Validated Solution of Initial Value Problems for ODEs with Interval Parameters

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Abstract

Initial value problems for ODEs arise naturally in many applications in science and engineering. It is often the case that the problem involves parameters and/or initial values that are not known with certainty but that can be expressed as intervals. For this situation it is desirable to be able to determine an enclosure of all possible solutions to the ODEs. Interval methods (validated methods) not only can determine such guaranteed error bounds on the true solution, but can also verify that a unique solution to the problem exists. An excellent review of interval methods for initial value problems has been given by Nedialkov et al. [1] Much work has been done for the case in which the initial values are given by intervals, and there are several available software packages (e.g., AWA, VNODE, COSY VI) that deal with this case. However, relatively little work has apparently been done on the case in which parameters are given by intervals. We concentrate here on the case of such parametric ODEs. However, the method developed will also account for interval-valued initial values.

Since available general-purpose validated ODE solvers are focused on dealing with uncertainties in the initial values, the presence of interval parameters can cause inefficiencies because they lead to a wrapping effect. An alternative approach is to treat time-invariant interval parameters as additional state variables, with zero first-order derivatives, as suggested by Lohner [2]. Since the parameters are now treated as independent variables, tighter enclosures can be obtained. However, the increase in the number of state variables, n, can result in a significant increase in the computational expense. For example, a matrix of order n must be factored at each time step in the usual methods (e.g., QR factorization) for controlling the wrapping effect. In the work presented, we will describe a method for efficiently determining validated solutions of ODEs with interval parameters, where instead of increasing the number of state variables, we will treat the parametric uncertainty directly. The method makes use, in a novel way, of the Taylor models described by of Makino and Berz [3]. The effectiveness of the method is demonstrated using several numerical examples involving interval-valued parameters.

References

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