On reliability of higher order finite element method in fluid-structure interaction problems

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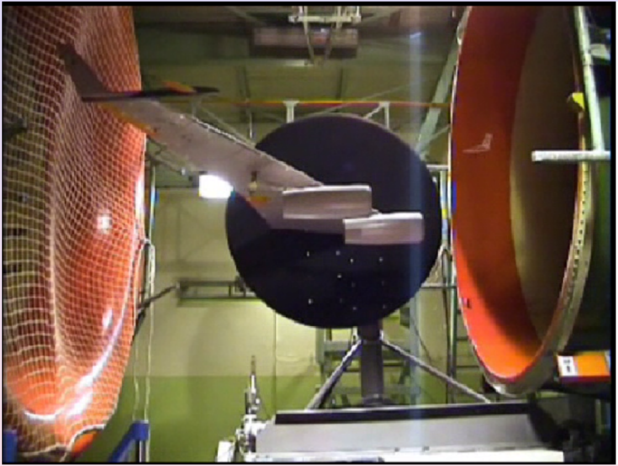
Outline

1. Introduction
2. Mathematical Model
    - General Model
    - Simplifications
    - Interface Conditions
3. Numerical Approximation
    - Time Discretization
    - Space discretization of Fluid Model
4. Numerical Results
    - Fluid Flow Approximation
    - Fluid Approximation over Moving Structure
    - Aeroelastic Simulations
5. Conclusions

Fluid-structure interaction problem



Air Flows Interacts with Elastic Wing
Wind Tunnel in Aeronautical Test and Research Institute, Prague

Computational Aeroelasticity

- numerical simulation of both fluid and structure motion
- fluid-structure mutual interaction

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  - aerodynamical forces → structural motion

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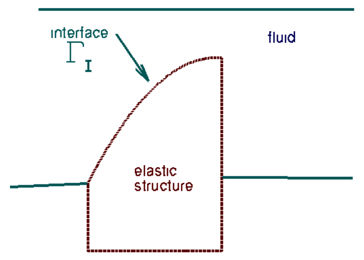
Mathematical Model



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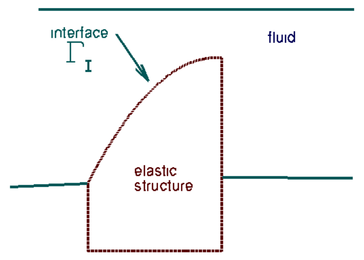

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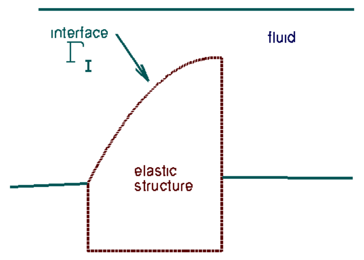

Computational Aeroelasticity

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Fluid Models

- incompressible viscous flow (NS eq.)
- RANS equations - turbulence models

Navier-Stokes System of Equations

Navier-Stokes system

$$\begin{aligned} \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} + \nabla p - \nu \triangle \mathbf{v} &= 0 \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in } \Omega_t \end{aligned}$$

- $\mathbf{v}$ - fluid velocity
- p - pressure
- Navier-Stokes system of equations

Reynolds Averaged Navier-Stokes equations

Navier-Stokes system

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Reynolds Averaging

- $\mathbf{v} = \mathbf{V} + \mathbf{v}'$, $p = P + p'$, such that $\overline{\mathbf{v}} = \mathbf{V}$

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- Reynolds-Stresses $\sigma_{ij}^R = -\overline{v_i' v_j'}$

Turbulence model - SA model

Reynolds-Stresses approximation $\sigma_{ij}^R = -\overline{v_i' v_j'}$

Reynolds stresses are approximated

$$\sigma_{ij}^R = \frac{2}{3} k \delta_{ij} + \nu_T \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right), \qquad \nu_T \approx \tilde{\nu},$$

Turbulence Modelling - Spallart-Almaras Model

$$\frac{\partial \tilde{\nu}}{\partial t} + (\mathbf{V} \cdot \nabla)\tilde{\nu} = \frac{1}{\sigma} \frac{\partial}{\partial x_i} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_i} \right) + \frac{c_{b_2}}{\sigma} (\nabla \tilde{\nu})^2 + G - Y,$$

Comparison of NS and RANS:

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  - describes (turbulent) fluid flow
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  - **the turbulent stresses requires further modelling**
  - (**any) turbulence model is still inexact !!!**

Structure Model

- elasticity equation

$$\rho\frac{\partial^2 u_i}{\partial t^2} - \sum_j \frac{\partial \sigma_{ij}(u)}{\partial x_j} = f_i$$

- u - structure deflection
- special cases: linear stationary elasticity

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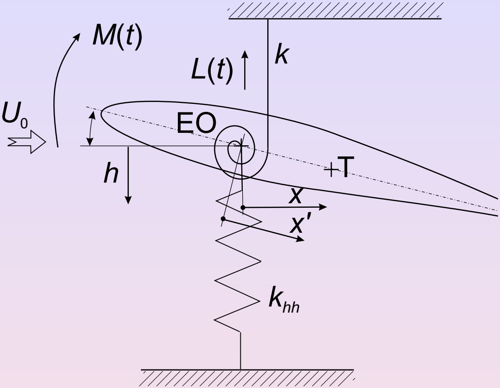
Flexibly Supported Airfoil



Airfoil motion equations

system ODEs

$$\begin{aligned} m\ddot{h} + S_\alpha \ddot{\alpha} + K_{hh} h &= -L \\ S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + K_{\alpha\alpha}\alpha &= M_3 \end{aligned}$$

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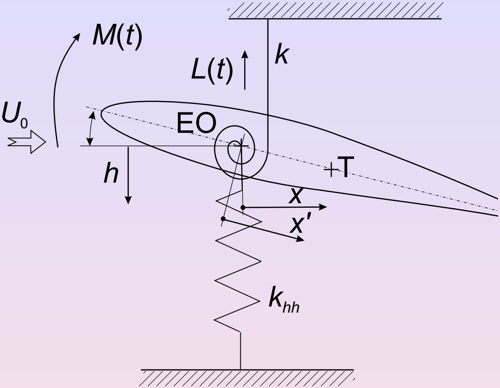

Airfoil motion equations

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Interface Conditions

- elastic structure model
  - interface velocity $\mathbf{w}_I$

$$\mathbf{v} = \mathbf{w}_I, \qquad \dot{\mathbf{u}} = \mathbf{w}_I$$

  - equality of fluid/elastic forces

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  - equality of fluid/elastic forces
- flexibly supported airfoil model
  - airfoil surface condition

$$\mathbf{v} = \mathbf{w}_I$$

  - aerodynamical fluid forces L - lift and M - torsional moment

Goals - revisited

Model

- fluid flow
- flexibly supported airfoil

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Goals

- determine the safe region (critical velocity)
- simulate post-critical regimes (nonlinear aeroelasticity)
- how can we verify our results ?
- compare numerical results to experimental data

Numerical Approximation

- time discretization
- space discretization
- interface conditions

Computations on Moving Meshes

How to approximat the time derivative ? **AVI format**

How to approximate the time derivative?

