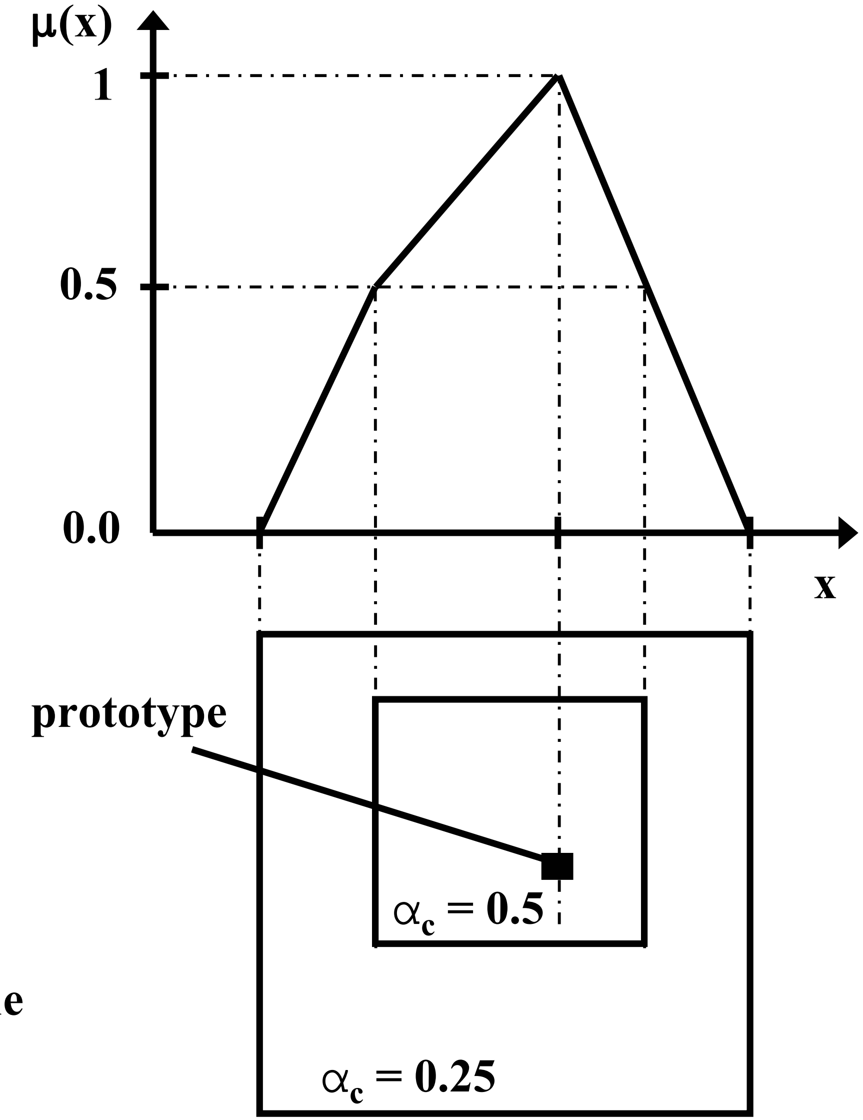
Design variants – modified fuzzy input variables

modeling

- selection of a cluster or cluster combination
- definition of the support of the fuzzy input variable based on the cluster boundaries
- definition of the mean value x with μ(x) = 1

construction of the membership function

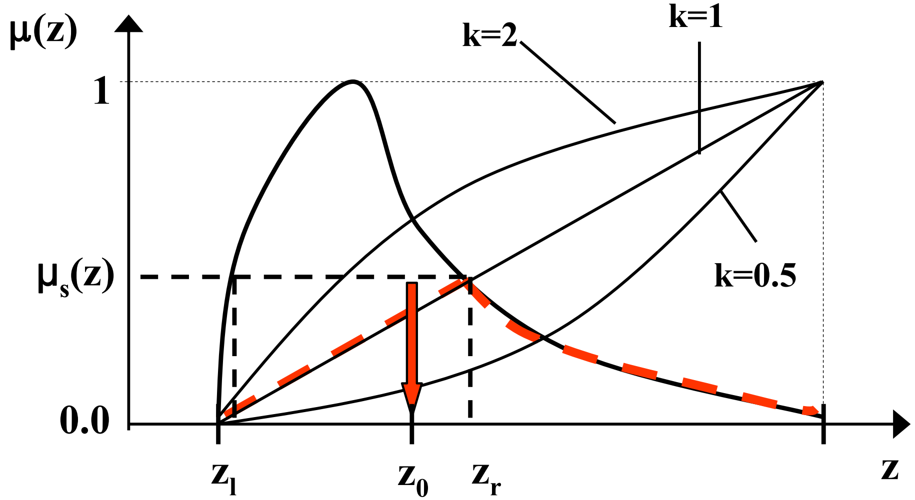
- oriented to the primary fuzzy input variable
- based on α_c-levels from fuzzy clustering
- representative object or prototype as mean value
- geometrical cluster center as mean value



Assessment of alternative design variants (1)

defuzzification

- **JAIN-algorithm**



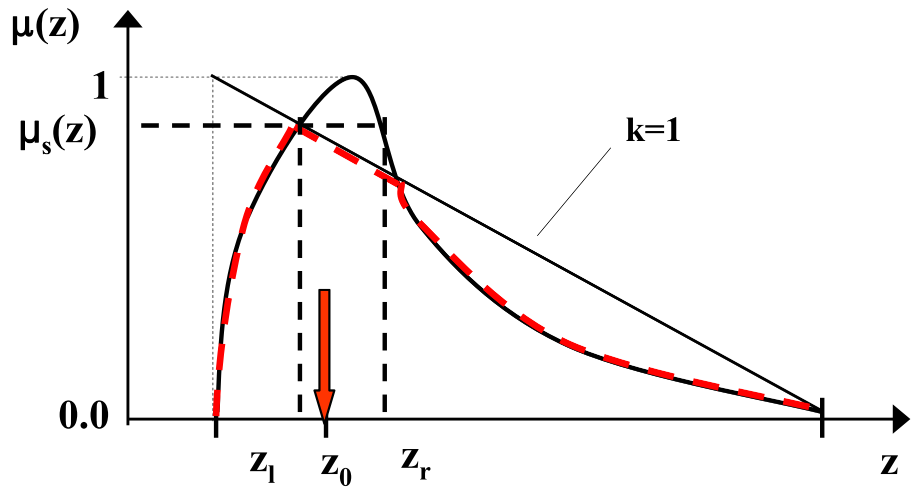
maximizing set $\tilde{z}_{max}$ **with:**

$$\mu(z_{max}) = \left[\frac{z - \inf\limits_{z \in \tilde{z}} [z]}{\sup\limits_{z \in \tilde{z}} [z] - \inf\limits_{z \in \tilde{z}} [z]} \right]^k$$

$$\mu_s(z) = \sup_{z \in \tilde{z}} \min [\mu(z); \mu(z_{max})]$$

$$z_0 = \mu_s |z_r - z_l| + z_l$$

- **CHEN-algorithm**



minimizing set $\tilde{z}_{min}$ **with:**

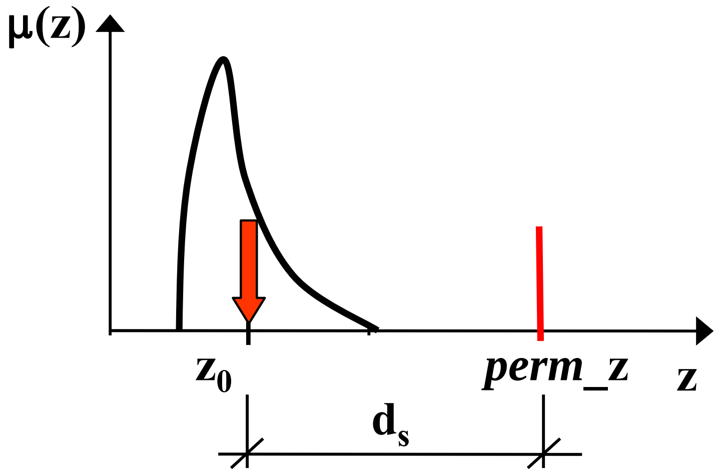
$$\mu(z_{min}) = \left[\frac{\sup\limits_{z \in \tilde{z}} [z] - z}{\sup\limits_{z \in \tilde{z}} [z] - \inf\limits_{z \in \tilde{z}} [z]} \right]^k$$

additional methods:

