Prediction of Deflection for Prestressed Concrete Girders Using a Bayesian Approach

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Abstract: To control the alignment of prestressed concrete bridge during construction, it is reasonable to treat the prediction of the deflection as an uncertainty. This paper presents a Bayesian updating approach to predict the deflection of prestressed concrete girders. A prior distribution is developed by using Monte Carlo stimulation with a proposed deterministic model under a variety of prestressing levels, material properties and environmental conditions. Then the posterior distribution is obtained by updating the prior distribution based on a limited number of initial measurements, thus greatly reducing the uncertainty of the deflection prediction. The method is applied to predict the camber of two actual prestressed T girders, and the predictions are satisfied.

Keywords: PC, deflection, Bayesian approach, prediction

1. Introduction

In order to obtain a good alignment design for a particular prestressed concrete (PC) bridge, the deflection is a very important aspect in its construction control. It is obvious that the deflection of PC girders varies with the different cases because of various random effects; these effects include concrete strength, elastic modulus, creep and shrinkage as well as section properties. All these make the deterministic prediction of the deflection unrealistic. It is therefore reasonable to treat the prediction of the deflection as an uncertainty. This paper provides an assessment of the variability of the deflection for PC girders with a Bayesian updating approach (Bazant and Wittman,1987; Zhang and Du, 1994) . Based on a deterministic model for deflection, the numerical calculation is repeated many times using Monte Carlo simulation (Li Jihua,1988), the results of which is used as a prior distribution. With a limited number of measurements available, a posterior distribution can be obtained by updating the prior distribution and a more believable long-term prediction for the deflection of the PC girders can be assessed.

2. Deterministic model for time-dependent deflection

Time-stepping approaches for deformation calculations based on the principle of superposition have appeared in many literatures. For the case of changing stress, the stress history can be divided into several sections [t_j, t_{j+1}], the stress is assumed as a series of stress increments $\Delta\sigma$ (t_j)applied at times t_j , then the creep strains can be expressed as follows based on the principle of superposition:

$$\varepsilon(t_{i+1}) = \sigma_0 \Phi(t_{i+1}, t_0) + \sum_{j=1}^{i} \Delta\sigma(t_j) \Phi(t_{i+1}, t_j) \tag{1}$$

The Eq.(1) can also be written in a form of a section curvature as follows:

$$\psi(\mathrm{t}_{i+1}) = \frac{M_0}{\mathrm{I}} \Phi(\mathrm{t}_{\mathrm{i+1}}, \mathrm{t}_0) + \sum_{\mathrm{j=1}}^{\mathrm{i}} \frac{\Delta M(\mathrm{t}_\mathrm{j})}{\mathrm{I}} \Phi(\mathrm{t}_{i+1}, \mathrm{t}_\mathrm{j}) \tag{2}$$

where $\varepsilon(t_{i+1})$ and $\psi(t_{i+1})$ are the total strain and section curvature at time t_{i+1} respectively; σ_0 and $\Delta\sigma$ (t_j)are the instantaneous stress at time t_0 and the increment at time t_j; M_0 and ΔM (t_j)are the instantaneous moment at time t_0 and the increment at time t_j respectively; I is the moment inertia of the section; and $\Phi(t_{i+1}, t_j)$ is a creep function for creep strain at time t_{i+1}. Among all the creep and shrinkage models, ACI Committee 209, CEB-FIP (MC78, MC90), BP2 (Bazant and Lisa, 1980) are often recommended. Although the BP2 is the most complex in form, it takes the effects of aggregation into account and it is adopted in this paper. The deflection then can be calculated by many approaches (Glali and Azarnejad, 1996) such as the finite element method. For a simply supported beam, the deflection f at the mid span can be accurately calculated using the following equation:

$$f = \frac{l^2}{96}\left[\psi_{\mathrm{E1}}(\mathrm{t},\tau) + 10\psi_{\mathrm{M}}(\mathrm{t},\tau) + \psi_{\mathrm{E2}}(\mathrm{t},\tau)\right] \tag{3}$$

where ψ is the section curvature. The subscripts E1 and E2 mean both the end sections of beam, and M the mid section.