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Arbitrary Lagrangian-Eulerian method

How to approximate the time derivative?

Arbitrary Lagrangian-Eulerian method

- Define ALE mapping $\mathcal{A}_t$

$$\mathcal{A}_t : \Omega_{\text{ref}} \mapsto \Omega_t$$

- Domain velocity (grid velocity)

$$\tilde{\mathbf{w}}_g(t, Y) = \frac{\partial \mathcal{A}_t(Y)}{\partial t}$$

- ALE derivative - time derivative on ALE trajectory

$$\frac{D^{\mathcal{A}}}{Dt} f = \frac{\partial f}{\partial t} + (\mathbf{w}_g \cdot \nabla) f$$

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Navier-Stokes system in ALE form

$$\begin{aligned} \frac{D^{\mathcal{A}}}{Dt}\mathbf{v} + (\mathbf{v} - \mathbf{w}_g) \cdot \nabla \mathbf{v} + \nabla p - \nu \triangle \mathbf{v} &= 0 \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in } \Omega_t \end{aligned}$$

Weak Formulation

- The ALE derivative is approximated

$$\frac{D^{\mathcal{A}}\mathbf{v}}{Dt} \approx \frac{3\mathbf{v}_{n+1} - 4\tilde{\mathbf{v}}_n + \tilde{\mathbf{v}}_{n-1}}{2\Delta t}$$

- Weak formulation: find $\mathbf{v}_{n+1} = \mathbf{v}$, p

Weak formulation

$$\left(\frac{3\mathbf{v}}{2\Delta t}, \varphi\right) + \left([(\mathbf{v} - \mathbf{w}_g)\cdot\nabla]\,\mathbf{v}, \varphi\right) + \nu(\nabla\mathbf{v}, \nabla\varphi)$$
$$-(p, \nabla^T\cdot\varphi) + (\nabla\cdot\mathbf{v}, q) = \left(\frac{4\tilde{\mathbf{v}}_n - \tilde{\mathbf{v}}_{n-1}}{2\Delta t}, \varphi\right)$$

Galerkin formulation

FEM gives unstable results?

- very high Reynolds numbers → convection dominated flows

$$Re_K^{loc} = \frac{h\|\mathbf{v}\|_K}{\nu} > 1$$

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- Babuška-Brezzi (inf-sup) condition needs to be satisfied

$$\sup_{\mathbf{v}_h \in X_h} \frac{(q_h, \nabla \cdot \mathbf{v}_h)}{\|\mathbf{v}_h\|_{1,2,\Omega}} \geq c \|q_h\|_{0,2,\Omega}$$

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$\Rightarrow$ use Galerkin/Least-Squares stabilization

Spatial discretization

- Stabilization $\psi = (\mathbf{w} \cdot \nabla)\varphi + \nabla q$

$$\mathcal{L}(U,V) = \sum_K \delta_K \left(\frac{3}{2\Delta t}\mathbf{v} - \nu\triangle\mathbf{v} + (\mathbf{w}\cdot\nabla)\,\mathbf{v} + \nabla p, \psi \right)_K ,$$

$$\mathcal{F}(V) = \sum_K \delta_K \left(\frac{4\mathbf{v}^n - \mathbf{v}^{n-1}}{2\Delta t}, \psi \right)_K ,$$

Stabilized problem

Galerkin terms, GALS stabilization, grad-div stabilization

$$a(U,V) + \mathcal{L}(U,V) + \sum_{K\in\tau_h} \tau_K \Big(\nabla\cdot\mathbf{v}, \nabla\cdot\varphi \Big) = f(V) + \mathcal{F}(V).$$

Other topics

- solution of nonlinear problem (NS) - linearization
- solution of linear problem (UMFPACK)
- approximation of ODEs
- interface conditions - coupling of fluid-structure models

Numerical Results

- Fluid Flow Approximation (fixed structure)
- Fluid Flow over Moving structure (validation)
- Aeroelastic Simulations

Numerical Results

- Fluid Flow Approximation (fixed structure)
- Fluid Flow over Moving structure (validation)
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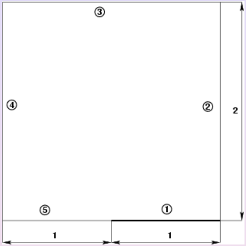
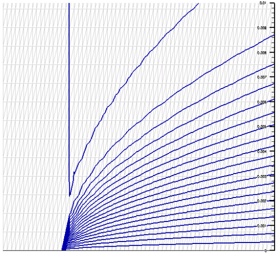
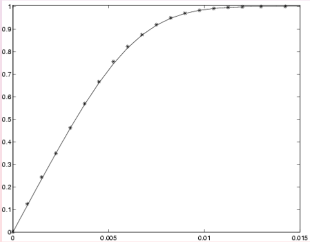
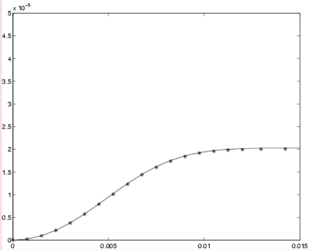
Boundary layer approximation

