An Algorithm for Bounding the Solution of a Fredholm Integral Equation of the Second Kind

Eldon Hansen

Abstract

Consider a Fredholm integral equation of the form

$$f(y) = g(y) + ba\int f(x)k(x,y)dx. \tag{1}$$

One class of methods for solving (1) involves approximating the kernel $k(x, y)$ by a sum of the form

$$ni = 1\sum p_i(x)q_i(y). \tag{2}$$

Both interval and non-interval methods have used this form. In these methods, the error in this approximation must be small over the entire interval $[a, b]$.

We introduce a method in which the interval $[a, b]$ is divided into small subintervals and a separate Taylor expansion of the kernel is used in each subinterval. This provides much simpler approximations both because the intervals of approximation are small and because a Taylor expansion is easily obtained.

A virtue of using (2) in an interval method is that no contraction procedure is required in order to obtain error bounds on the solution. Our method does involve contraction; and will generally require iteration.

When using an approximation of the form (2), the error in the solution depends on the error in (2). If bounds on the solution are not sufficiently sharp, a new, more accurate, approximation must be derived. This can involve considerable effort. In our method, we have two methods for improving accuracy. We can simply increase the order of the Taylor expansion or we can refine the subdivision of $[a, b]$.

Other simplifications derive from our formulation.v