- **level rank method**
- **centroid method**

Assessment of alternative design variants (2)

constraint distance



$$d_s \longrightarrow \textbf{max}$$

robustness

- **modified SHANNON's entropy**

$$H(\tilde{z}) = -k \cdot \int_{z \in \tilde{z}} \left[\mu(z) \cdot \ln(\mu(z)) + (1-\mu(z)) \cdot \ln(1-\mu(z))\right] dz$$

relative sensitivity measure

uncertainty of the fuzzy result variables
in relation to the
uncertainty of the fuzzy input variables

$$B(\tilde{z}_j) = \sum_i \frac{H(\tilde{z}_j)}{H(\tilde{x}_j)} \longrightarrow \textbf{min}$$

Deterministic computational model

geometrically and physically nonlinear numerical analysis of plane (prestressed) reinforced concrete bar structures

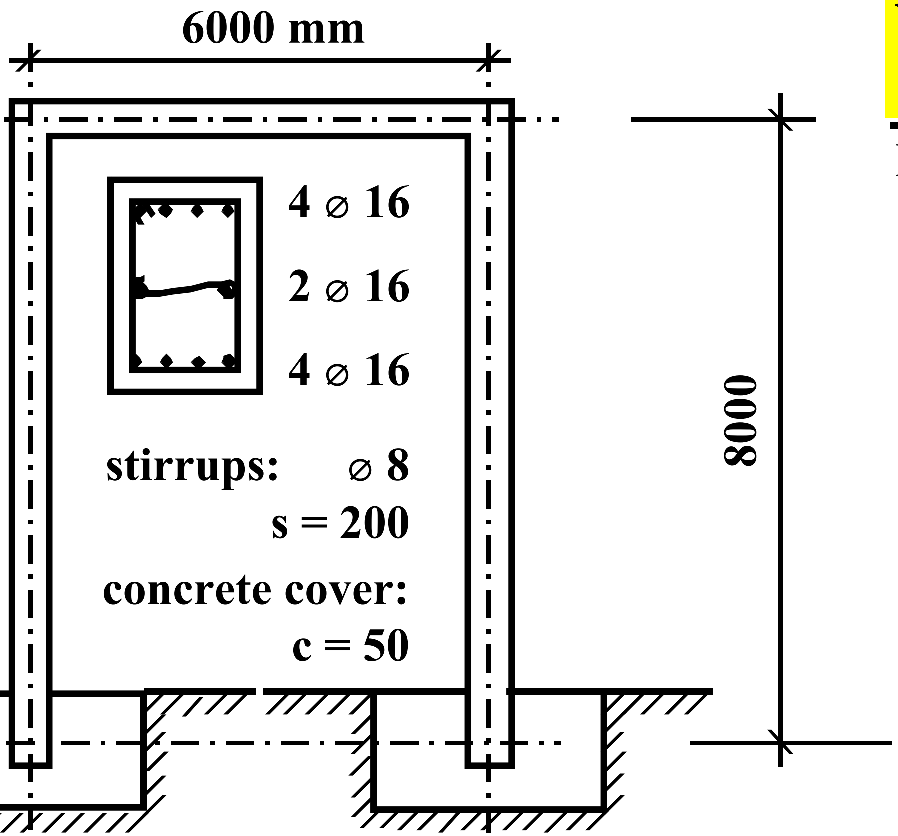
- imperfect straight bars with layered cross sections
- numerical integration of a system of 1st order ODE for the bars
- interaction between internal forces
- incremental-iterative solution technique
  to take account of complex loading processes
- consideration of all essential geometrical and physical nonlinearities:
  - large displacements and moderate rotations
  - realistic material description of reinforced concrete
    including cyclic and damage effects