$Re = 2 \cdot 10^5$, comparison with the analytical Blasius solution



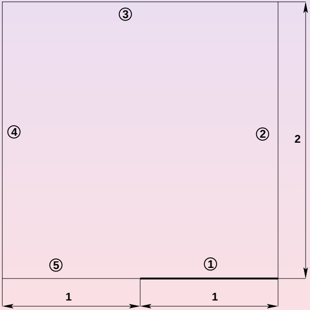


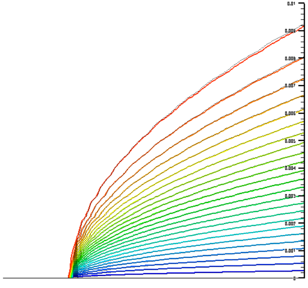




Approximation of Boundary Layer - Taylor Hood

- laminar flow ($Re = 2 \cdot 10^5$)
- FE Dimension: 16683 x 2 + 4242 = 37608
- Nodes: 4242
- Elements: 8200





Blasius solution - Taylor-Hood

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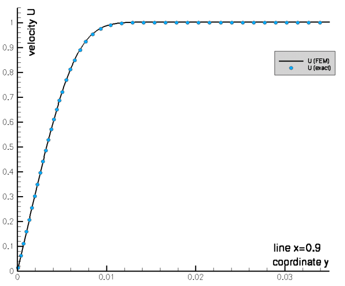

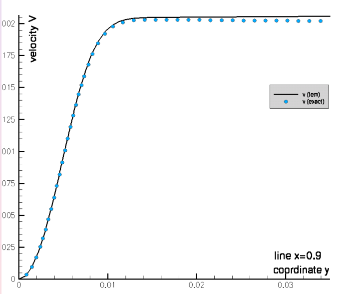

Blasius solution - Taylor-Hood

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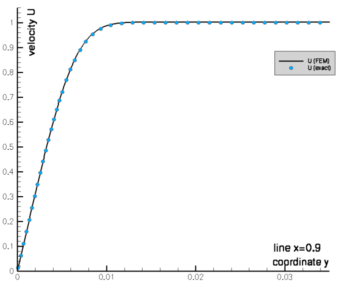
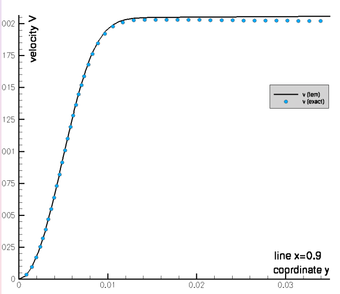
- Can we improve that?





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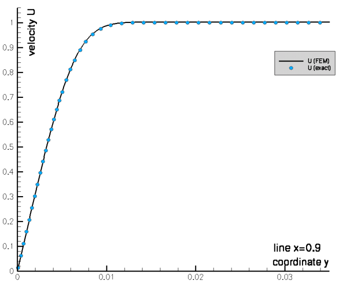
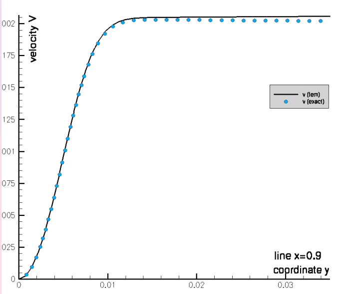
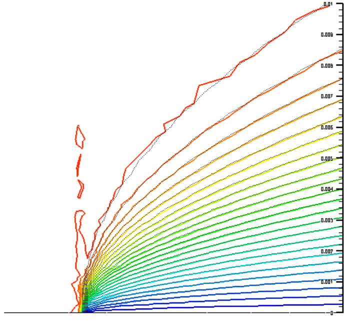
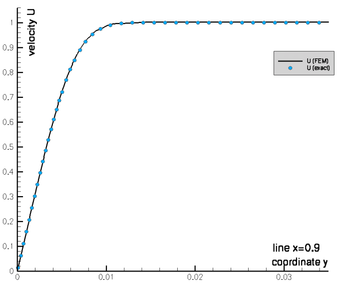




Blasius solution - P3/P2 elements

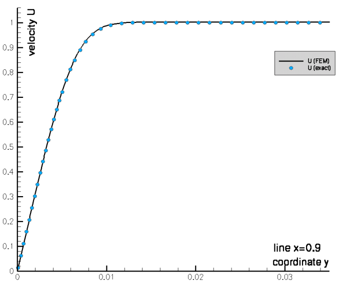
- laminar flow ($Re = 2 \cdot 10^5$)
- uniform p-distribution
- FE Dimension: $4012 \times 2 + 1809 = 9833$
- Nodes: 472
- Elements: 866

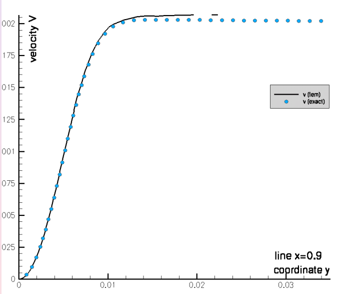




Blasius solution - P4/P3 elements

- laminar flow ($Re = 2 \cdot 10^5$)
- FE Dimension: 2x13235 (Velocity) + 7483 (Pressure) = 33953
- Nodes: 866, Elements: 1629





Flow over NACA $63_2 - 415$

Fluid velocity isolines, $Re = 5 \cdot 10^5$, **AVI format**

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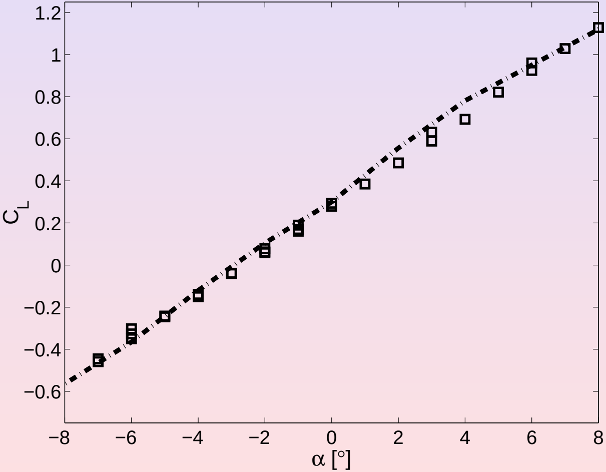
Flow over NACA $63_2 - 415$

- aerodynamical lift coefficient (time averaged values)
- comparison with experimental data for NACA $63_2 - 415$



