Effects of Error, Variability, Testing and Safety Factors on Aircraft Safety

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Abstract: In this paper we aim to clarify the interaction between error, variability, testing and safety factors on the safety of aircraft structures by using an error model that includes errors made in the calculation of loads and stresses, and also errors in material and geometric parameters. The effect of various representative safety measures taken while designing aircraft structures following the deterministic approach codes in the FAA regulations is investigated. Uncertainties include errors, such as in predicting the response (stress, deflection etc.) of the structure and variability in materials, loading and geometry. Two error models, one is simple and the other is more detailed, are used and the results of these two models are compared. We use a simple model of failure of a representative aircraft structure. In addition, we explore the effectiveness of certification tests for improving safety. It is found that certification tests reduce the calculated failure probabilities by updating the modeling error. We find that these tests are most effective when safety factors are low and when most of the uncertainty is due to systemic errors rather than variability.

Nomenclature

c.o.v.	=	Coefficient of variation
e_m, e_P, e_σ, e_t and e_w	=	Error factor for material failure stress, load, stress, thickness and width
e_{total}	=	Cumulative effect of various errors
P_{act}, P_{calc} and P_d	=	Actual, calculated and design load
$\sigma_{f\,design}$, t_{design} and w_{design}	=	Design values of failure stress, thickness and width
$\sigma_{f\,built}$, t_{built} and w_{built}	=	Average values of failure stress, thickness and width of the components built by an aircraft company
$\sigma_{f\,actual}$, t_{actual} and w_{actual}	=	Actual values of failure stress, thickness and width
$S_{F\,avg}$	=	Fleet-average safety factor
k	=	Error multiplier
$\overline{P}_{nt}$ and $\overline{P}_t$	=	Average value of probability of failure without and with certification
MEF and SEF Cases	=	Multiple Error Factor and Single Error Factor Cases

1. Introduction

Aerospace structures have traditionally been designed using a deterministic approach based on FAA regulations. The safety of the structures has been achieved by combining safety factors with tests of material and structural components. There is a growing interest to replace safety factors by reliability-based design. However, there is no consensus on how to make transition from deterministic design to reliability-based design. An important step in this transition is to understand the way safety is built into aircraft structures now, via explicit safety factors, use of conservative material properties and by testing. Safety measures are intended to compensate for errors and variability. Errors reflect inaccurate modeling of physical phenomena, errors in structural analysis, errors in load calculations, or use of materials and tooling in construction that are different from those specified by the designer. Thus, the errors affect all copies of structural components in the entire fleet of aircraft of the same model. On the other hand, variability reflects variation in material properties, geometry, or loading between different copies of the same structure on different aircraft in the fleet.

Our previous paper (Kale et al, 2004) sought to clarify the interaction between the error, variability and testing on the overall probability of failure. We started with a structural design employing all considered safety measures. The effect of variability in geometry, loads, and material properties was incorporated by the appropriate random variables. For errors we used a simplified model that represented the overall error by a single random variable used in the calculation of stress. In this paper, we use a more detailed model in which we consider individual error components in load calculation, stress calculation, material properties and geometry parameters. The objective of the paper is to observe differences between the use of the simple model and the more detailed model.

As in our previous paper, we transform the errors into random variables by considering the design of multiple aircraft models. As a consequence, for each model the structure is different. It is as if we pretend that there are hundreds of companies (Airbus, Boeing, Bombardier, Embraer etc.) each designing essentially the same airplane, but each having different errors in their structural analysis and manufacturing. For each model we simulate certification testing. If the airplane passes the test, then an entire fleet of airplanes with the same design is assumed to be built with different members of the fleet having different geometry, loads, and material properties based on assumed models for variability in these properties. That is, the uncertainty due to variability is simulated by considering multiple realizations of the same design, and the uncertainty due to errors is simulated by designing different structures to carry the loads specified by the FAA.

We consider only stress failure due to extreme loads, which can be simulated by an unstiffened panel designed under uniaxial loads. No testing of components prior to certification is analyzed for this simple example.

2. Structural uncertainties

A good analysis of different sources of uncertainty is provided by Oberkampf et al. (2000). Here we simplify the classification with a view to the question of how to control uncertainty. We propose in Table 1 a classification that distinguishes between (1) uncertainties that apply equally to the entire fleet of an aircraft model and (2) uncertainties that vary for the individual aircraft. The distinction is important because safety measures usually target one or the other.

Similarly, the uncertainty in the failure of a structural member can also be divided into two types: systemic errors and variability. Systemic errors reflect inaccurate modeling of physical phenomena, errors in structural analysis, errors in load calculations, or use of materials and tooling in construction that are different from those specified by the designer. Systemic errors affect

all the copies of the structural components built using the same model and are therefore fleet-level uncertainties. The other type of uncertainty reflects variability in material properties, geometry, or loading between different copies of the same structure and is called here individual uncertainty.