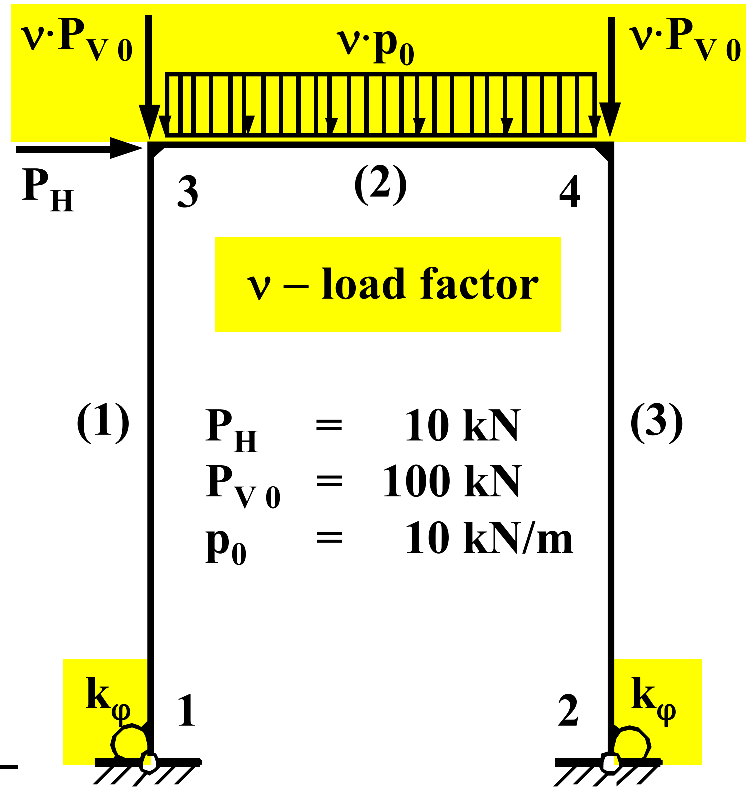
Reinforced concrete frame

cross sections:
500/350

concrete:
C 20/25

reinforcement
steel:
BSt 420





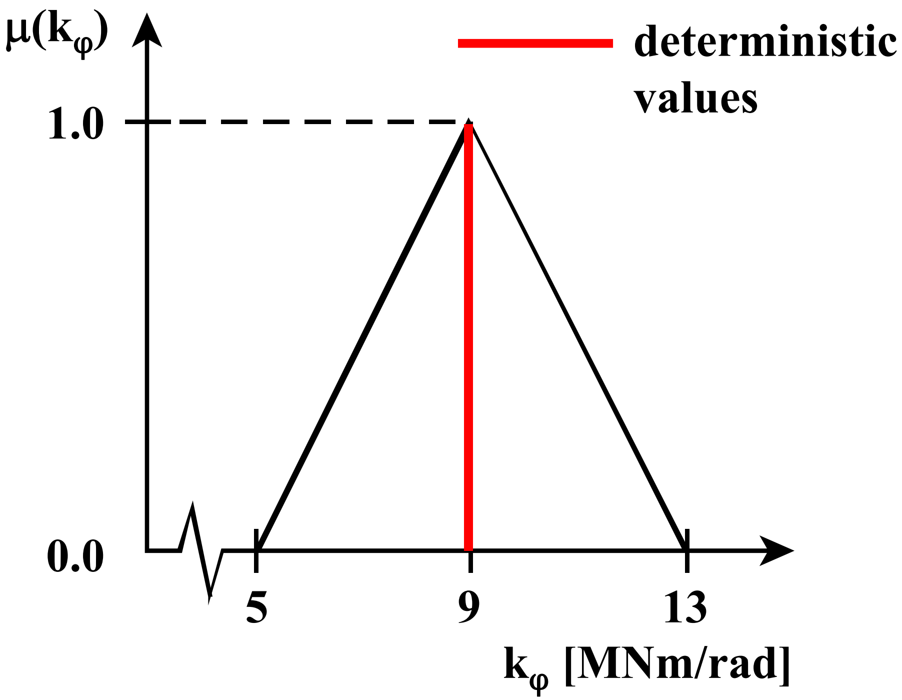
- loading process: 1) dead load
  2) horizontal load P_H
  3) vertical loads $\nu \cdot P_{V\,0}$ and $\nu \cdot p_0$

Fuzzy structural analysis (1)

fuzziness

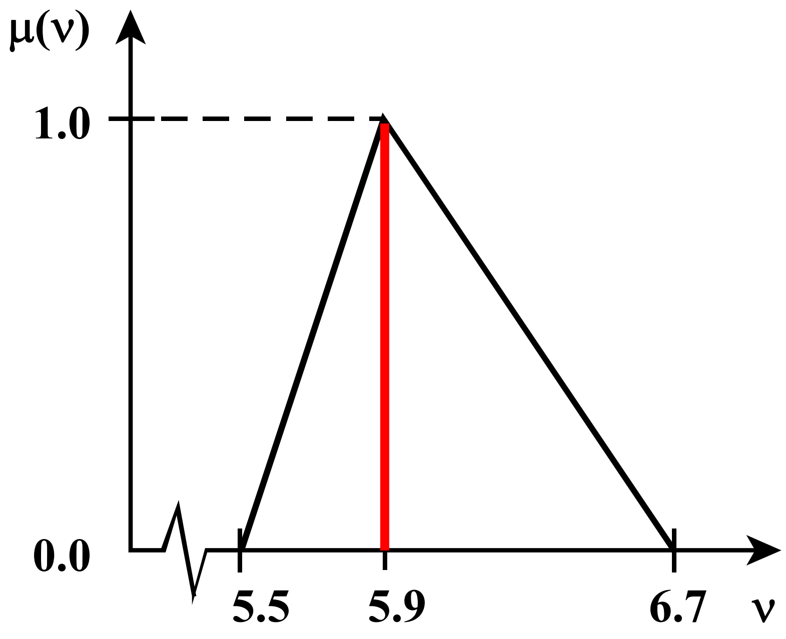
- **fuzzy rotational spring stiffness**

$\tilde{k}_{\varphi} = < 5;\ 9;\ 13 >$ MNm/rad



- **fuzzy load factor**

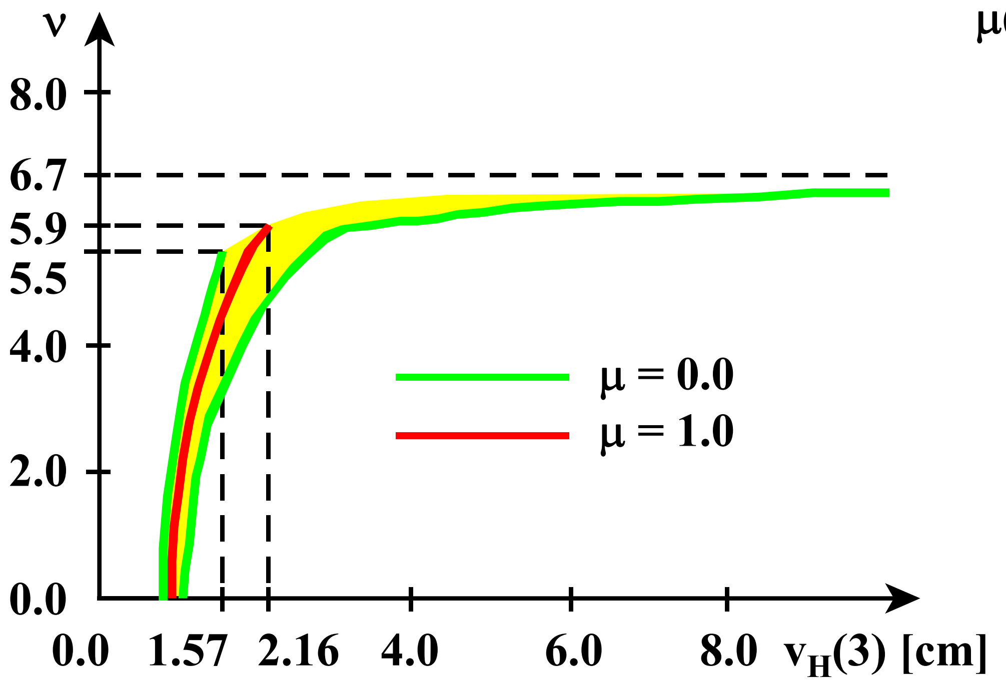
$\tilde{\nu} = < 5.5;\ 5.9;\ 6.7 >$



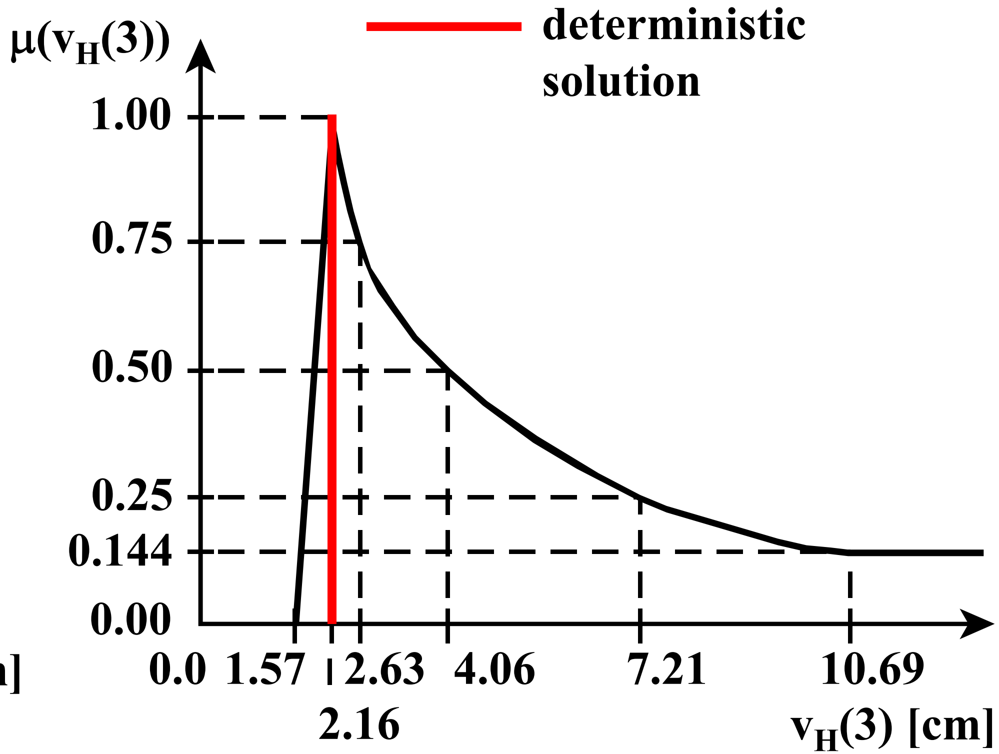
Fuzzy structural analysis (2)

fuzzy structural response

- fuzzy load-displacement dependency (left-hand frame corner, horizontal)