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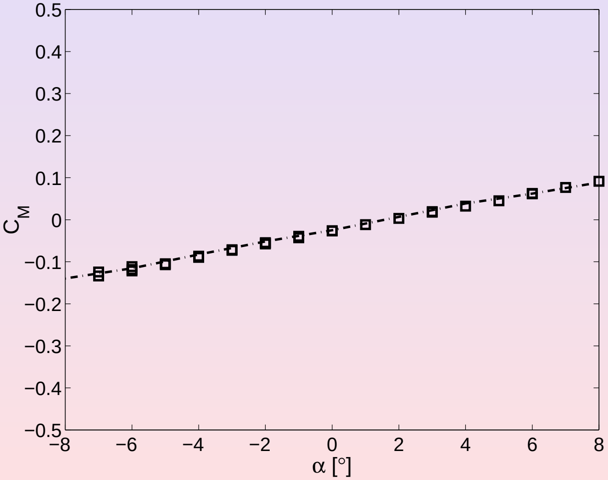

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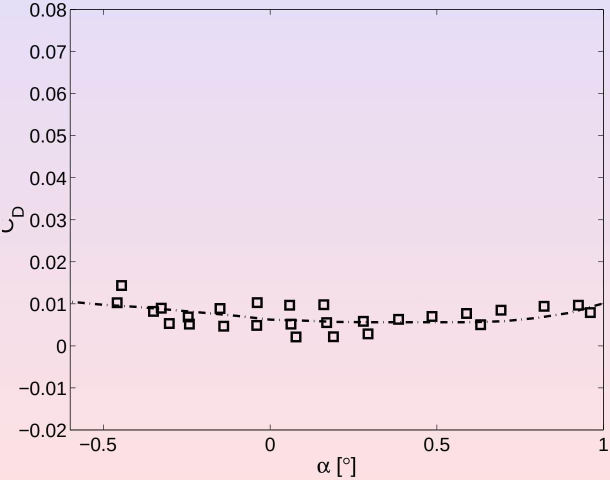
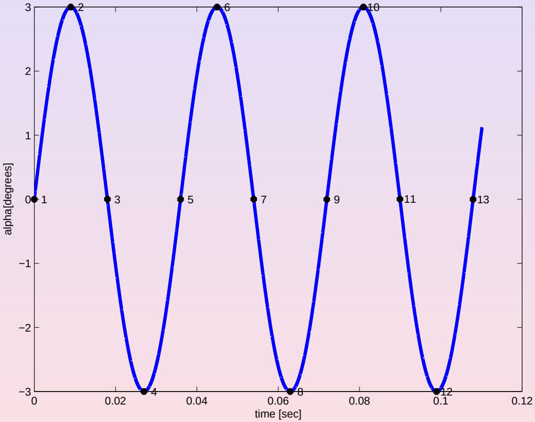
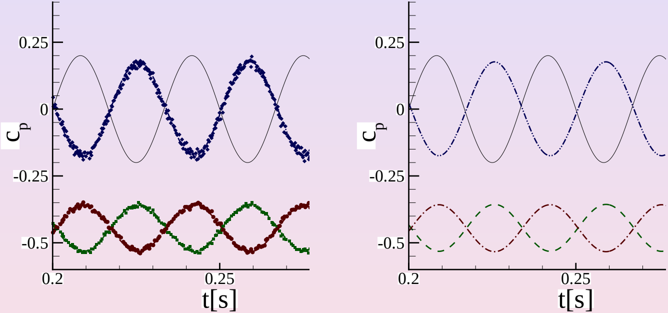


Prescribed vibrations of α, 30 Hz, amplitude 3,2,1 degrees.



Prescribed vibrations of α, 30 Hz, amplitude 1 degree.



Pressure coeficient (up/down) at $x/c = 0.15$ - dependence on time.

Pressure Coefficient

- What is pressure coefficient?

$$c_p = \frac{p - p_0}{\frac{1}{2}\rho U_\infty^2}$$

- for prescribed vibrations

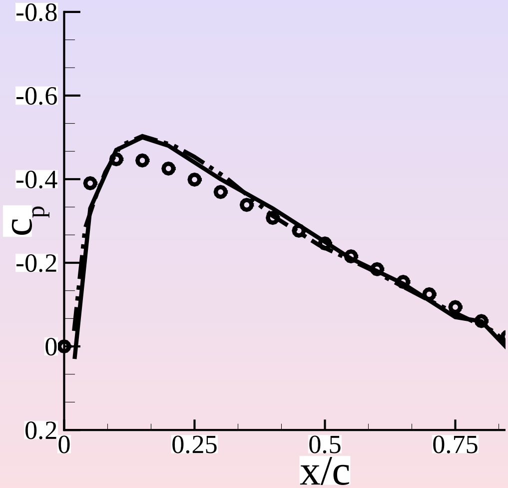
$$\alpha = \alpha_0 \cdot \sin(2\pi f t)$$

- the pressure at airfoil surface is expected to behave like

$$c_p = c_p^{\text{mean}} + c_p' \sin(2\pi f t) + c_p'' \cos(2\pi f t)$$

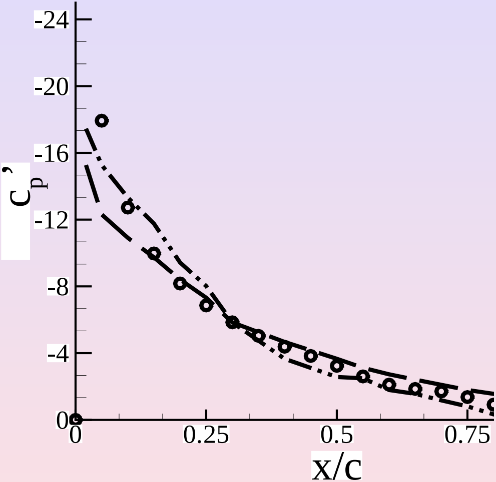
- comparison with experimental data
  Benetka, J. et al, Tech. report 3418/02, ARTI, 2002 ,
  Triebstein, H., 1986., J. Aircraft 23.