Table 1. Uncertainty Classification

Type of uncertainty	Spread	Cause	Remedies
Systemic error (modeling errors)	Entire fleet of components designed using the model	Errors in predicting structural failure and differences between properties used in design and average fleet properties.	Testing and simulation to improve math model and the solution.
Variability	Individual component level	Variability in tooling, manufacturing process, and flying environments.	Improve tooling and construction. Quality control.

3. Safety Measures

Aircraft structural design is still done by and large using code-based design rather than probabilistic approaches. Safety is improved through conservative design practices that include use of safety factors and conservative material properties. It is also improved by tests of components and certification tests that can reveal inadequacies in analysis or construction. In the following we detail some of these safety measures.

Safety Margin: Traditionally all aircraft structures are designed with a safety factor to withstand 1.5 times the limit load without failure.

A-Basis Properties: In order to account for uncertainty in material properties, the Federal Aviation Administration (FAA) recommends the use of conservative material properties. This is determined by testing a specified number of coupons selected at random from a batch of material. The A-basis property is determined by calculating the value of a material property exceeded by 99% of the population with 95% confidence.

Component and Certification tests: Component tests and certification tests of major structural components reduce stress and material uncertainties for given extreme loads due to inadequate structural models. These tests are conducted in a building block procedure. First, individual coupons are tested, and then a sub assembly is tested followed by a full-scale test of the entire structure. Since these tests cannot apply every load condition to the structure, they leave uncertainties with respect to some loading conditions. It is possible to reduce the probability of failure by performing more tests to reduce uncertainty or by extra structural weight to reduce stresses. If certification tests were designed together with the structure, it is possible that additional tests would become cost effective because they would allow reduced structural weight.

4. Errors in Stress, Load, Geometry and Material Allowable

4.1. Errors in design

We assume that different aircraft companies like Airbus, Boeing, Bombardier, Embraer, etc. essentially design the same airplane. Before performing a stress analyses, we first assume that these companies perform aerodynamic analyses to determine the loads acting on aircraft. However, the loads calculated by an aircraft company, P_{calc}, differ from the loads corresponding to FAA design specifications, P_d. The error made in load calculation, e_p, is different from one company to another. Throughout all error factor definitions we consistently formulate the expressions such that positive error factor implies a conservative decision. Based on this, P_{calc} is expressed in terms of P_d as:

$$P_{calc} = (1 + e_P) P_d \tag{1}$$

Besides the error in load calculation, an aircraft company has also errors in stress calculation. Considering a small part of the aircraft structure, we can represent it as an unstiffened panel such that the value of stress calculated by stress analysis team, σ_{calc}, is expressed in terms of the load values calculated by the aerodynamics team, P_{calc}, the design width, w_{design}, and thickness, t. Hence, introducing the term e_σ representing error in the stress analysis we can write

$$\sigma_{calc} = (1 + e_\sigma) \frac{P_{calc}}{w_{design}\, t} \tag{2}$$

Equation (2) is used by the designer to calculate the design thickness t_{design} required to carry the calculated design load times the safety factor S_F. That is

$$t_{design} = (1 + e_\sigma)(1 + e_P) \frac{S_F P_d}{w_{design}\, \sigma_a} \tag{3}$$

where σ_a is the value of allowable stress used in the design. This allowable stress is based on A-basis properties (see Appendix 1) for the design material.

4.2. Errors in implementation (difference between design value and fleet average)

The error factors e_σ and e_σ represent the errors made in the design stage. In addition, there will be some implementation errors in the geometric and material parameters. These implementation errors represent the difference between the values of these parameters in an average airplane (fleet-average) built an aircraft company and the design values of these parameters. Since we represent a small part of the aircraft structure as an unstiffened panel, the geometry parameters are the width and the thickness of the panel. Errors in panel width, e_w, represent the deviation of the values of panel width designed by an individual aircraft company, w_{design}, from the average value of panel width and thickness of panels built by the company, w_{built}. Thus we have

$$w_{built} = (1 + e_w)\, w_{design} \tag{4}$$

Similarly the built thickness value will differ from the design value such that

$$t_{built} = (1 + e_t)\, t_{design} \tag{5}$$

In addition average built material parameters and the design material parameters will be different from each other. In particular, the failure stresses σ_f are related as

$$\sigma_{f\,built} = (1 - e_m)\, \sigma_{f\,design} \tag{6}$$

The relationship between the allowable and failure stresses is that the allowable stress is the A-basis value of failure stress. The detailed explanation on the computation of A-basis value is given in Appendix 1. The formulation of Eq.(6) is different from Eqs. (1, 2, 4 and 5) in that the sign in front of the error factor e_m is negative. The reason is that we consistently formulate the expressions such that positive error factor implies a conservative decision.