- fuzzy displacement



Fuzzy structural analysis (3)

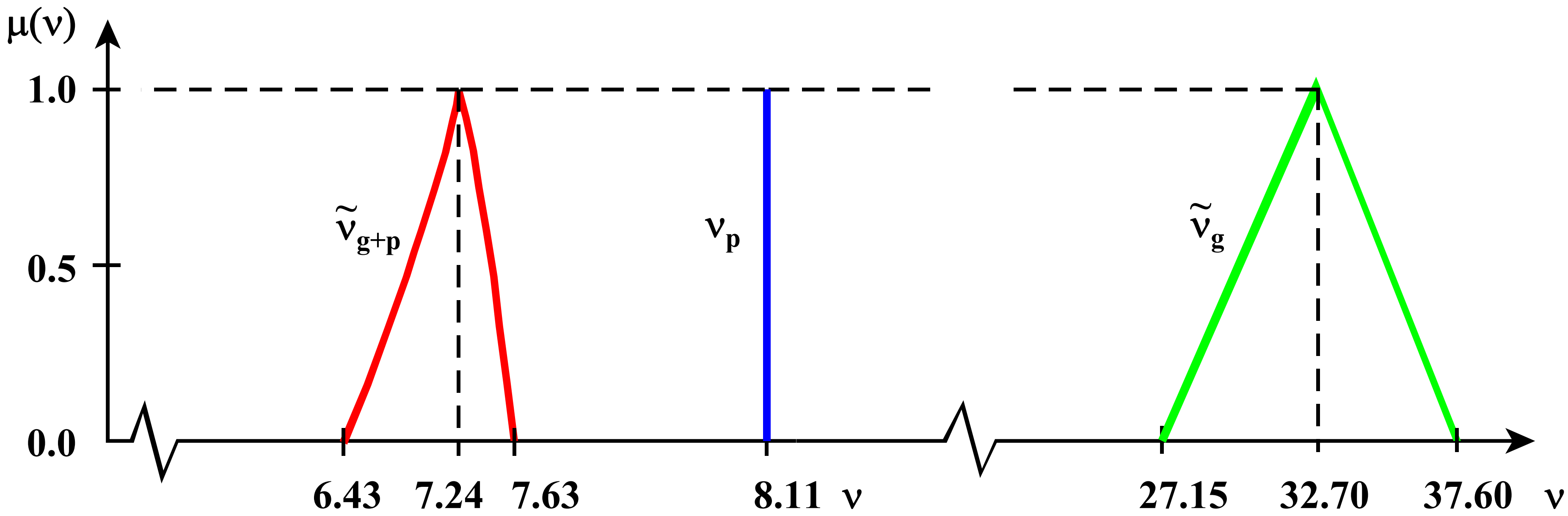
- **fuzzy failure load**
- **influence of the deterministic fundamental solution**

nonlinearities considered:

g + p = geometrical and physical

p = only physical

g = only geometrical



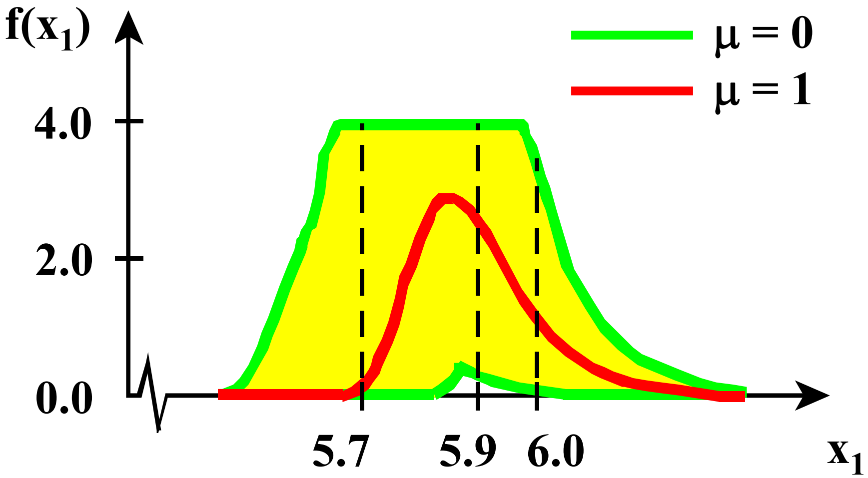
FFORM – analysis I (1)

fuzzy randomness

- load factor ν: $\tilde{X}_1$

  extreme value distribution
  Ex-Max Type I (GUMBEL)

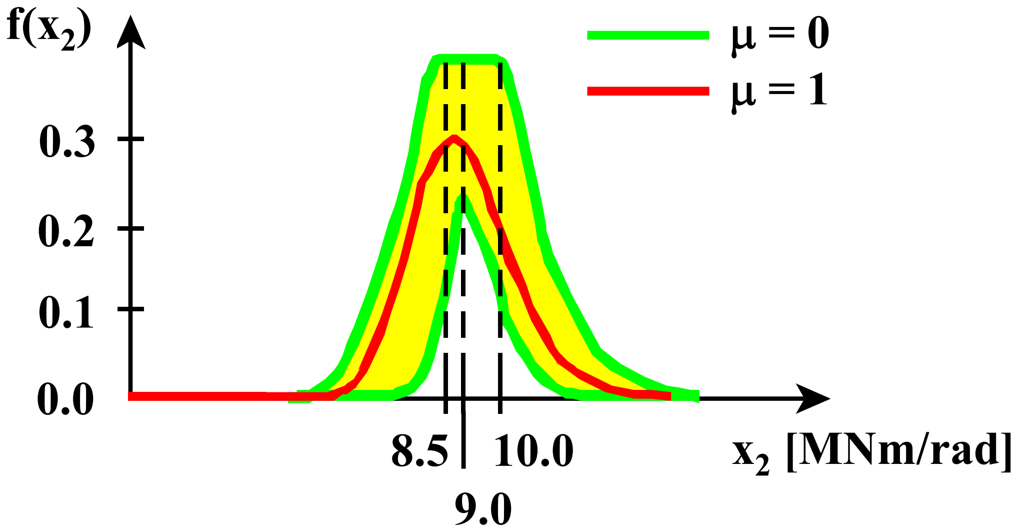
  $\tilde{m}_{x_1} = < 5.7; 5.9; 6.0 >$
  $\tilde{\sigma}_{x_1} = < 0.08; 0.11; 0.12 >$