Comparison of c_p^{mean} with experimental data



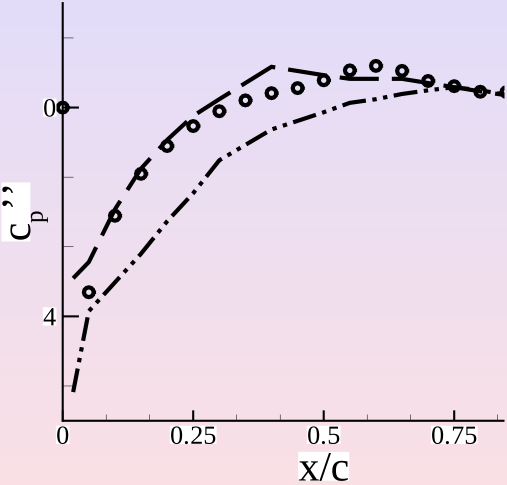
Comparison of c_p^{mean} with experimental data

Comparison of c_p' with experimental data



Comparison of c_p' with experimental data

Comparison of c_p'' with experimental data



Comparison of c_p'' with experimental data

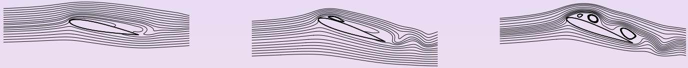
Stall Flutter Approximation

$$\alpha = (10 + 10 \cdot \sin(2\pi f t)), Re = 5000$$

Naudasher, E., Rockwell, D., *Flow-Induced Vibrations*, 1994

Stall Flutter Approximation

$$\alpha = (10 + 10 \cdot \sin(2\pi f t)), Re = 5000$$



$\alpha = 10.859$ $\alpha = 18.3407$ $\alpha = 19.9988$

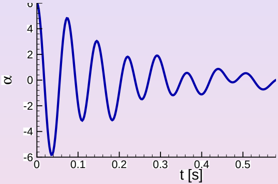
$\alpha = 19.4553$ $\alpha = 12.045$ $\alpha = 8.06$

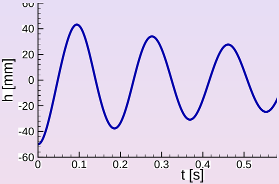
Naudasher, E., Rockwell, D., *Flow-Induced Vibrations*, 1994