4.3. Fleet average safety factor

The fleet average of stress experienced by a panel under the correct design loads is

$$\sigma_{d-avg} = \frac{P_d}{t_{built}\, w_{built}} \tag{7}$$

Substituting from Eqs.(3-5) into Eq. (7) we have

$$\sigma_{d-avg} = \frac{1}{(1 + e_\sigma)(1 + e_P)(1 + e_w)(1 + e_t)} \frac{\sigma_a}{S_F} \tag{8}$$

Then, we can define a fleet average safety factor

$$S_{F\,avg} = \frac{\sigma_{f\,built}}{\sigma_{d-avg}} \tag{9}$$

Combining Eqs. (6) and (9) yields

$$S_{F\,avg} = S_F \frac{\sigma_{f\,design}}{\sigma_a} \left(1 + e_{total}\right) \tag{10}$$

where

$$e_{total} = \left[(1 + e_\sigma)(1 + e_P)(1 + e_w)(1 + e_t)\ (1 - e_m)\right] - 1 \tag{11}$$

Here e_{total} represents the cumulative effect of the various errors on the safety factor for the <u>average</u> airplane (fleet average) built by a company. Equation (10) shows that when there are no errors, the average safety factor is larger than S_F due to conservative allowable stress (A-Basis properties). The error factors are random variables represented by distribution type, their average values and their bounds as given in Table 2. In addition there is variability in the material and

geometric properties and the load experienced in actual flight between individual aircraft in the fleet. This will be discussed next.

Table 2. Distribution of error factors and their bounds

Error factors	Distribution Type	Mean	Bounds
Error in stress calculation, e_σ	Uniform	0.0	± 5%
Error in load calculation, e_P	Uniform	0.0	± 10%
Error in width, e_w	Uniform	0.0	± 1%
Error in thickness, e_t	Uniform	0.0	± 2%
Error in material allowable, e_m	Uniform	0.0	± 20%

Table 2 presents nominal values for the error factors. In the Results section of the paper we will vary these error bounds and investigate the effects of these changes on the probability of failure. As seen in Table 2, the error having the largest bound in its distribution is the error in material failure stress, because it includes also the likelihood of unexpected failure modes.

4.4. Variability

In the previous sections, we analyzed the different types of errors made in the design and implementation stages representing the differences between the fleet average values of geometry, material and loading parameters and their corresponding design values. These parameters, however, vary from one aircraft to another in the fleet. For instance, we assume that the actual value of thickness of a panel in an aircraft is defined by the fleet average thickness value by

$$t_{act} = U\left(t_{built}; 3\% \, bounds\right) \tag{12}$$

Here '*U*' indicates that the distribution is uniform, 't_{built}' is the average value of thickness (fleet average) and '3% bounds' defines that the lower bound for thickness value is the average value minus 3% of the average and the upper bound for thickness value is the average value plus 3% of the average. Note that the thickness error in Table 2 is uniformly distributed with bounds of ± 2%. Thus the difference between all thicknesses over the fleets of all companies is up to ± 5%. However, the combination of error and variability is not a uniform distribution. Table 3 presents the assumed distributions for variabilities.

Table 3. Distribution of random variables having variability

Variables	Distribution type	Mean	Scatter
Actual service load, P_{act}	Lognormal	$P_d = 100$	(10%) c.o.v.*
Actual panel width, w_{act}	Uniform	$w_{built} = 1$	(1%) bounds
Actual panel thickness, t_{act}	Uniform	t_{built}	(3%) bounds
Actual failure stress, $\sigma_{f\ act}$	Lognormal	$\sigma_{f\ built} = 150$	(10%) c.o.v.*

* c.o.v. = coefficient of variation

5. Certification Tests

We simulate the effect of safety measures mentioned in Section 3 by assuming the statistical distribution of the uncertainties and incorporating them in approximate probability calculations and a two-level Monte Carlo simulation (see Figure 1), with different aircraft models being considered at the upper level, and different instances of the same aircraft at the lower level. To simulate the epistemic uncertainty, we assume that we have a large number of nominally identical aircraft being designed (e.g. by Airbus, Boeing, Bombardier, Embraer etc.), with the errors being fixed for each aircraft. We consider a simple example of an unstiffened panel designed for strength under uniaxial tensile loads. The Monte Carlo simulation works as follows.

After the structural component has been designed with random errors in stress, load, width, allowable stress and thickness (step A in Fig. 1), we simulate certification testing for the aircraft (step B in Fig. 1). Here we assume that the component will not be built with complete fidelity to the design due to variability in geometric properties. That is, the actual values of these parameters w_{act} and t_{act} and will be different from the fleet-average values w_{built} and t_{built} due to variability. The panel is then loaded with the design axial force of S_F times P_{calc}, and the stress in the panel is recorded (step C in Fig. 1). If this stress exceeds the failure stress $\sigma_{f\ act}$ (itself a random variable with an average value $\sigma_{f\ built}$, see Table 3.) then the design is rejected, otherwise it is certified for use. That is, the airplane is certified if the following inequality is satisfied and we can build multiple copies of the airplane.

$$\sigma - \sigma_f = \frac{S_F P_{calc}}{w_{act}\, t_{act}} - \sigma_{f\ act} \le 0 \tag{13}$$

or

$$S_F \left(1 + e_P\right) P_d \le R_{act} = w_{act}\, t_{act}\, \sigma_{f\ act} \tag{14}$$

where the left side denotes the applied load, and the right side the load bearing capacity or "resistance" R_{act}. As noted earlier the terms w_{act}, t_{act} and $\sigma_{f\ act}$ in Eq. (14) reflect the variability in geometric and material properties (see Table 3). The distribution types and the distribution parameters of the random variables used in design and certification are listed in Table 3.