Table 1. Parameter statistical distributions for prestressed concrete girder

Variables	Mean	Std. Dev.	Cov.
C50 concrete strength（MPa）	39.9088	4.9088	—
Unit weight of concrete（kg/m^3）	2400	80	—
permissible prestressσ_k（MPa）	1.00σ_k	—	0.055
Prestressing aere A_p（cm^2）	1.01176A_p	—	0.0125
Moment inertia I（cm^4）	1.006I	0.0107	—
Location of duct d（cm）	1.00d	1.20	—
Volume surface ratio V/S	1.00V/S	—	0.0528
Coarse aggregate W_1(kg)	1.00W_1	—	0.100
Fine aggregate W_2(kg)	1.00 W_2	—	0.100
Cement W_3(kg)	1.00 W_3	—	0.050
Water W_4(kg)	1.00 W_4	—	0.050
Humidity	70%	—	13.3%

Based on the deterministic approaches mentioned above, Monte Carlo simulation can be used. The basic idea of a Monte Carlo analysis is repeatedly to simulate random input parameters. These statistical parameters are listed in Table 1 based on related information. (Zhao and Jin, 2000). All variables are considered to be normally distributed for simplicity.

3. Bayesian approach for deflection prediction

According to the Bayesian formula, if the initial probability P (X_{ik}) of all hypotheses X_{ik} are known, the posterior probability $P(X_{ik})$ then could be obtained in conjunction with a set of limited measurements S_M which were taken during the early life of the girder. As stated in the former section, the probability obtained by Monte Carlo stimulation can be taken as a prior probability:

$$P'(X_{ik}) = C \cdot L(S_M | X_{jk}) \cdot P(X_{ik}) \qquad (4)$$

where X_{ik} represents the predicted deflection at time t_i on the k^{th} Monte Carlo run; S_M is a set of measured deflection; C is a normalizing constant and $L(\cdot)$ represents the likelihood function, which means the likelihood of obtaining the measured values. Assuming that the deflection is normally distributed, $N(\mu_{ik},\sigma_i)$ and the statistical independence is appropriate, then,

$$L(S_j | X_{jk}) = \prod_{j=1}^{m} p_j(S_j | X_{jk}) \qquad (5)$$

in which,

$$p_j(S_j | X_{jk}) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left[-\frac{1}{2}\left(\frac{s_j - \mu_{jk}}{\sigma_j}\right)^2\right] \qquad (6)$$

let $$w_k = \exp\left[-\sum_{j=1}^{m} \frac{1}{2}\left(\frac{s_j - \mu_{jk}}{\sigma_j}\right)^2\right] \qquad (7)$$

replacing them into Eq.(3) and since C should ensure the total probability to be unity, the probability of the deflection X_{ik} appeared at the time t_i in the k^{th} Monte Carlo run becomes,

$$P'(X_{ik}) = \frac{\sum w_k P(X_{ik})}{\sum w_k} \qquad (8)$$

The mean and standard deviation of the posterior distribution for the deflection at time t_i can be further written as

$$\overline{X_i'} = \frac{\sum w_k X_{ik}}{\sum w_k} \qquad (9)$$

$$V_i' = \sqrt{\frac{\sum w_k \left(X_{ik} - \overline{X}_i'\right)^2}{\sum w_k}} \qquad (10)$$

4. Experiments on T girders

The cambers of two of prestressed concrete T girders were measured during the early phase of the bridge. Both of the girders (T1 and T2) are the same in section and span with a height and a span 1.68m and 30m respectively, prestresing strand 270(low relaxation), the total prestressing steel area 0.0028m^2, permissible prestress 0.75R_y^b, concrete C50. Area and moment of inertia of the section are 0.615m^2 and 0.2115m^4. The prestresing ages for T1 and T2 girders are 5 and 4 days, respectively. The camber measurement was taken before sunrise so as to reduce the effect of the temperature.