Stall Flutter

Fluid velocity isolines, $Re = 5 \cdot 10^5$ **AVI(1) format**

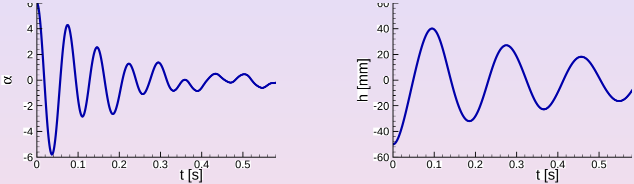
Aeroelastic simulations - laminar





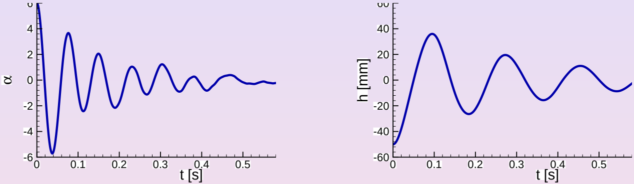
$$U_\infty = 4 \text{ m/s}$$

Aeroelastic simulations - laminar



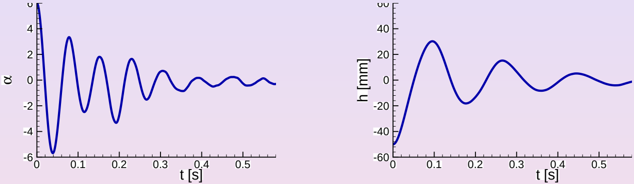
$$U_\infty = 8 \text{ m/s}$$

Aeroelastic simulations - laminar



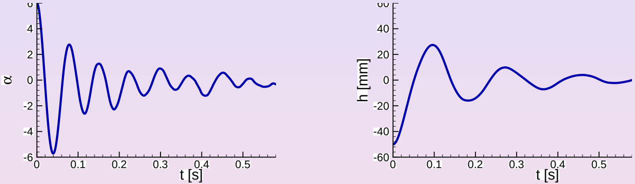
$$U_\infty = 12 \text{ m/s}$$

Aeroelastic simulations - laminar



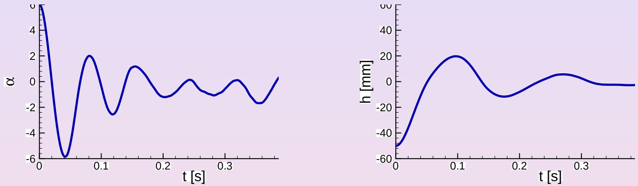
$$U_\infty = 16 \text{ m/s}$$

Aeroelastic simulations - laminar



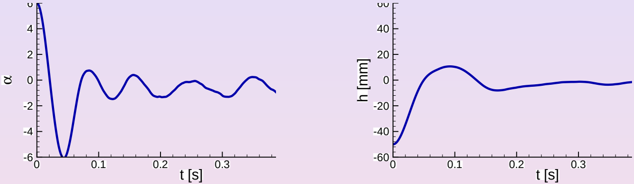
$$U_\infty = 18 \text{ m/s}$$

Aeroelastic simulations - laminar



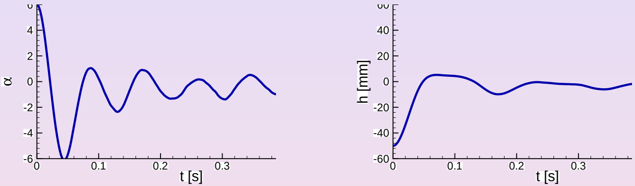
$$U_\infty = 22 \text{ m/s}$$

Aeroelastic simulations - laminar



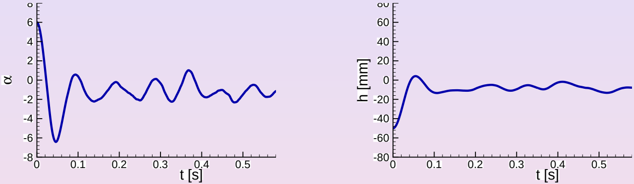
$$U_\infty = 26 \text{ m/s}$$

Aeroelastic simulations - laminar



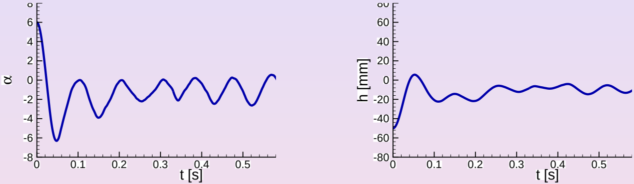
$$U_\infty = 28 \text{ m/s}$$

Aeroelastic simulations - laminar



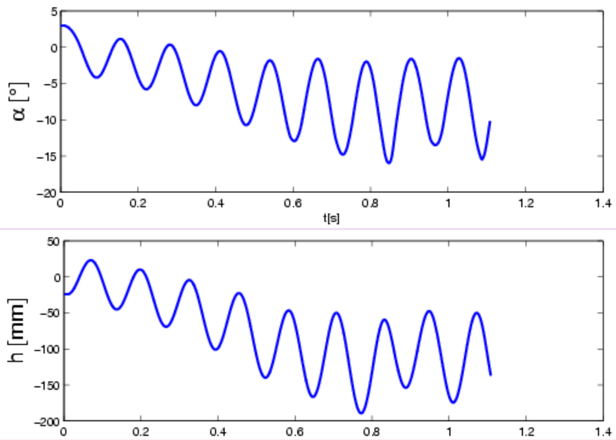
$$U_\infty = 32 \text{ m/s}$$

Aeroelastic simulations - laminar



$U_\infty = 34$ m/s

Aeroelastic simulations - laminar

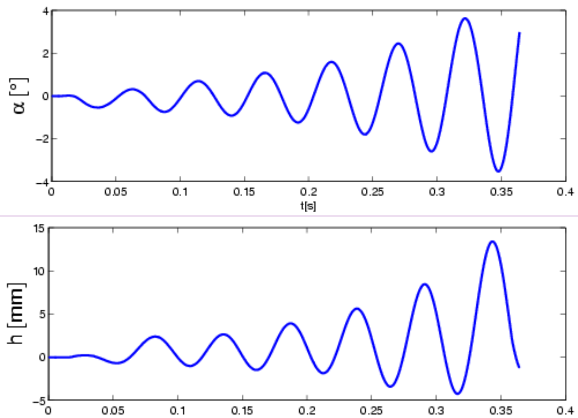


$$U_\infty = 40 \text{ m/s}$$

Aeroelastic simulations - laminar

$$U_\infty = 40 \text{ m/s}$$

Aeroelastic simulations - laminar

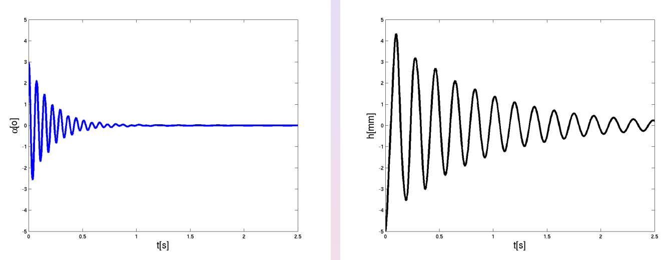


$$U_\infty = 45 \text{ m/s}$$

Aeroelastic model - Turbulent Flow

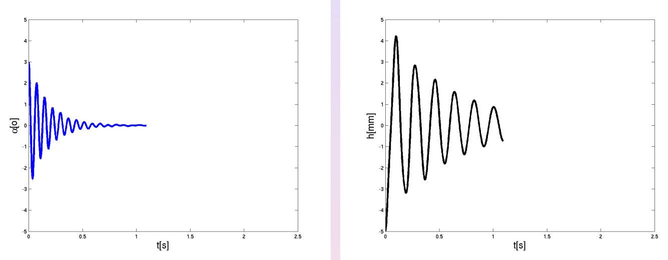
- flexibly supported airfoil NACA 0012
- RANS + Spallart-Almaras turbulence model
- NASTRAN computation with the STRIP model critical speed $U_\infty = 37.7 m/s$
- frequencies and damping comparison

Aeroelastic model - Turbulent Flow



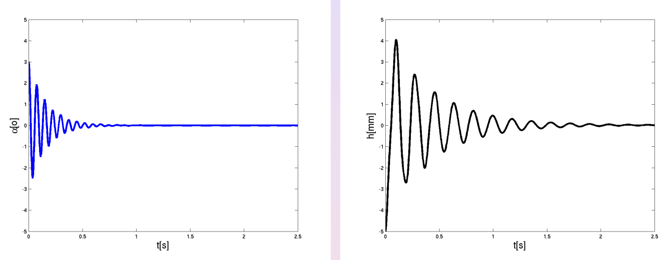
Solution of the coupled aeroelastic model (h,α), $U = 5m/s$

Aeroelastic model - Turbulent Flow



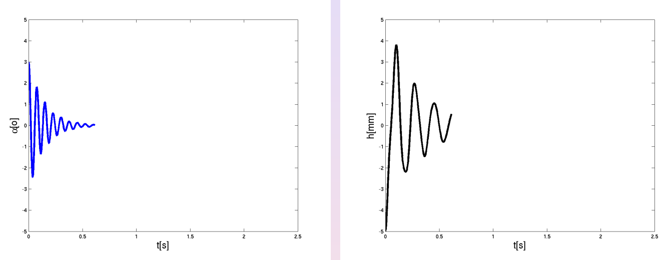
Solution of the coupled aeroelastic model (h,α), $U = 7.5m/s$

Aeroelastic model - Turbulent Flow



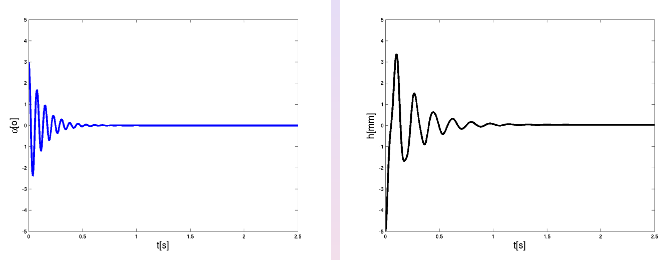
Solution of the coupled aeroelastic model (h,α), $U = 10 m/s$