Effect of Certification Tests on Distributions of e_{total} and $S_{F\ avg}$

The fleet-average safety factor (see Eq. (10)) is defined in terms of safety factor, failure stress ratio and total error property. Amongst those terms only the error term is subject to change due to certification testing. One can argue that the way certification tests reduce the probability of failure is by changing the distribution of the error factor e_{total}. Without certification testing, we assume uniform distributions for the error factors in stress, load, width and thickness. However, designs based on unconservative models are more likely to fail certification, and so the distribution of e_{total} becomes conservative for structures that pass certification. In order to quantify this effect, we calculated the updated distribution of the error factor e_{total}. The updated distribution is calculated by Monte Carlo simulations.

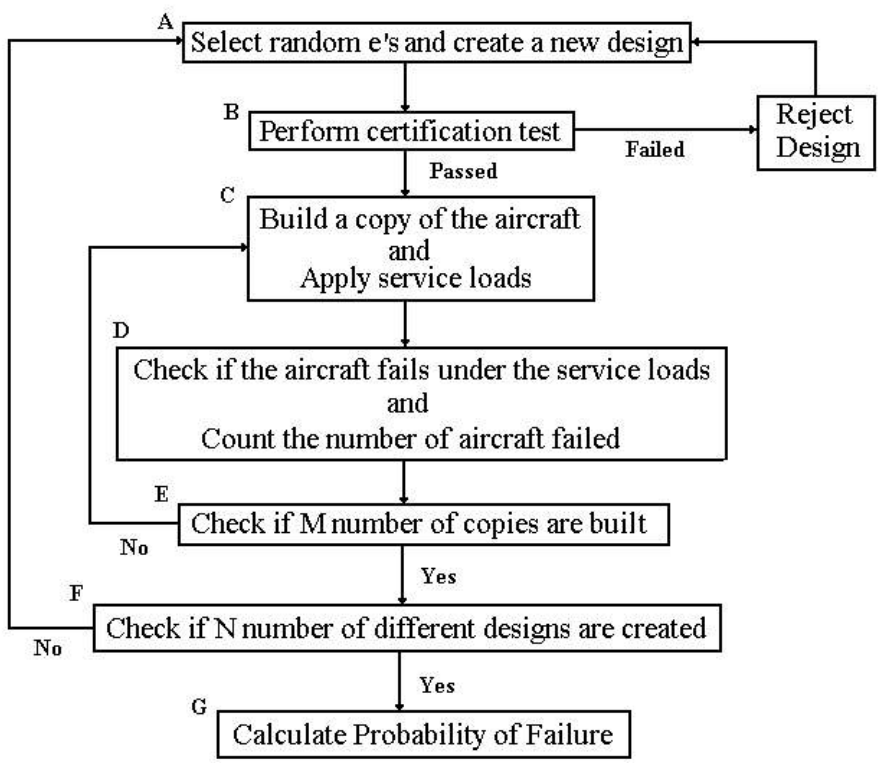

Figure 1. Flowchart for Monte Carlo simulation of component design and failure

As noted earlier, in our previous paper (Kale et al, 2004) we represented the overall error with a single parameter, hereinafter the "Single Error Factor Case (SEF case)", and used uniform distribution for the initial distribution of this error. However, in the present work we use a more complex representation of error with individual error factors, hereinafter the "Multiple Error Factor Case (MEF case)", and we represent initial distribution of each error factor with uniform distribution. For the SEF Case we obtained updated the distribution of error term using Bayesian updating. However, since we use a more complex model to represent error in this study, updating by analytical means is quite difficult. In addition, we prefer to update average-safety factor, $S_{F\ avg}$. Revisiting the expression for average safety factor (see Eq. (10)), we see that only random variables are σ_a and e_{total}. Since the variability in σ_a is very small compared to e_{total}, the distribution of

$S_{F\,avg}$ and e_{total} are nearly the same. We calculated the updated distribution of average safety factor thru Monte Carlo simulations of sample size 10,000. Initial and updated distribution of $S_{F\,avg}$ are shown in Fig.2.

Figure 2 shows us how certification tests update the distribution of average safety factor for SEF and MEF cases. For SEF case the uniform initial distribution is updated such that the likelihood of higher values of average safety factor is increased. That is the components built with low safety factor are rejected in certification tests. Similarly for the MEF case, the initial distribution of average safety factor is shifted to up and right indicating that the components with high safety factors are favored via certification testing.

