RELIABILITY OF COMPUTATIONAL ENGINEERING

Ivo Babuška
Institute for Computational Engineering and Sciences
University of Texas at Austin

R. Tempone
Florida State University

ENGINEERING ACCIDENTS

1. Tacoma Narrows Bridge: The first Suspension Bridge Across Puget- Sound (Washington State) collapsed Nov. 7th 1940.

   Reason: Incorrect Model – Not respecting aerodynamic forces. (Effect of von Kármán vortices)

2. Collapsed roof of The Hartford Civic Center Jan. 18th 1978.

   Reason: Linear model and model of the joints was not adequate.

3. Collapsed roof in Katowice (Poland) Jan. 28th 2006, 65 dead. Design was not adjusted for heavy snow and avoiding total collapse.

4. The Columbia Space Shuttle accident caused by a piece of foam broken off the fuel tank. After the hit was observed the potential damage was computationally judged non-serious.

   Reason: The used model was based on the effect of small meteorites and not on a large piece of foam.

5. The Sleipner accident: Offshore platform made of reinforced concrete sank during ballast test operations Aug. 23rd 1991.

   Reason: Inaccurate FEM analysis.

6. Failure of the ARIANE 5 Rocket June 1996. Implementation round-off. Problem of computer science

7. Loss of Mars Climate Orbiter:

   Reason: Unintended mixture of English and metric units.

Some Basic Relevant Papers

- Guide for Verification and Validation of Computational Fluid Dynamics Simulation; American Institute of Aerodynamics and Astronautics. Tech. Dep. AIAA G-077-1998.
- W. L. Oberkampf, T. G. Trucano, Verification and Validation in Computational Fluid Dynamics; Progress in Aerospace Sciences, 38 (2002), 209-272.
- D. J. Roache, *Verification and Validation in Computational Science and Engineering*; Hermosa Publ. 1998.
- D.E. Post, The Coming Crisis in Computational Sciences; Los Alamos National Laboratory, Rep. LA-UR-03-88 2004.

- I. Babuška, T. Strouboulis; *The Finite Element Method and its Reliability*, Oxford Press, 2001
- I. Hlavácek, J. Chleboun, I. Babuška; *Uncertain Input Data Problem and the Worst Scenario Method*; Elsevier 2004
- I. Babuška, J. T. Oden; Verification and Validation in Computational Engineering and Science: Basic Concepts; Comp. Meth. Appl. Mech. Engrg. 193 (2004) 4057-4061
- I. Babuška, J. T. Oden; The Reliability of Computer Predictions: Can They be Trusted; Int. J. Num. Anal. Model 3 (2005), 255-273.
- I. Babuška, F. Nobile, R. Tempone; Worst Case Scenario Analysis for Elliptic Problems With Uncertainity; Numerische Mathematik 101 (2005), 185-219.
- I. Babuška, R. Tempone, G. Zouraris; Solving Elliptic Boundary Value Problems With Uncertain Coefficients by the Finite Element Method: The Stochastic Formulation, Comp. Meth. Appl. Mech. Eng. 194 (2004) 1251-1294.

- I. Babuška, F. Nobile, R. Tempone; Model Validation Challenge Problem: Static Frame Problem, Sandia Nat. Lab. Workshop, May 22-23, 2006.
- I. Babuška, K. M. Liu, R. Tempone; Solving Stochastic Partial Differential Equations Based on Experimental Data; Math. Models & Method in Applied Sciences 13, (2003), 715-745.
- I. Babuška, R.Tempone, G. Zouaris; Galerkin Finite Element Approximation of Stochastic Partial Differential Equations; SIAM J. Num. Anal. 42 (2004), 800-825.

WHY DO WE COMPUTE?

To provide the data (quantities of interest)=Prediction for the engineering decision (or understanding of various effects)..

REALITY →MATH. MODEL → COMP. MODEL →PREDICTION →DECISION

REALITY → MATH. MODEL: VALIDATION

MATH. MODEL → COMP. MODEL: VERIFICATION

The Mathematical Model consists of the input data, the structure and desired quantity of interest (for the decision).

