Propagating Uncertainties in Modeling Nonlinear Dynamic Systems

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Abstract

Engineering analysis and design problems, either static or dynamic, frequently involve uncertain parameters and inputs. Propagating these uncertainties through a complex model to determine their effect on system states and outputs can be a challenging problem, especially for dynamic models.

Probability distributions describing the uncertainties may not be known precisely, if known at all. If there are only upper and lower bounds on the uncertainties but no known probability distribution, then this can be represented by an interval. If there is some knowledge of the probability distribution, but it is uncertain, then this can be represented by a probability box (p-box). A p-box [1] is an ordered pair of monotonically increasing functions which bound a set of cumulative probability distribution functions. Thus, a p-box bounds a set of cumulative probability distribution functions in the same way that an interval bounds a set of real numbers.

Recently, Lin and Stadtherr [2] have described and implemented a new verifying solver for parametric ODEs (VSPODE). Using VSPODE, it is possible to obtain a Taylor model representation [3] of the state variables and outputs in terms of the uncertain quantities. A Taylor model consists of a Taylor polynomial function and an interval remainder bound.

In this presentation, we demonstrate the use of Taylor models for propagating uncertainties through nonlinear ODE models. The case in which uncertainties are interval-valued has been discussed by Lin and Stadtherr [2]. We concentrate here on uncertainties represented by p-boxes, and show how to use p-boxes in the context of

Taylor models. This allows us to obtain p-box representations of the uncertainties in the state variables and outputs. Examples are used to demonstrate the potential of this approach for studying the effect of uncertainties with imprecise probability distributions.

References

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