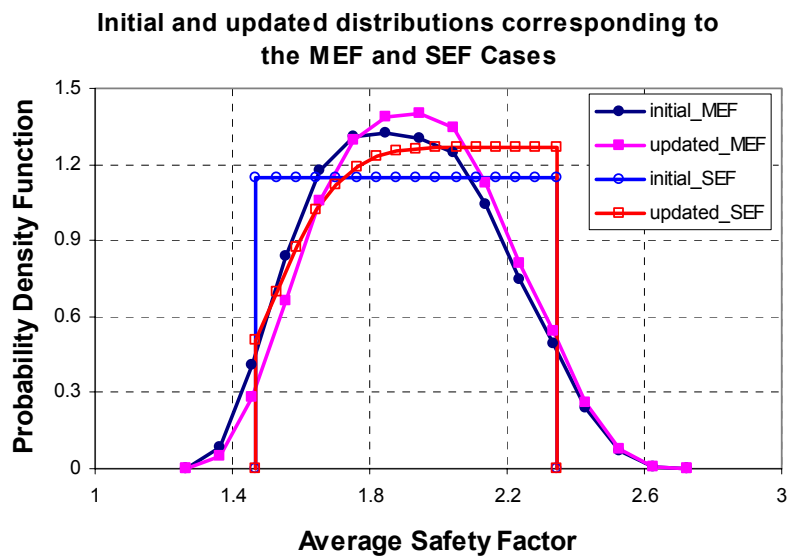

Figure 2. Comparing initial and updated distribution of S_{Favg} between SEF and MEF cases. The single error is chosen as to match the standard deviation of the safety factor MEF.

6. Probability of Failure Calculation

After the component passes the certification test, we subject the component in each airplane to actual random maximum (over a lifetime) service loads (step D in Fig. 1) and decide whether it fails using Eq. (15).

$$P_{act} \geq R = t_{act}\, w_{act}\, \sigma_{f\ act} \tag{15}$$

Here, P_{act} is the actual load acting under service, and R is the resistance or load capacity of the structure in terms of the random width w_{act}, thickness t_{act} and failure stress, $\sigma_{f\ \mathrm{act}}$.

This procedure of design and testing is repeated (steps A-B in Fig.1) for N different aircraft models. Here N different design is the representative of different designs of different aircraft

companies (outer loop of Monte Carlo simulation). For each new model, different random error factors e_σ, e_P, e_w e_t and e_m are picked for the design representing the different error factors for the different aircraft companies.

The inner loop in Figure 1 (steps C-E in Fig.1) represents the simulation of a population of M airplanes (hence structural components) that all have the same design. However, each component is different due to variability in geometry, failure stress, and loading (step D). We subject the component in each airplane to actual random maximum (over a lifetime) service loads (step E) and calculate whether it fails using Eq. (15).

We count the number of panels failed for each airplane, and add up all the failures. The failure probability is calculated by dividing the number of failures by the number of airplane models that passed certification, times the number of copies of each model.

7. Results

We first investigate the effect of error bounds on the probability of failure of panels. Since we have 5 different contribution to total error in the analysis, we scale all error components with a single multiplier, k,

$$e_{total} = \left[(1+ke_\sigma)\ (1+ke_P)\ (1+ke_w)\ (1+ke_t)\ (1-ke_m)\right]-1 \tag{16}$$

and explore the effect of k on failure probability. We calculated the average value and coefficient of variation of probability of failure values for the panels designed with A-basis properties and safety factor of 1.5.

Table 4. Average and coefficient of variation (over N=500 companies) of probability of failure for the components designed with A-basis properties and S_F=1.5. Monte Carlo simulations with N=500, M=20,000.

k	$\overline{P}_{nt}$ and c.o.v.(P_{nt}^*)	$\overline{P}_t$ and c.o.v.(P_t^*)	$\overline{P}_{nt} - \overline{P}_t$	$\overline{P}_t / \overline{P}_{nt}$
0.5	1.510 x10^{-5} (330 %)	1.483 x10^{-5} (336 %)	2.740 x10^{-7}	0.982
0.75	9.000 x10^{-5} (289 %)	8.053 x10^{-5} (313 %)	9.474 x10^{-6}	0.895
1	3.626 x10^{-4} (448 %)	2.464 x10^{-4} (596 %)	1.163 x10^{-4}	0.679
1.5	5.275 x10^{-3} (366 %)	1.294 x10^{-3} (496 %)	3.981 x10^{-3}	0.245
2	3.106 x10^{-2} (318 %)	1.905 x10^{-3} (694 %)	2.916 x10^{-2}	0.061

* P_{nt} and P_t are the probability of failure without and with certification testing, respectively.

Table 4 shows that as the error in analyses increases, i.e. k increases, the average values probability of failures (both with and without certification) of the components are also increases. The coefficient of variation of failure probability is very large. With N=500, the coefficient of variation of the average between repeated Monte Carlo simulations should be reduced by $\sqrt{500}$=22. This would still indicates variations of up to 30% in the values in Table 4.