Aeroelastic model - Turbulent Flow



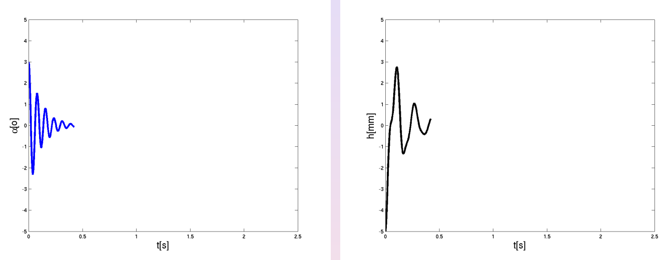
Solution of the coupled aeroelastic model (h,α), $U = 12.5 m/s$

Aeroelastic model - Turbulent Flow



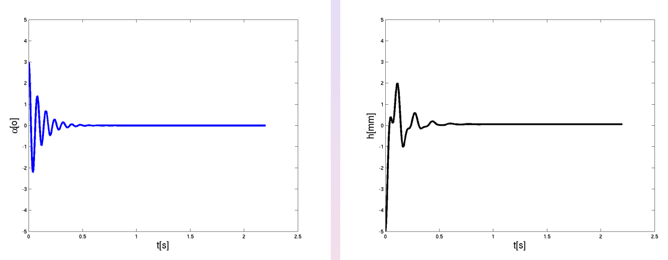
Solution of the coupled aeroelastic model (h,α), $U = 15m/s$

Aeroelastic model - Turbulent Flow



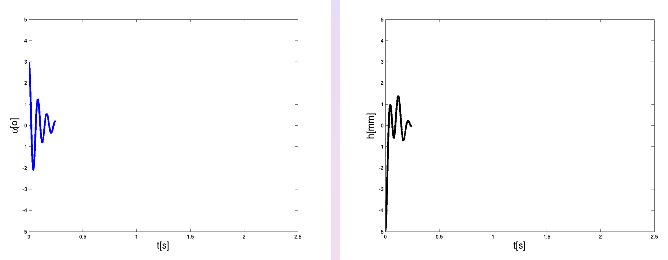
Solution of the coupled aeroelastic model (h,α), $U = 17.5 m/s$

Aeroelastic model - Turbulent Flow



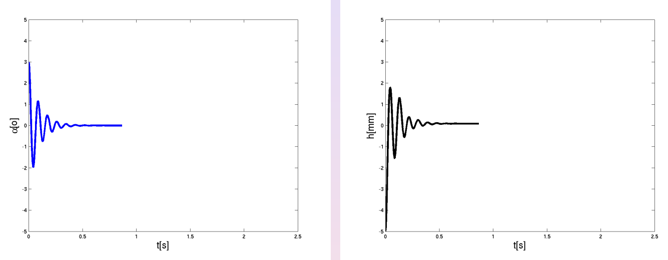
Solution of the coupled aeroelastic model (h,α), $U = 20m/s$

Aeroelastic model - Turbulent Flow



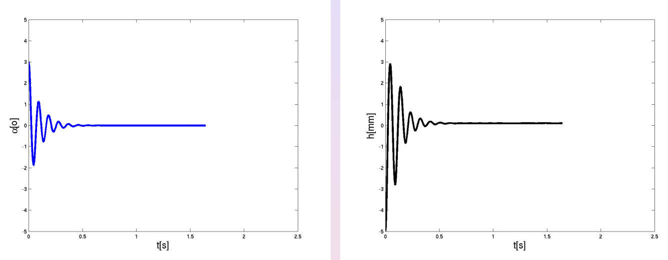
Solution of the coupled aeroelastic model (h,α), $U = 22.5m/s$

Aeroelastic model - Turbulent Flow



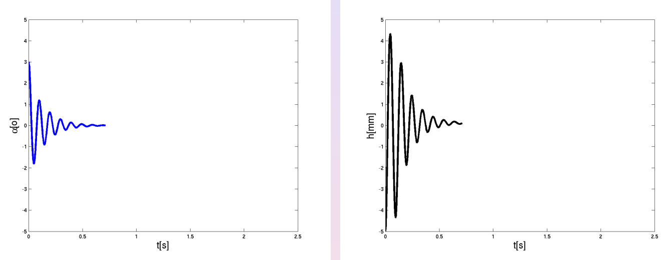
Solution of the coupled aeroelastic model (h,α), $U = 25 m/s$

Aeroelastic model - Turbulent Flow



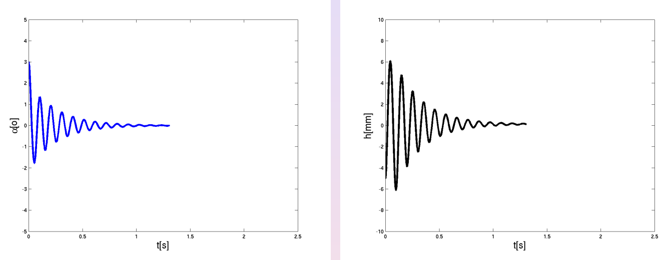
Solution of the coupled aeroelastic model (h,α), $U = 27.5m/s$

Aeroelastic model - Turbulent Flow



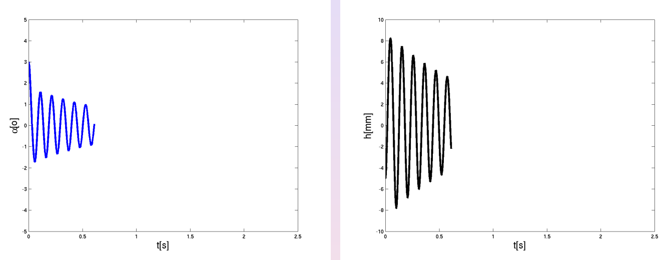
Solution of the coupled aeroelastic model (h,α), $U = 30m/s$

Aeroelastic model - Turbulent Flow



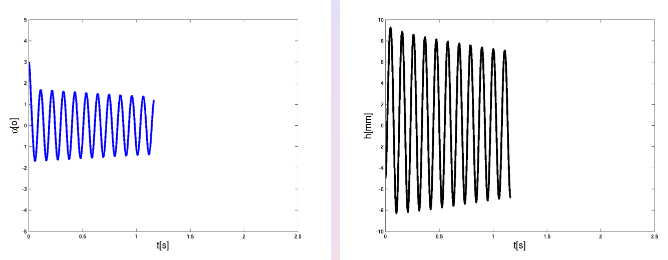
Solution of the coupled aeroelastic model (h,α), $U = 32.5 m/s$