Input data have epistemic and aleatory uncertainty – determined usually by the experimental data.

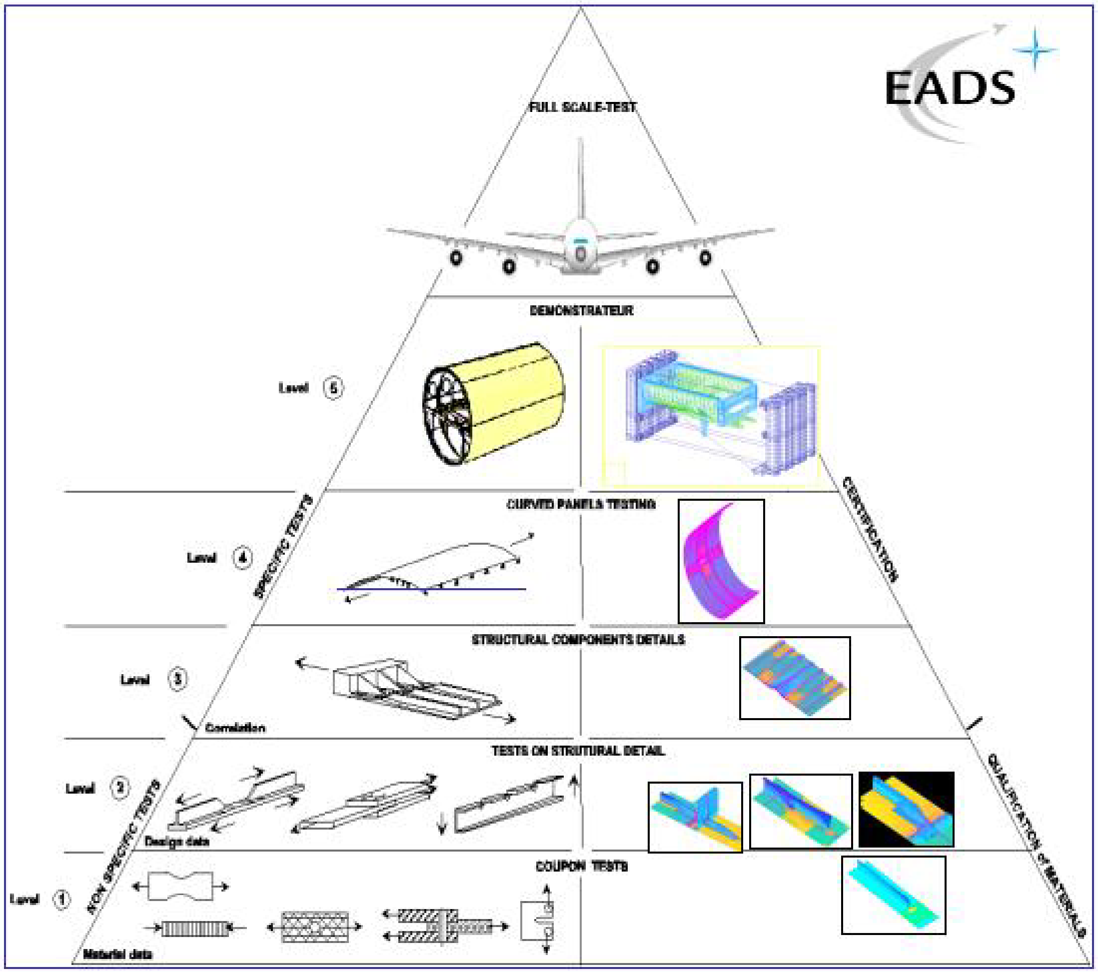
Structure has uncertainty, depends typically on expert’s opinion, usually the structure is based on the physical laws.

Uncertainty in the input includes the uncertainty in the measurement technique. In practice, it is usually not easy to distinguish between what we want to measure and what we actually measure.

Validation is the process of determining if the mathematical model leads to the decision with sufficient reliability.

