Extreme probability distributions of random/fuzzy sets and p-boxes

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Abstract

Uncertain information about a system variable described by a random set or an equivalent Dempster-Shafer structure on a finite space of singletons determines an infinite convex set of probability distributions, given by the convex hull of a finite set of extreme distributions. Although general procedures to derive extreme distributions from Believe/Plausibility set functions (Choquet capacities and respectively Alternate Choquet capacities of ∞ order) are well known in the literature (see for example [1]), in the applications little attention is given to the calculation of (upper/lower bounds on) the expectation of functions of uncertain variables by using extreme distributions, for example in reliability evaluation of engineering systems. The procedure described in the paper is based on the Choquet integral, and is particularly useful in the case of non-monotonic functions because in these cases it gives the exact bounds on the expectation. With reference to the simple case of a single variable, the paper presents a brief summary of the subject with emphasis on random sets corresponding to a nested structure of focal elements (consonant random sets or the equivalent fuzzy set) and to p-boxes: in both cases “indexable-type random sets”[2].
A simple direct procedure to derive extreme distributions from a p-box is described through simple numerical examples.

References

1. Klir, G.J., *Uncertainty and Information. Foundations of generalized Information Theory*. 2005, Hoboken, New Jersey: Wiley & Sons, Inc. XVIII+500.
2. Alvarez, D.A., *On the calculation of the bounds of probability of events using infinite random sets.* International Journal of Approximate Reasoning, 2006. **43**(3): p. 241-267.