- rotational spring stiffness k_φ: $\tilde{X}_2$

  logarithmic normal distribution

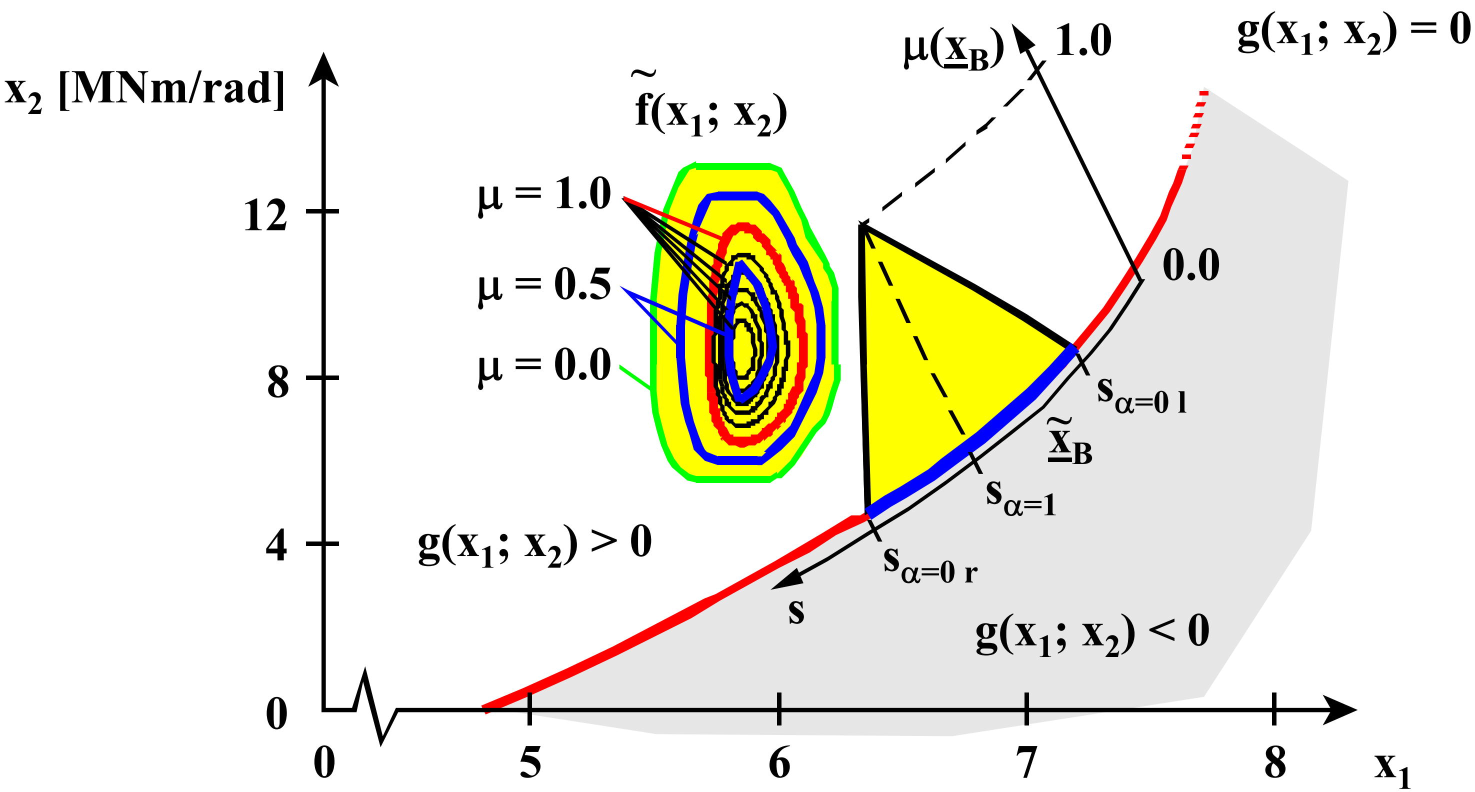
  $x_{0,2} = 0$ MNm/rad
  $\tilde{m}_{x_2} = < 8.5; 9.0; 10.0 >$ MNm/rad
  $\tilde{\sigma}_{x_2} = < 1.00; 1.35; 1.50 >$ MNm/rad



FFORM – analysis I (2)

original space of the fuzzy probabilistic basic variables

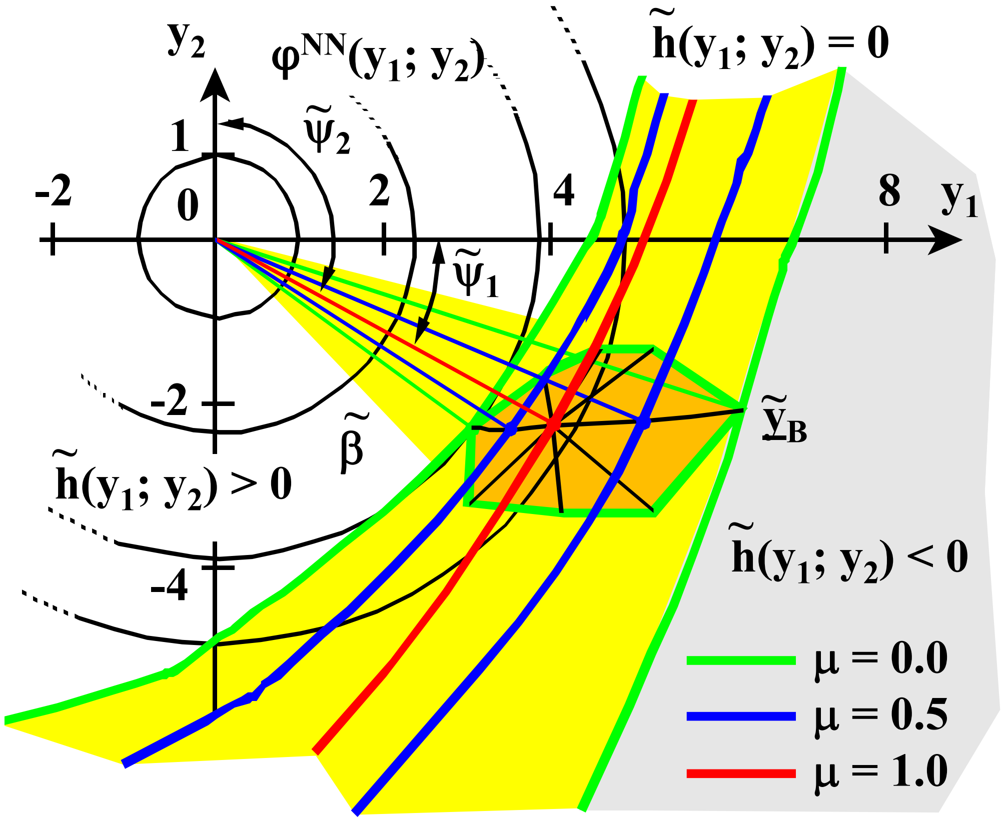
- joint fuzzy probability density function
- crisp limit state surface
- fuzzy design point



