Sampling Without Probabilistic Model

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Abstract

In this paper a novel technique for random vector sampling starting from rare data is presented. This model-free sampling provides a basis for increasing the numerical efficiency of Monte Carlo simulations [2] in computational stochastic mechanics. If the structural response data can only be obtained to a very limited extent, for instance, due to a high numerical cost of the underlying deterministic computational model, the novel technique can be attached as a postprocessor to the stochastic structural analysis to generate a proper estimation of stochastic structural responses and, thanks to a sound reproduction of distribution tails, of structural reliability.

The model-free sampling technique is developed to operate without a probability distribution function. Instead of estimating a distribution function, the information contained in a given small sample is extracted directly to produce the sampling result. Specifically, starting from the given small sample $\underline{S}_0$ of size n_0, a second sample $\underline{S}_1$ of considerably larger size $\mathrm{n}_1 \gg \mathrm{n}_0$ that completely reflects the properties of the original sample $\underline{S}_0$ is numerically produced. A specification of a probabilistic model for describing the random properties of the given sample $\underline{S}_0$, which is affected by subjectivity, is omitted. This means compliance of model-free sampling with the basic assumption of mathematical statistics that all information is contained in the sample.

The numerical procedure for generating the sampling result $\underline{S}_1$ is formulated as an iteration. Starting from an initial, general estimation $\underline{S}_1^{[0]}$ for $\underline{S}_1$, repeated random modifications are applied until it is no longer possible to obtain an improvement of $\underline{S}_1$ beyond $\underline{S}_1^{[n]}$ from the n-th iteration step. This requires the definition of a measure $\mathrm{G}^{[r]}$ for assessing the quality of $\underline{S}_1^{[r]}$ in each iteration step r. The measure $\mathrm{G}^{[r]}$ is composed of two nonprobabilistic criteria formulated on the basis of the Euclidean distance between sample elements from $\underline{S}_0$ and from $\underline{S}_1$.

The non-traditional quality assessment of $\underline{S}_1$ via $\mathrm{G}^{[r]}$ allows for an extension of the model-free sampling to processing imprecise data, which finally leads to a prediction of uncertain stochastic structural responses and of uncertain structural reliability. As a basis for dealing with imprecise data, the generalized uncertainty model fuzzy randomness [1] is employed.

In the full paper the model-free sampling technique is presented in detail and demonstrated by means of numerical examples.

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References

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