The last two columns of Table 4 present the effect of certification tests on failure probabilities. For this purpose we used two measures; the difference of failure probabilities and the ratio of failure probabilities. In our previous work, we have shown that the difference may be more meaningful when the probability of failure is high since it indicates the amount of aircraft that is saved by the use of certification tests. As we can see from the 4th column when the error increases, the difference between the two failure probabilities also increases pointing out that the certification

tests become more effective. The results given in the last column demonstrate that the trend in the probability ratio is also similar to the previous trend; when the error increases the ratio of the two probabilities decreases showing the increase in the effectiveness of the certification tests.

As noted earlier, in our previous paper (Kale et al, 2004) we represented the overall error with a single error factor (SEF), and used uniform distribution for the initial distribution of this error. However, in this work we use a more complex representation of error with multiple error factors (MEF), we represent initial distribution of each error factor with uniform distribution. In this case, the distribution of total error is no more uniform (see Figure 2). In order to compare the two approaches, we first calculate the mean and standard deviation of the initial total error factor, e_{total}^{ini}. The mean value of the total error factor is close to zero so that we use zero mean value and equal standard deviation value for a uniform distribution of total error. The equivalent error bound s for uniform distribution corresponding to different error multiplier *k* is listed of Table 5.

Table 5. Equivalent error bounds for the SEF case corresponding to the same standard deviation in the MEF case

k	Average e_{total}^{ini}	Standard Deviation of e_{total}^{ini}	from the SEF Case → to the MEF Case	Average e_{total}^{ini}	Bound of error for e_{total}^{ini}
0.5	1.00 x10^{-3}	0.064		0	11.1
0.75	1.42 x10^{-3}	0.100		0	17.3
1	1.12 x10^{-3}	0.132		0	22.9
1.5	2.97 x10^{-3}	0.200		0	34.6
2	1.07 x10^{-2}	0.271		0	46.9

Using the equivalent error bounds of SEF Case given in the right portion of Table 5 we calculate the average values of probabilities of failure without and after certification test for SEF case and we compare them in Table 6 with corresponding failure probabilities of MEF case from Table 4. In addition, the comparison of the probability of failures for the two cases is presented in Fig. 3.

Table 6. Comparison of failure probabilities for SEF and MEF case

k	$\overline{P}_{nt}^{MEF}$ (x10^{-4})	$\overline{P}_{t}^{MEF}$ (x10^{-4})	P_f Ratio*	$\overline{P}_{nt}^{SEF}$ (x10^{-4})	$\overline{P}_{t}^{SEF}$ (x10^{-4})	P_f Ratio*
0.5	0.151	0.148	0.982	0.147	0.139	0.948
0.75	0.900	0.805	0.895	0.620	0.525	0.848
1	3.626	2.464	0.679	2.579	1.592	0.617
1.5	52.57	12.94	0.245	37.06	5.671	0.153
2	310.6	19.05	0.061	314.6	5.733	0.018

* P_f Ratio is the ratio of failure probabilities; $\overline{P}_t / \overline{P}_{nt}$

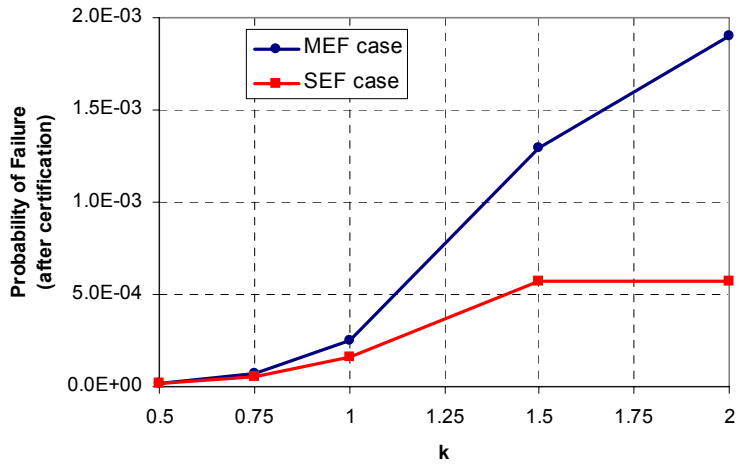

Figure 3. After certification failure probabilities for SEF and MEF case

When we compare the probability of failure without certification the results are similar for both the MEF Case and SEF Case (see columns 2 and 5, Table 6). Note that the differences between the corresponding columns are of the same order as the scatter in the Monte Carlo simulations. Comparing the failure probabilities after certification, we notice that the MEF Case leads to higher probability of failure values hence $\overline{P}_t / \overline{P}_{nt}$ ratios. That is the additional detail of the MEF reduces the effectiveness of the certification testing. This is due to the fact that in the SEF case (Kale et al, 2004) the certification testing is performed with the average value of actual load, P_d (see Table 3 for the definition of P_d). However, in the MEF case certification testing is performed with the calculated load, P_{calc} (see Eq. (1) for the expression for P_{calc}). Therefore, one component of the error can not be exposed by certification testing. This effect is also apparent when we compare the average safety factor values for these two cases in Table 7 and in Fig. 4.