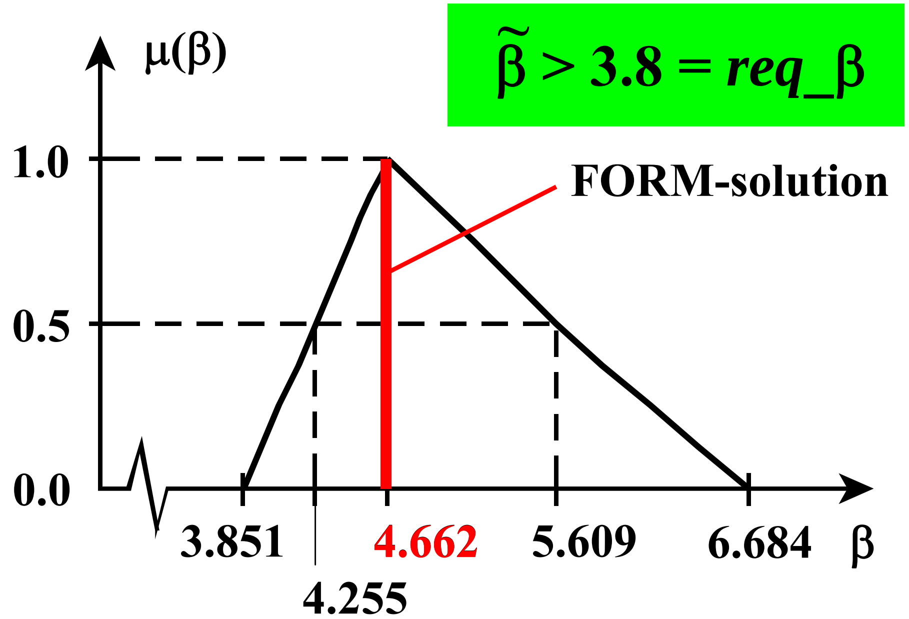
FFORM – analysis I (3)

standard normal space

- crisp standard joint probability density function
- fuzzy limit state surface
- fuzzy design point



- fuzzy reliability index
- safety verification



FFORM – analysis II (1)

fuzziness and fuzzy randomness

- **fuzzy probabilistic basic variable $\tilde{X}_1$: load factor ν**

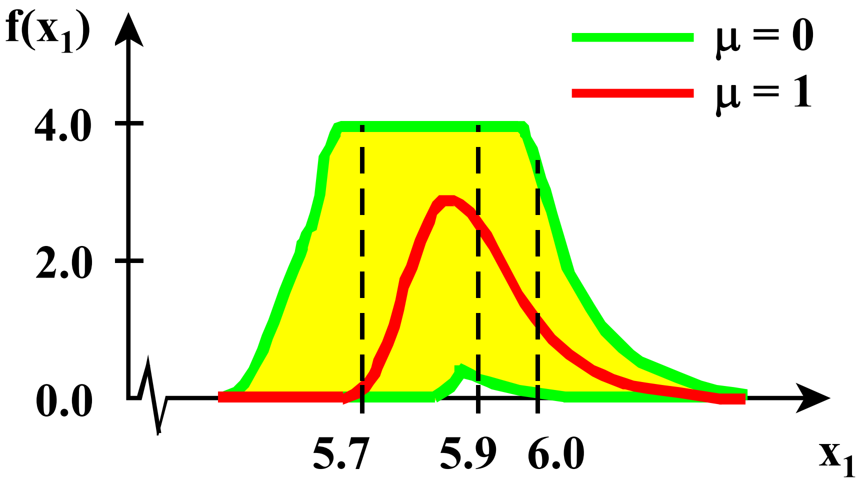
  extreme value distribution Ex-Max Type I (GUMBEL)

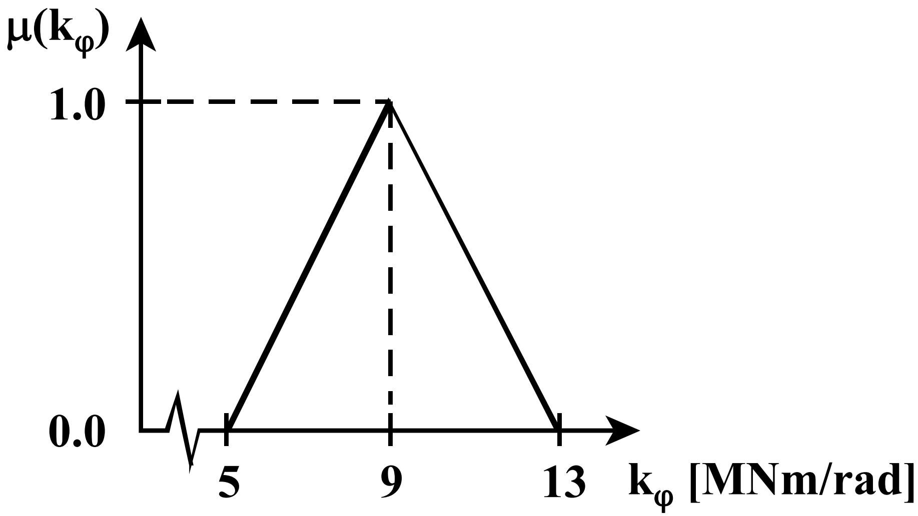
  $\tilde{m}_{x_1} = < 5.7; 5.9; 6.0 >$
  $\tilde{\sigma}_{x_1} = < 0.08; 0.11; 0.12 >$

- **fuzzy structural parameter: rotational spring stiffness $\tilde{k}_\varphi$**

