Introductory Remarks on Reliable Engineering Computing

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“In a physical system model, uncertainties can originate from several sources”(Rafi Muhanna and Robert Mullen):

1. “The appropriateness of the mathematical model to describe the physical system”

Examples of the use of interval methods in this connection:

1)A physics problem at Lockheed (ca. 1960): Q: Is the strange behavior of the computer model due to round-off errors? A: After converting the program to run in interval arithmetic with outward rounding, it was determined that round-off error was very small in the original program. Result: the physicist took another look at the model equations and found that there was a missing term.

2)A long controversy between research groups at MIT and Cal Tech concerned whether the observed behavior of computer simulations was due to roundoff error or defects in the mathematical model, in the case of computer solutions of the Birkhoff-Rota complex partial differential/integral equations modeling the onset of turbulence in wind-shears. My graduate student Jeffrey Ely wrote a program for variable-precision interval arithmetic with outward rounding, and finally by using about 300 decimal place (nearly 1000 bits) in outwardly rounded interval arithmetic, was able to settle the controversy. It was NOT roundoff error, but the model itself. The model realistically determined the onset of turbulence at a reproduceable, finite time after the initial appearance of the wind-shear. I have always found it odd to suppose that anything we want to compute can be done carrying only some fixed number of digits or bits. Is 40 bits enough? 80 ? A thousand? It depends on what we are trying to compute. [J. Ely and G. R. Baker. High precision calculations of vortex sheet motion. J. Comp. Phys., 111:275-282, 1994].

The point of these examples is that outwardly rounded interval arithmetic automatically bounds roundoff error in any computation. As a consequence, if the interval results are adequately narrow, then no repetition is needed using higher precision arithmetic, carrying more digits. If the results are not narrow enough, then the computations can be repeated carrying more digits. If roundoff is the only source of error, we may eventually obtain satisfactorily narrow interval results containing the corresponding infinite precision results that would come from using exact real arithmetic. Floating-point arithmetic by itself cannot provide such answers.

More important is the fact that interval computational methods can answer questions about the ”appropriateness of the mathematical model” even when roundoff is not the only nor the main source of computational error in simulating the behavior of a proposed mathematical model on a computer. I will expand upon this in the following discussions of items 2,3, and 4 listed by the organizer of this workshop as origins of uncertainties in physical system models:

2. "The discretization of the mathematical model into a computational framework"

If there is an analytic expression for the discretization error, for example the mean value form of the remainder in a truncated Taylor series approximation, that too can be bounded by interval computation. In addition to that, a finite element method or a finite difference method may be looked at as a computational model of a physical process or structure, and interval methods can provide intervals containing the exact behavior of the computational model. In this way we test for suitable computational parameters (such as mesh size or the number of nodes) for which the model produces results in agreement with observed behavior of the real process or characteristics of the the real physical structure.

3. "The inexact knowledge of input parameters of a problem"

Suppose we have measured that input parameters fall within certain upper and lower limits, then we can use interval inputs for them, and interval computation is designed for just that sort of thing.

For example, if we measure a width as $w = 7.2 \pm 0.1$, length as $l = 14.9 \pm 0.1$, and height as $h = 405.6 \pm 0.2$, then the volume $V = w \times l \times h$ is within in the interval

$([7.1, 7.3] \times [14.8, 15.0]) \times [405.4, 405.8] = [42599.432, 44435.1] = 43517.266 \pm 917.834$ If desired, we could use outward rounding to retain containment with numbers having only one digit after the decimal point, and find that the volume is contained in the interval of numbers $[42599.4, 44435.1] = 43517.25 \pm 917.85$. This in turn is contained in the interval $[42599.4, 44435.2] = 43517.3 \pm 917.9$.

All I am saying in the above example is that IF all we know about w, l and h is that their values lie in the intervals given, THEN all we can say about V is that it is in the interval obtained by the appropriate multiplications of the given endpoints of the input variables w, l and h. If we need V more accurately, then we have to measure w, l and h more accurately.

There are successful interval algorithms for doing this sort of thing for real engineering computations involving uncertain values of parameters, if those uncertain values are at least known to lie in certain intervals. See for example papers of Muhanna and Mullen; Stadtherr et al; G. Fichtner, H. J. Reinhart, and D. W. T. Rippin;R. P. Broadwater, H. E. Shaalan and W. J. Fabrycky; C. H. Dou, W. Woldt, I. Bogardi, and M. Dahab; M. Hurme, M. Dohnal, and M. Jaervalaeinen; X. Lin, O. T. Melo, D. R. Hastie, et al; H. U. Koyluoglu, A. S. Cakmak, and S. R. K. Neilson; H. E. Shaalan and R. P. Broadwater; and many others, see e.g. http://www.cs.utep.edu/interval-comp/abstracts/list.html A recent noteworthy book on interval methods for global optimization is: Global Optimization Using Interval Analysis, E.

Hansen and G. William Walster, published by Marcel Dekker, 2004. More generally, interval arithmetic, which is very simple, is incorporated as a tool in more sophisticated methods for analyzing uncertainty in engineering design, in which more information is available than simple upper and lower bounds on uncertainties of inputs. For example some recent works of Erik Antonsson and others use the "level interval algorithm", which is itself used internally by their "Method of Imprecision". There are frequent uses of interval arithmetic as a tool in such fuzzy systems analyses, see for example the special issue on Interfaces between Fuzzy Set Theory and Interval Analysis, of the journal Fuzzy Sets and Systems, Vol 135, No 1, April 2003. See also two special issues on Interval Analysis and Fuzzy Sets of the journal Reliable Computing, Vol 10, No. 4, 2004 and Vol. 10, No. 5 (to appear).

Interval arithmetic is also coming into use in probabilistic handling of uncertainty, see for example the two special issues of the journal Reliable Computing devoted to Dependable Reasoning about Uncertainty, guest-edited by D. Berleant, Vol 9, No.6, (Dec. 2003) and Vol 10, No. 2 (April 2004); and D. Berleant and J. Zhang, Representation and problem solving with the distribution envelope determination (DEnv) method, Reliability Engineering and System Safety, in press. By using step-function interval envelopes around probability density functions, we can sometimes compute useful envelopes for cumulative distributions without using costly Monte Carlo methods.

4. "Errors introduced by the nature of computer finite arithmetic"

Interval arithmetic with outward rounding (lower bounds rounded to the left on the number line and upper bounds rounded to the right) yields computer results which contain both the unknown infinite-precision real arithmetic results as well as the results which would be obtained by ordinary floating-point arithmetic for the same sequence of computer operations.

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