

Defect-based ODE Solvers based on Taylor Model Methods

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Abstract

An new method is presented that provides verified enclosures of the flow of ODEs. It is based on establishing the defect of an approximation of the flow by its Taylor polynomial of order n in time and k in initial conditions, and verified integration of this defect using Taylor model methods. One advantage of the method is that the remainder error of a single step is expressed in terms of an error polynomial, which helps reduce dependency even in the calculation of this by nature small term. Another advantage of the approach is that it is possible to perform the determination of the flow using only a single suitable TM evaluation of the right hand side, which entails performance advantages for complicated ODEs.

A comparison with the conventional TM-based verified integration algorithm is performed, showing the merits of the new approach for various problems. Some applications of the method, in particular in the field of solar system dynamics and the field of beam physics, will be presented.