  fuzzy triangular number

  $\tilde{k}_\varphi = < 5; 9; 13 >$ MNm/rad

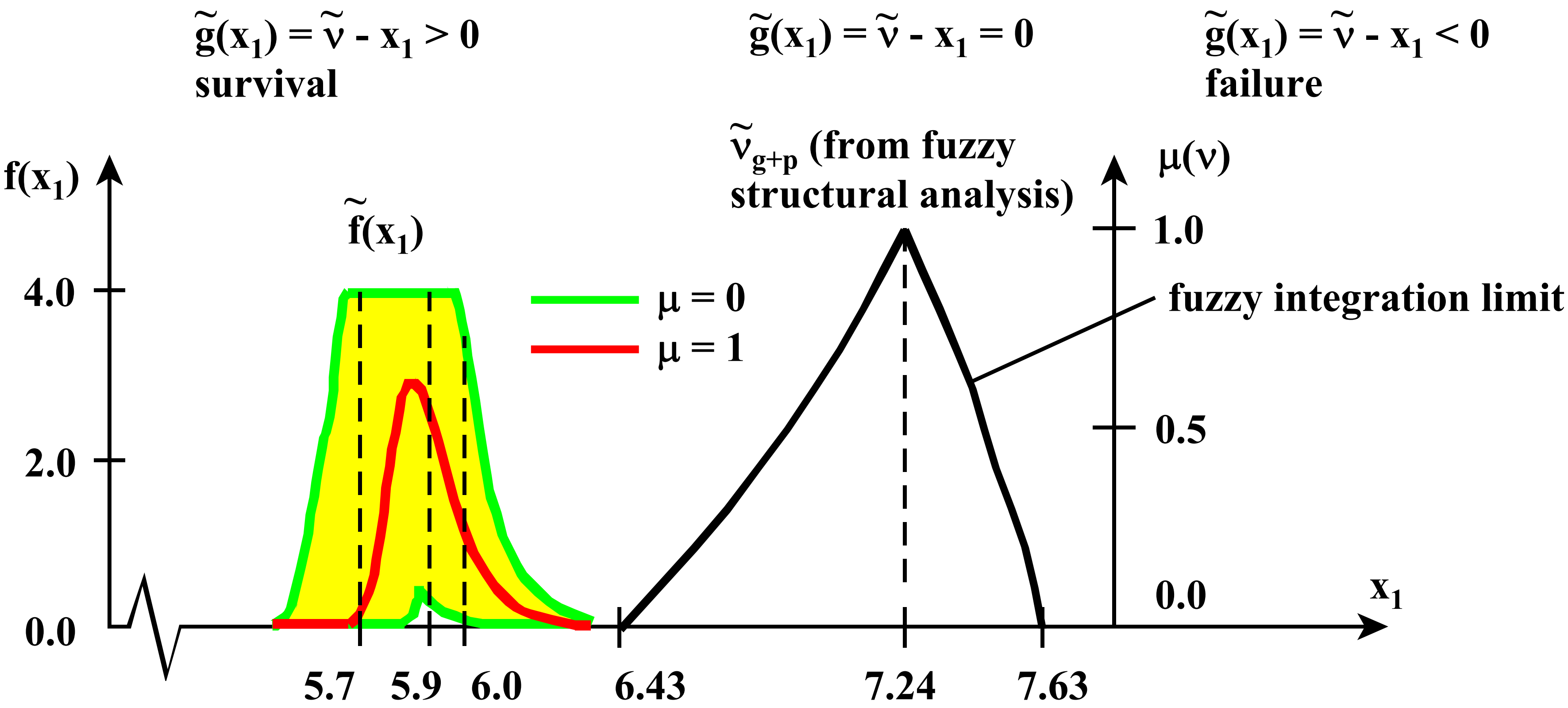




FFORM – analysis II (2)

original space of the fuzzy probabilistic basic variables

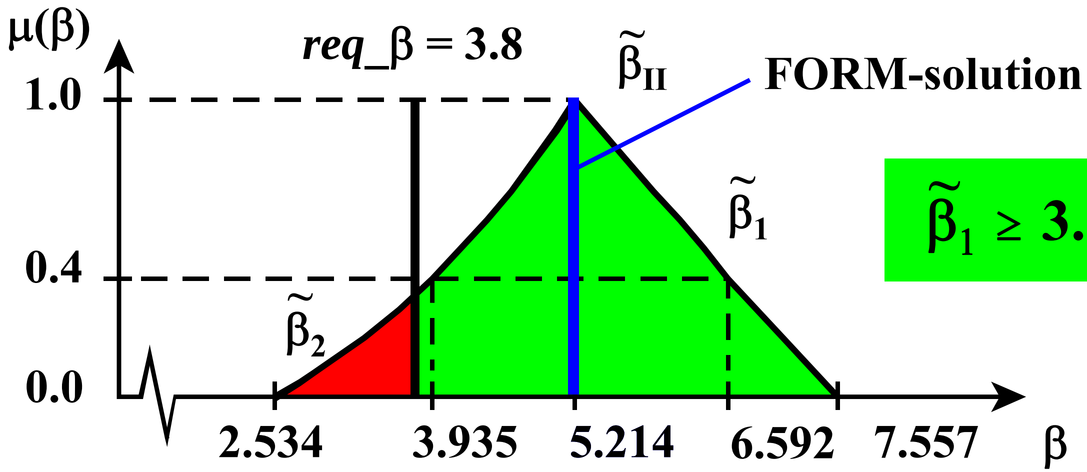
- fuzzy probability density function
- fuzzy limit state surface



FFORM – analysis II (3)

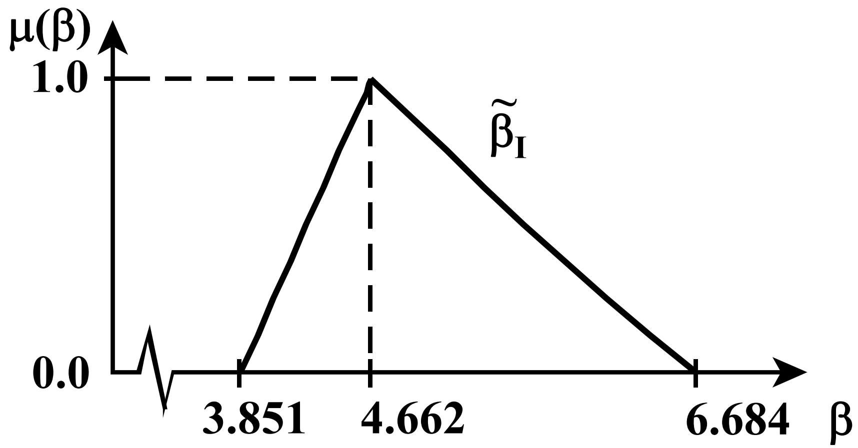
fuzzy reliability index, safety verification

$\tilde{\beta}_2 < 3.8 = erf\_\beta$



$\tilde{\beta}_1 \geq 3.8 = req\_\beta$

- comparison with the result from FFORM-analysis I



modified SHANNON's entropy

$$H_u(\tilde{\beta}_I) = 1.41 \cdot k < 2.48 \cdot k = H_u(\tilde{\beta}_{II})$$

Structural design based on clustering (1)

- **load factor ν: $\tilde{X}_1$**

  extreme value distribution Ex-Max Type I (GUMBEL)

  $\tilde{m}_{x_1} = <5.7;\ 5.9;\ 6.0>$

  $\tilde{\sigma}_{x_1} = <0.08;\ 0.11;\ 0.12>$

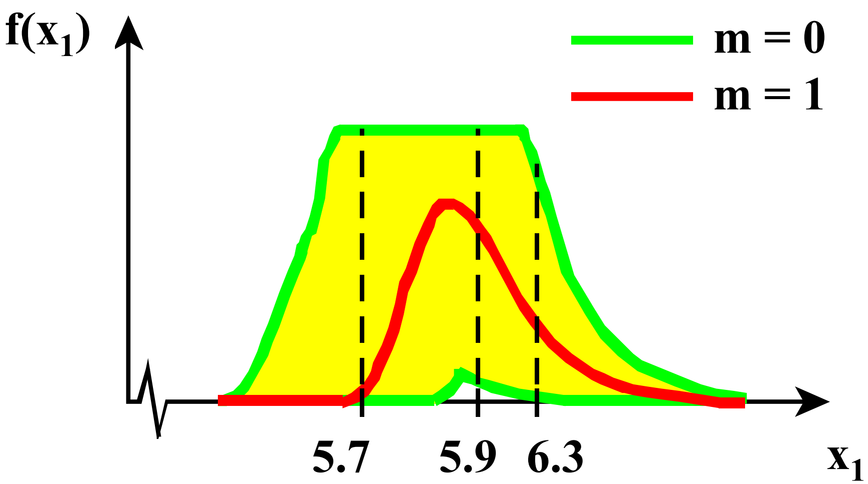
- **rotational spring stiffness k_φ: $\tilde{X}_2$**