Verification is the process of determining if the computational model and the code implementation leads to the prediction with sufficient accuracy, i.e. the difference between the exact and computed prediction is sufficiently small.

VALIDATION PYRAMID IN THE ENGINEERING – AIRBUS



VALIDATION PYRAMID IN THE ENGINEERING – AIRBUS

Left hand side are the experiments.
On the right the computational analysis, the results of which are compared with the experiments.
The complexity of the experiments and their cost grow when going up the pyramid.
Very often the lowest level is called calibration, higher the validation and the top of the pyramid is called – accreditation (or certification). In the Airbus pyramid it consists by testing two entire airplanes.
The prediction is sometimes called regulatory assessment.

Sandia National Laboratory: Workshop on Validation Challenge May 22-23, 2006

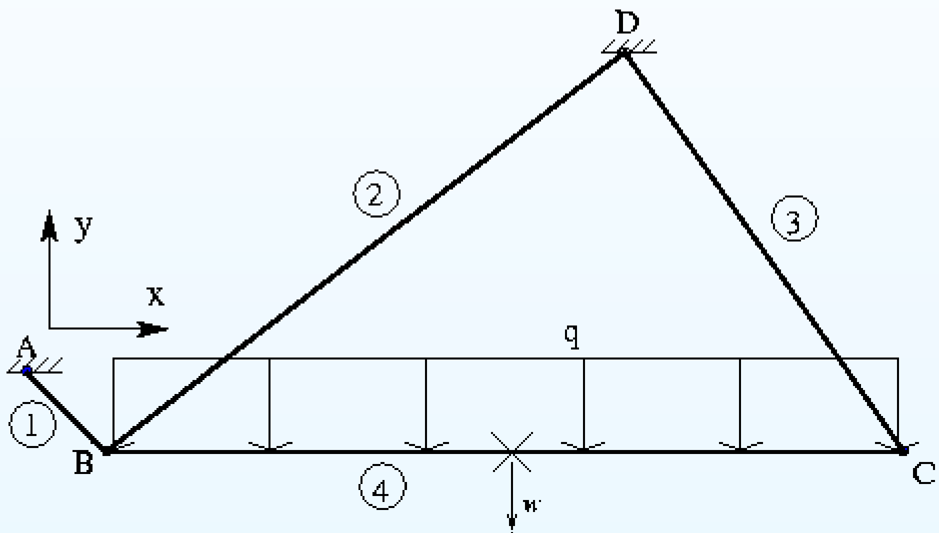
Problem 1: I. Babuška, F. Nobile, R. Tempone
Static Frame Problem

Problem 2: K.J. Dowding, R.G. Hills, W. Pilch
Thermal Problem

Problem 3: J.R. Red-Horse, T.L. Paez
Structural Dynamic Problem

Let us show now the problem of the Sandia Workshop as a simple illustration. For a detailed formulation see I. Babuška, F. Nobile, R. Tempone and the detailed report that will appear in the summer 2006.

PROBLEM



PREDICTION

probability that

$$w \leq \alpha$$

w: vertical displacement

at midpoint of bar #4

regulatory assessment α=3mm

PRELIMINARY ANALYSIS

The following basic principles are assumed valid:

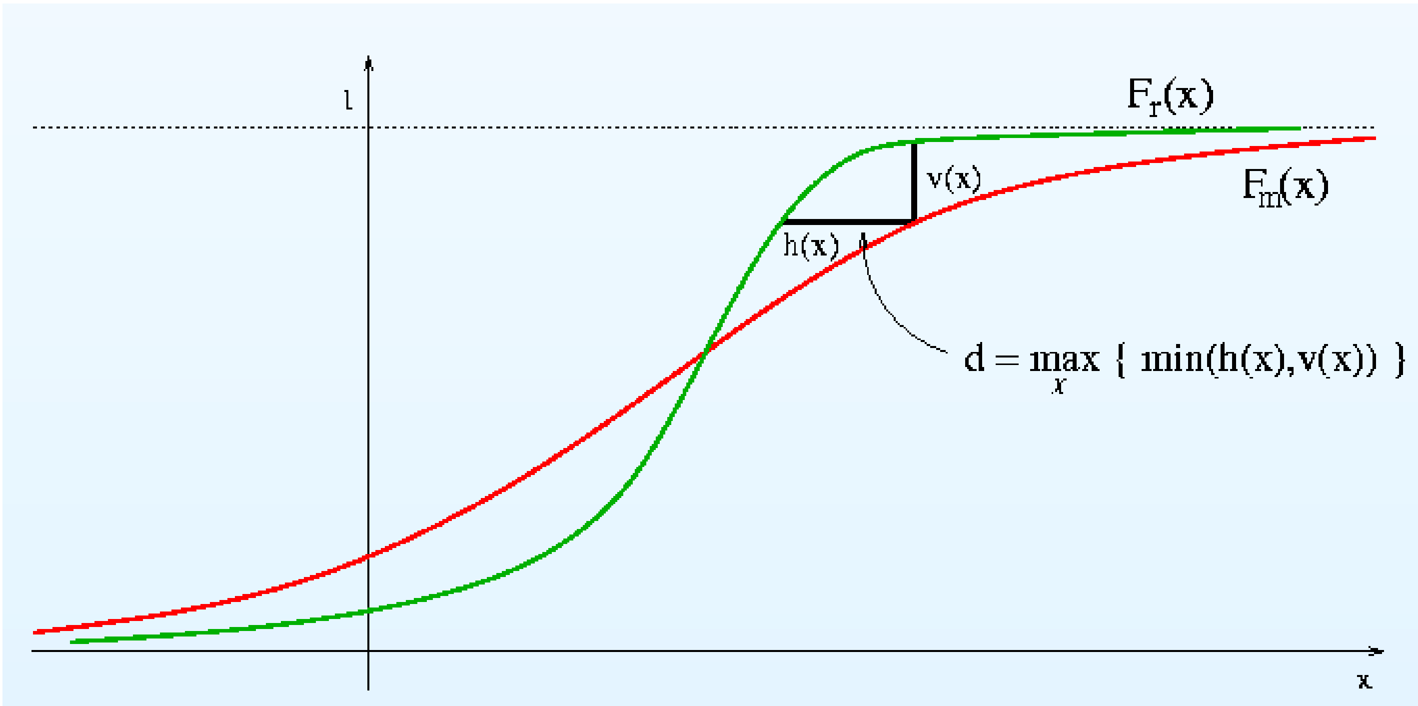
- Bar and Beam Theory
- Perfect joints
- Geometry and load completely known
- Heterogeneous material – linear constitutive law
- Stochastic stationary modulus of elasticity
- Bars and beams are independent