Table 7. Comparison of Average Safety Factor for two cases

k	$\left(\overline{S}_{F_{avg}}\right)_{nt}^{MEF}$	$\left(\overline{S}_{F_{avg}}\right)_{t}^{MEF}$	S_F Ratio*	$\left(\overline{S}_{F_{avg}}\right)_{nt}^{SEF}$	$\left(\overline{S}_{F_{avg}}\right)_{t}^{SEF}$	S_F Ratio*
0.5	1.909	1.911	1.001	1.907	1.910	1.002
0.75	1.910	1.920	1.005	1.907	1.920	1.007
1	1.909	1.938	1.015	1.907	1.942	1.018
1.5	1.912	2.015	1.054	1.907	2.031	1.065
2	1.907	2.093	1.097	1.907	2.149	1.127

* S_F Ratio is the ratio of average safety factors without and with certification

Comparing the average safety factors, $S_{F\,avg}$, after certification corresponding to the MEF and SEF Cases (columns 3 and 6, Table 7), we see that average safety factor values corresponding to SEF Case is larger which will in turn lead to smaller probability of failure (see Table 6).

Looking at the columns 4 and 7 we see and expected trend in the values of S_F Ratios. Both the ratios corresponding to the MEF Case and SEF Case increases with the increase of error bounds, rendering certification tests more effective.

The main reason for lower safety in the MEF case is the reduced effect of certification on design thickness as seen in Table 8.

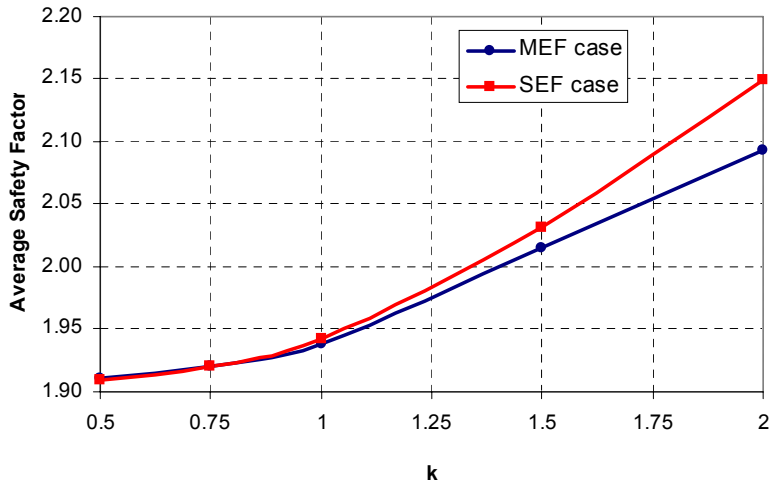

Figure 4. After certification failure probabilities for SEF and MEF case

Table 8. Comparison of design thicknesses for two cases

k	$\left(\bar{t}_{design}\right)_t^{MEF}$	$\left(\bar{t}_{design}\right)_t^{SEF}$	t_{design} Ratio*
0.5	1.273	1.274	1.001
0.75	1.274	1.280	1.005
1	1.276	1.295	1.015
1.5	1.282	1.354	1.056
2	1.282	1.434	1.118

* t_{design} Ratio is the ratio of average design thickness for the MEF and SEF Cases

Table 8 illustrates the effect of error multiplier k on the average design thicknesses after certification of the components corresponding to the MEF and SEF Cases. When we compare average design thicknesses, we see that components corresponding to SEF Case are designed thicker compared to MEF Case leading to low probability of failure values.

Finally, we change the variability in failure stress and investigate the effect of this change in probability of failure. The results are presented in Table 9.

Table 9. Comparison of Failure Probabilities for the MEF Case corresponding to different variability in failure stress σ_f

c.o.v. (σ_f)	$\left(\bar{S}_{F_{avg}}\right)_{nt}$	$\left(\bar{S}_{F_{avg}}\right)_{t}$	Average $\left(t_{design}\right)_{nt}$	Average $\left(t_{design}\right)_{t}$	$\bar{P}_{nt}$ (x10^{-4})	$\bar{P}_{t}$ (x10^{-4})	P_f Ratio
0 %	1.503	1.650	1.001	1.007	79.19	0.306	0.004
5 %	1.691	1.761	1.127	1.131	10.50	1.297	0.124
10 %	1.909	1.938	1.275	1.276	3.626	2.464	0.679
15 %	2.152	2.166	1.434	1.435	3.624	3.231	0.892
20 %	2.450	2.458	1.628	1.628	3.576	3.370	0.943

Table 9 displays the effect of variability in failure stress, σ_f, on the average safety factor and probability of failure for the MEF Case. We observe that the average safety factor and design thickness increases with the increase of variability in failure stress. On the other hand, probability of failure increases with the increase of variability. Comparing the design thicknesses with and without certification cases and also from P_f ratio given in the last column of Table 10 we observe that certification tests become less effective as variability increases. Figure 5 also shows the diminishing of the efficiency of testing as variability grows.