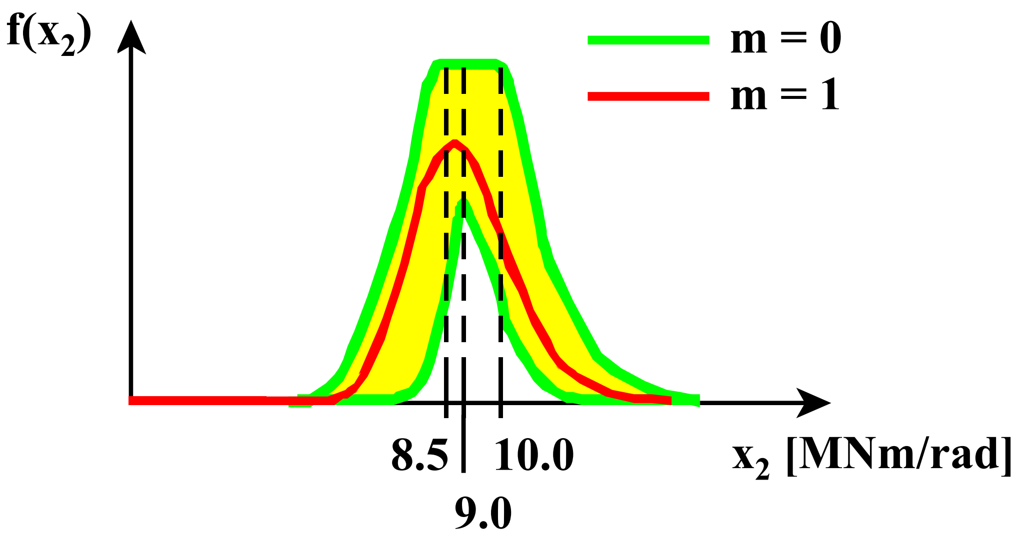
  logarithmic normal distribution

  $x_{0,2} = 0$ MNm/rad

  $\tilde{m}_{x_2} = <8.5;\ 9.0;\ 10.0>$ MNm/rad

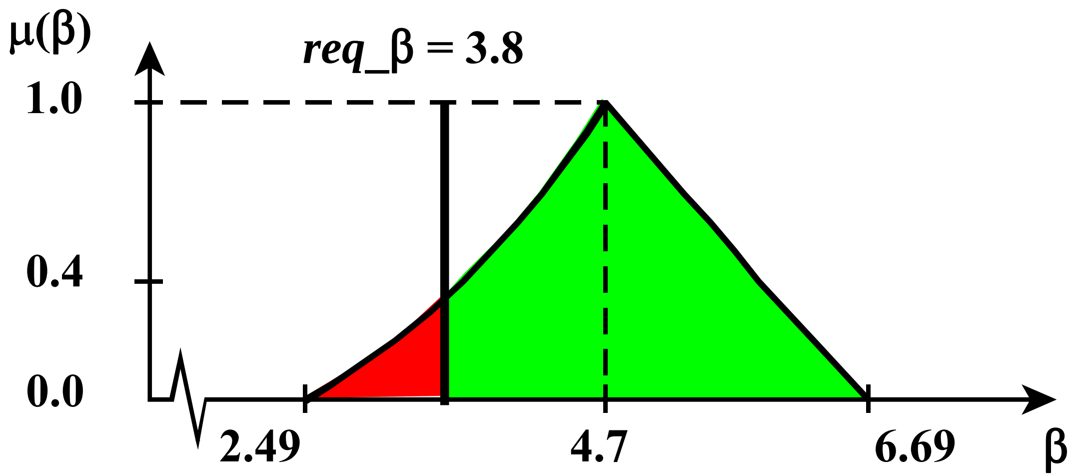
  $\tilde{\sigma}_{x_2} = <1.00;\ 1.35;\ 1.50>$ MNm/rad





Structural design based on clustering (2)

fuzzy reliability index



design constraint

reliability index
***req*_β ≥ 3.8**

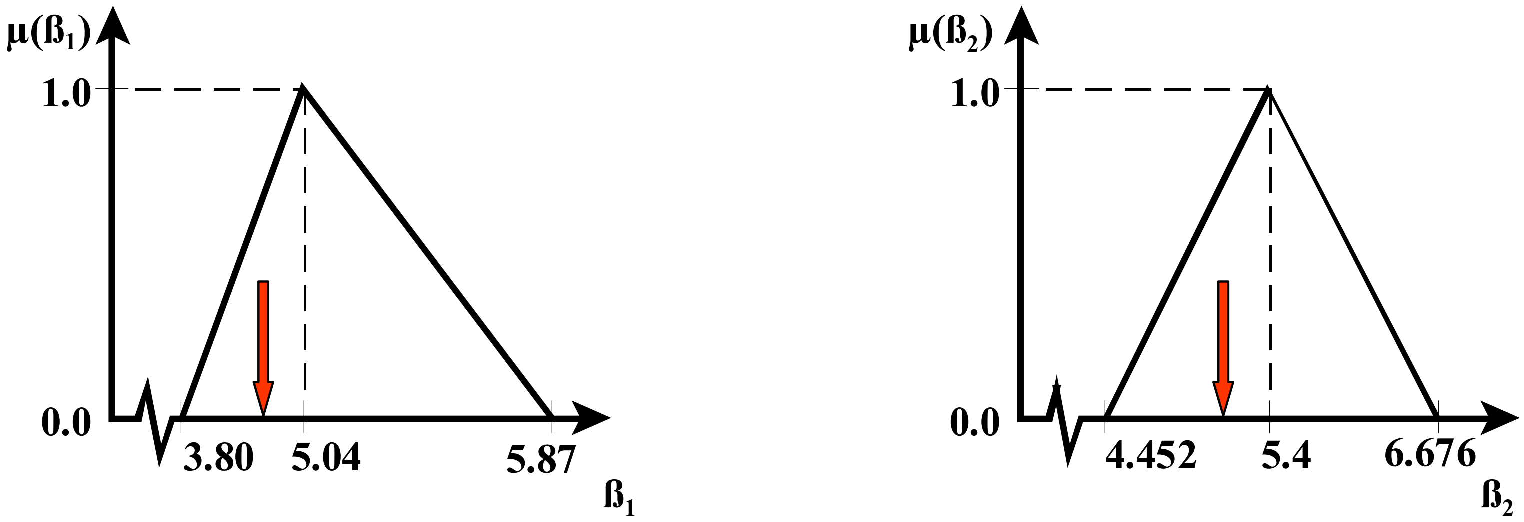
fuzzy cluster analysis ⟶ **modified fuzzy probabilistic basic variables**

design variant	load factor ν		rotational spring stiffness k_φ [MNm/rad]	
	$\tilde{m}_{x_1}$	$\tilde{\sigma}_{x_1}$	$\tilde{m}_{x_2}$	$\tilde{\sigma}_{x_2}$
1	< 5.85; 5.90; 6.0 >	< 0.08; 0.10; 0.12 >	< 8.5; 9.0; 9.6 >	< 1.30; 1.40; 1.50>
2	< 5.70; 5.85; 6.0 >	< 0.08; 0.10; 0.11 >	< 8.9; 9.5; 10.0 >	< 1.02; 1.14; 1.26 >

Structural design based on clustering (3)

FFORM-analysis with the modified fuzzy probabilistic basic variables

design variant 1 | **design variant 2**



defuzzification after CHEN	$z_{01} = 4.60$	$z_{02} = 5.12$
relative sensitivity	$B_1 = 92.67$	$B_2 = 77.48$

Conclusions

- **consideration of non-stochastic uncertainty in structural analysis, design, and safety assessment**
- **enhanced quality of prognoses regarding structural behavior and safety**
- **direct design of structures by means of nonlinear analysis**

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A detailed explanation of all concepts presented and much more may be found in the book:

Bernd Möller
Michael Beer

Fuzzy Randomness

Uncertainty Models in Civil Engineering and Computational Mechanics

Springer-Verlag, 2004