SOLUTION – MAIN IDEAS

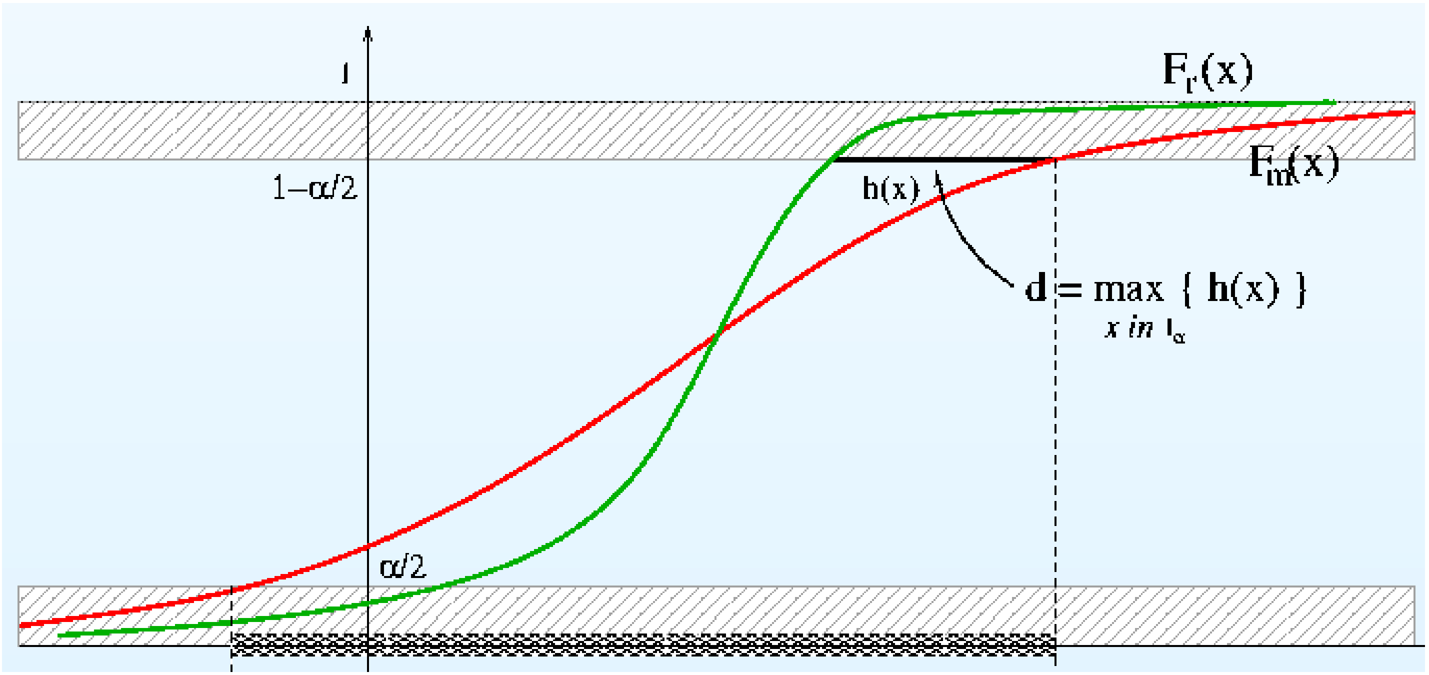
- It is impossible to elaborate on any details because of the time limitation. The solution with all the settings and data will be in the Sandia report and the aim here is to show the general direction. Note that the solution approach is not unique.
- The goal of the solution is to get an approximation of the cumulative probability distribution (CDF).
- Prediction = displacement in the given point. The distance between two CDF is defined as the accuracy of the approximate solution.
- The distance between the exact and approximate CDF is the accuracy of the prediction.

- The accuracy has to be estimated. The entire CDF is used although the goal is only to determine the probability of the event, in our case the displacement in a specific point of the frame.
- The Bayesian solution as a more accurate solution is used for the a-posteriori error estimation.
- Uncertainty = Distance between predicted and Bayesian solution in the calibration, validation and accreditation have to be (and are) comparable so that accuracy in the prediction can be assumed to be the same.

Distance 1



Distance 2



THE PYRAMID PROBLEM

- Prediction
- Accreditation
- Validation
- Calibration

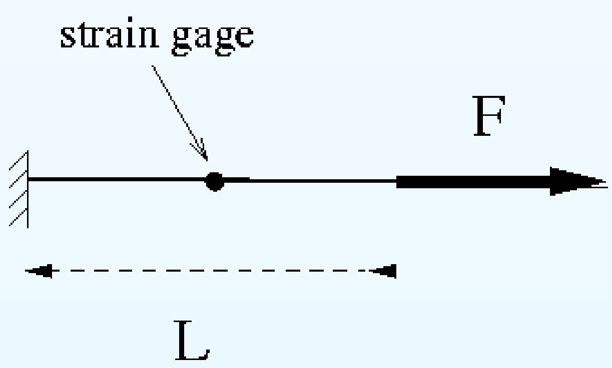
This is a simplified pyramid of the airbus pyramid.