Table 2. The mean and standard deviation of the girders at loading age of 100 days

State	Number of samples used	T1		T2	
		mean	std. dev.	mean	std. dev.
Before updating	0	2.8742	0.4873	2.9816	0.5095
After updating	5	2.8679	0.2057	3.0616	0.2146
	15	2.9211	0.1268	3.0273	0.1367

Numerical calculations were conducted with the Bayesian updating algorithm outlined above using only the five data values at the early period. The results for both T1 and T2 girders are shown in Figure 1 and 2. These figures show the camber (prior) mean of the girders based on the Monte Carlo simulation and the updated (posterior) mean based on the limited measured data and their 95% confidence limits are also indicated. In both experiments, the measured data produce a significant narrowing of the confidence limit band as shown in Table 2, which demonstrates an improvement in the confidence of long-term prediction. It is also noticed that the later measured data are all fallen within the narrowed limit band which verifies the confidence of the proposed method. From the comparison of the two girders, the deviation of initial measured data has great effects on the long-term prediction, so correctly measured data should be ensured.

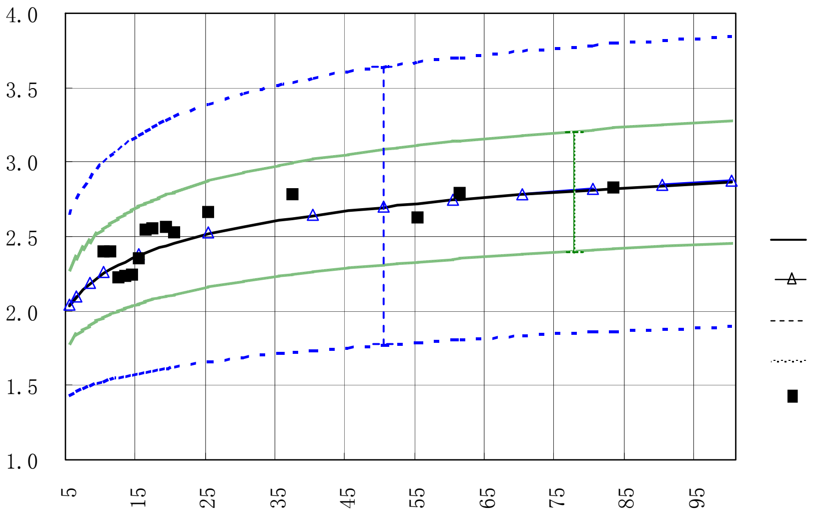

Figure 1. Camber of T1 at midspan (cm)

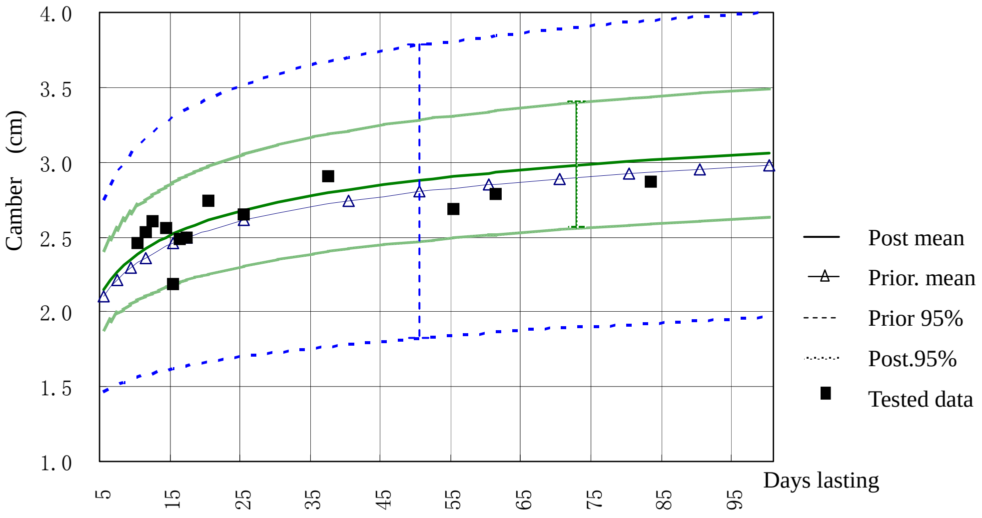

Figure 2. Camber of T2 at midspan (cm)

5. Conclusion

This paper presents a method to reduce the uncertainties in the long-term prediction of a prestressed concrete girder. By updating the prior distribution based on a limited number of measurements in the early stage of the girder, the uncertainty of its deflection prediction can be greatly reduced. Two practical experiments show that the results are accurate. It is also noticed that the deviation of the measurements has great effects on the long-term prediction.

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