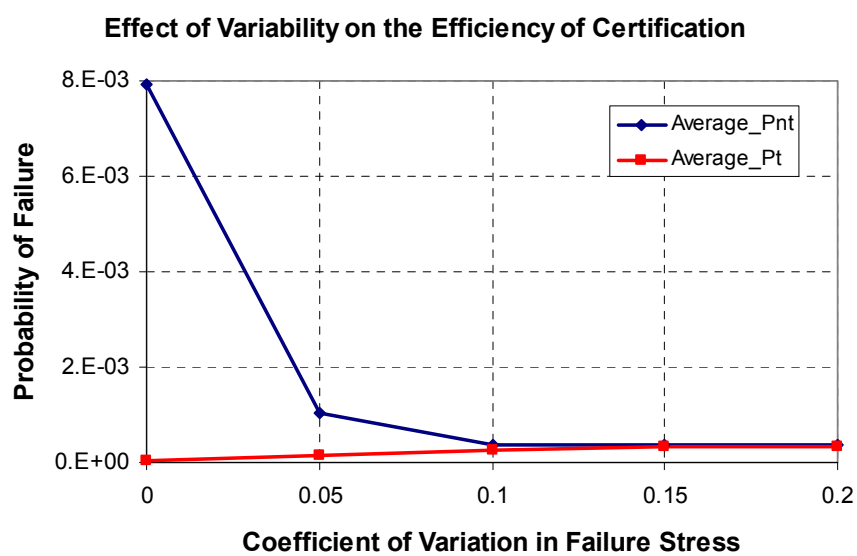

Figure 5. Effect of variability in failure stress on MEF case

8. Concluding Remarks

The interaction between error, variability and testing on the probability of failure of aircraft structures was analyzed. We used stress failure due to extreme loads, which can be simulated by an unstiffened panel designed under uniaxial loads. Monte Carlo simulations were performed to account for both fleet-level uncertainties (such as errors in analytical models) and individual uncertainties (such as variability in material properties).

In our previous paper (Kale et al, 2004) we sought to clarify the interaction between error, variability and testing by the use of a simple model of error, lumping it into a single error component in the calculation of stresses.. In this paper we used a more realistic error model such that errors in load and stress calculation, and also errors in material and geometric properties were modeled using uniform distributions for their initial distributions and compared the results with our previous paper's results. The same as in our previous paper, the variability in the material and geometric properties and in the loading was included in the analysis by modeling the variabilities with random numbers and their distributions.

In our previous paper, we had found that the effect of tests is most important when errors in analytical models are high and when the variability between airplanes is low. These observations also apply to the results obtained in this paper. We expressed the effectiveness of the certification tests is expressed by the ratio of the probability of failure with the test, P_t, to the probability of failure without tests, P_{nt}. Using this ratio we have shown that the effectiveness of certification tests increases when the error in the analysis is large. We changed the bound of error in material properties, e_m, in which we included the likelihood of occurrence of unexpected failure modes and the difference in the behavior of material in coupon tests and in the actual service, and have shown that the reduction in bounds in e_m is also an indication of safer designs. It was an expected result since the safer the design, the lesser the need for testing. In addition, we played with the variability of failure stress and have shown that the increase of variability increased the probability of failure and made certification tests less effective.

Another observation from study is that this new more realistic error model led to an increase in average safety factor (fleet-average) thereby an increase in the probability of failure. In addition, the certification testing for this new case, we called as MEF Case, found to be less effective since we used the calculated load values in testing of components instead of using actual loads as we did in our previous paper.

The effect of building-block type tests that are conducted before certification was not assessed here. However, these tests reduce the errors in the analytical models, and on that basis we determined that they can reduce the probability of failure by one or two orders of magnitude.

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Appendix 1

A-Basis property – A-basis value is the value exceeded by 99% of the population with 95% confidence. This is given by

$$\text{A-basis} = \mu - \sigma \times k_1 \tag{A1}$$

where μ is the mean, σ is the standard deviation and k_1 is the tolerance coefficient for normal distribution given by Equation A2

$$k_1 = \frac{z_{1-p} + \sqrt{z_{1-p}^2 - ab}}{a}$$
$$a = 1 - \frac{z_{1-\gamma}^2}{2(N-1)} \; ; \; b = z_{1-p}^2 - \frac{z_{1-\gamma}^2}{N} \tag{A2}$$

where N is the sample size and z_{1-p} is the critical value of normal distribution that is exceeded with a probability of $1- p$. The tolerance coefficient k_1 for a lognormal distribution is obtained by first transforming the lognormally distributed variable to a normally distributed variable. Equation A1 and A2 can be used to obtain an intermediate value. This value is then converted back to the lognormally distributed variable using inverse transformation.

In order to obtain the A-basis values, 15 panels are randomly selected from a batch. Here, the uncertainty in material property is due to allowable stress. The mean and standard deviation of 15 random values of allowable stress is calculated and used in determining the A-basis value of allowable stress.