CALIBRATION

- Analysis of the parametric and non-parametric models.
- Notion of the discrepancy and its relation to the distance.
- Limitation of the parametric approach and bootstrapping approach
- Calibration of the covariance function

Calibration experiments

Goal Determine the relation between stress and elongation depending on the length of the bar: marginal probability and covariance



elongation measured on

- very small length: strain gage
- specimen length (dog bone)

Amount of data available

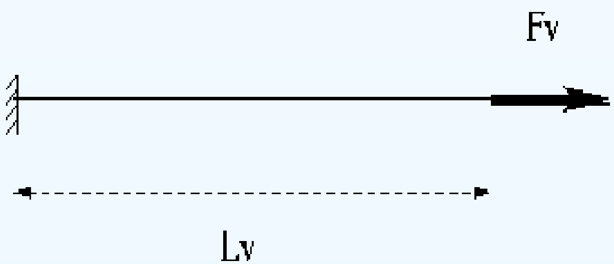
Case 1:	Small number of experiments	(5)
Case 2:	Moderate number of experiments	(20)
Case 3:	Larger number of experiments	(30)

VALIDATION

- Metric and the criterion based on the distance
- Comparison between calibration and validation

Validation experiments

Goal Validation of the probability field constructed in the calibration: marginal distribution and correlation length.



elongation at the end of the bar

Number of validation tests

Case 1: (2)

Case 2: (4)

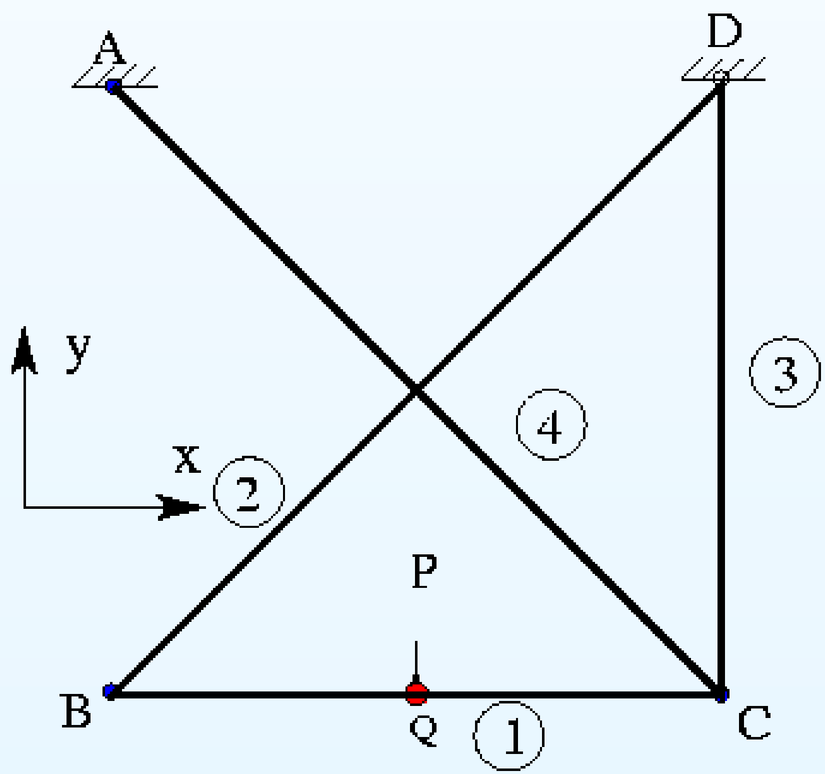
Case 3: (10)

ACCREDITATION

- Metric and criterion
- Comparison of the accuracies of validation

Accreditation experiments

Goal Validation of the model in an environment that has some similarities to the prediction.



vertical displacement at midpoint of bar #1

Number of accreditation tests

Case 1: (1)

Case 2: (2)

Case 3: (3)

PREDICTION

- Results for the 3 cases (number of measurements) and their accuracy.
- Interpretation.

CONCLUSIONS

- The mathematical model is a transformation of the available information into the desired one.
- Available information has always uncertainties that have to be specified and quantified.
- Uncertainty is in the input and the structure of the mathematical model. It leads to the uncertainty in the prediction that has to be specified.
- The uncertainty can be epistemic or aleatory.
- The model can be based on the probability theory, fuzzy sets, worst scenario, etc.
- The prediction has to be reliable, otherwise the decision can lead to very serious mishaps (accidents).