Aeroelastic model - Turbulent Flow



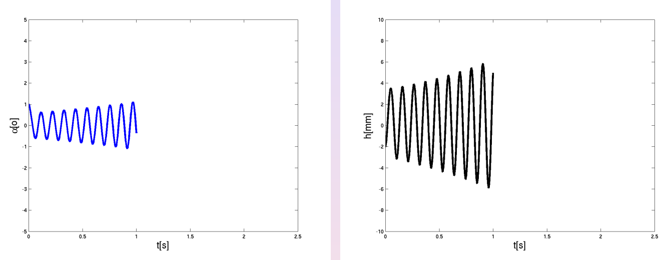
Solution of the coupled aeroelastic model (h, α), $U = 35m/s$

Aeroelastic model - Turbulent Flow



Solution of the coupled aeroelastic model (h,α), $U = 36m/s$

Aeroelastic model - Turbulent Flow

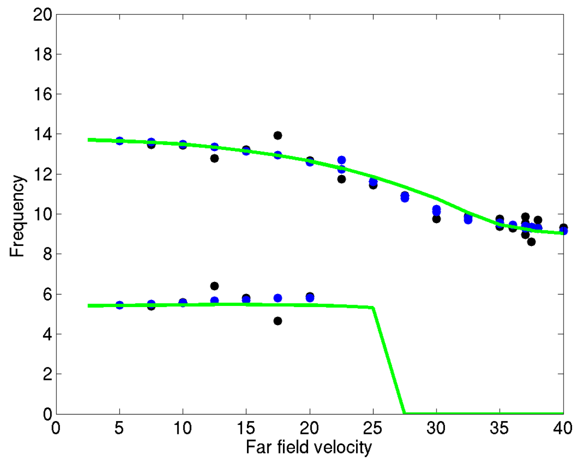


Solution of the coupled aeroelastic model (h,α), $U = 37 m/s$

Aeroelastic model - Turbulent Flow

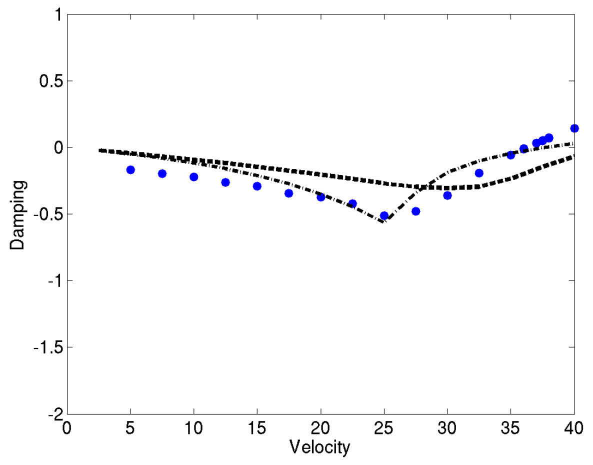
- U=37 m/s
- velocity isolines, **AVI format**

Comparison with NASTRAN computation



Frequencies comparison

Comparison with NASTRAN computation



Damping

Summary

- NS solver numerical results were compared to experimental/computational data.
- NS solver and RANS solver results for aeroelastic problem were compared each to other.
- Performence: RANS x NS (?)

Conclusion

- RANS and NS shows good agreement with NASTRAN computations.

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- NS solver numerical results were compared to experimental/computational data.
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Conclusion

- RANS and NS shows good agreement with NASTRAN computations.
- We must provide: careful mesh design, time step value, ...

Summary

- NS solver numerical results were compared to experimental/computational data.
- NS solver and RANS solver results for aeroelastic problem were compared each to other.
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Conclusion

- RANS and NS shows good agreement with NASTRAN computations.

- How to increase reliability?

Can the grid motion “pollute” the solution?

Can the grid motion "pollute" the solution?

AVI format Test Problem: constant fluid velocity $\mathbf{v} = (1, 0)$ on rectangle.

Use anallytical derivative of ALE mapping ...

$$\mathbf{w}_g = \frac{\partial \mathbf{x}}{\partial t}$$

Approximately 10 % error (!) **AVI format**

Use finite difference formula ...

$$\mathbf{w}_g \approx \frac{\mathbf{3}x^{(n+1)} - \mathbf{x}^{(n)} + \mathbf{x}^{(n-1)}}{2\Delta t}$$

Grid Motion Influence on Solution (finite difference

The ALE derivative approximation

- use $\mathbf{v}_{n-1}$ - defined on Ω_{n-1}
- $\mathbf{v}_n$ - defined on Ω_n

The ALE derivative approximation

- use $\mathbf{v}_{n-1}$ - defined on Ω_{n-1}
- $\mathbf{v}_n$ - defined on Ω_n
- and $\mathbf{v}_{n+1}$ - defined on Ω_{n+1}

$$\frac{D^{\mathcal{A}}\mathbf{v}}{Dt} \approx \frac{3\mathbf{v}_{n+1} - 4\tilde{\mathbf{v}}_n + \tilde{\mathbf{v}}_{n-1}}{2\Delta t}$$

The ALE derivative approximation

- use $\mathbf{v}_{n-1}$ - defined on Ω_{n-1}
- $\mathbf{v}_n$ - defined on Ω_n

- where $\tilde{\mathbf{v}}_n$ and $\tilde{\mathbf{v}}_{n-1}$ lives on Ω_{n+1}

Spatial discretization

- Stabilization $\psi = (\mathbf{w} \cdot \nabla)\varphi + \nabla q$

$$\mathcal{L}(U, V) = \sum_K \delta_K \left(\frac{3}{2\tau} \mathbf{v} - \nu \triangle \mathbf{v} + (\mathbf{w} \cdot \nabla) \mathbf{v} + \nabla p, \psi \right)_K ,$$

$$\mathcal{F}(V) = \sum_K \delta_K \left(\frac{4\mathbf{v}^n - \mathbf{v}^{n-1}}{2\tau}, \psi \right)_K ,$$

Stabilized problem

- Gelhard, T., Lube, G., Olshanskii, M. A., 2004. Stabilized finite element schemes with BB-stable elements for incompressible flows. *Journal of Computational and Applied Mathematics* (accepted).
- Gelhard, T., Lube, G., Olshanskii, M. A., 2004. Stabilized finite element schemes with LBB-stable elements for incompressible flows. *Journal of Computational and Applied Mathematics* .