Uncertain processes and numerical monitoring of structures

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Abstract. A structure is subjected to numerous alterations and modifications during its lifetime. The entirety of the modifications of structures constitutes the process of modifications. Numerical monitoring of a structure during its lifetime close to reality requires considering the complete load and modification processes simultaneously. Both processes run discontinuously. They cause time dependent, discontinuous result values. The parameters of the load and modification process are usually uncertain parameters. Due to their predominantly informal and lexical uncertainty, they are described as fuzzy processes, respectively fuzzy functions. Taking account of this uncertainty in the numerical simulation of the load and modification process requires a fuzzy structural analysis in the time domain. The fuzzy variables and the fuzzy functions are mapped on the fuzzy result variables with the aid of a crisp or uncertain analysis algorithm. The numerical simulation is based on an optimization procedure. This procedure searches for special points in the input space of the fuzzy variables. Each point of the input space represents a deterministic parameter data set, which is introduced in a deterministic fundamental solution. In this paper the geometrically and physically nonlinear analysis of plane reinforced concrete, prestressed concrete, textile concrete, and steel bar structures is chosen as deterministic fundamental solution. The algorithms are demonstrated by way of examples.

Keywords: uncertainty modeling, numerical monitoring, nonlinear numerical analysis

1. Numerical Monitoring of Structures – Conceptual Idea

Numerical monitoring of structures is the numerical simulation of the behaviour of structures during the lifetime. A structure is subject to numerous alterations during its lifetime. These modifications may result from:

- Sequence of different states during construction
- Changes in material, e.g., the change of material behavior due to physical or chemical processes
- Structural alteration resulting from, e.g., refurbishing, bonding of prestressing elements, strengthening
- Changes in load, described by a loading process

For structural alterations and the sequence of different states during construction the term "system modification" is adopted. The system modification comprises cross section modification, modification of structural members, and modification of support conditions [Bartzsch, Graf, Möller & Sickert 2004]. The change of prestressing forces may also be understood as system modification. The entirety of the system modifications constitutes the modification process. Analyzing a structure during the lifetime close to reality requires considering the complete load and modification processes simultaneously. Both processes run discontinuously. These processes must be described by means of suitable mechanical models. They cause time dependent, discontinuous result values $\underline{z}(\underline{t})$:

$$\underline{z}(\underline{t}) = f(\underline{g}(\underline{t}), \underline{p}(\underline{t}), \underline{F}_p(\underline{t}), \underline{T}(\underline{t}), \underline{A}(\underline{t}), \underline{I}(\underline{t}), \underline{E}(\underline{t})) \tag{1}$$

with

$\underline{z}$	vector of structural responses (e.g., displacements and internal forces)
$\underline{g}(\underline{t})$	dead load
$\underline{p}(\underline{t})$	statically and dynamic external loads
$\underline{F}_p(\underline{t})$	prestressing forces (internal and external prestressing)
$\underline{T}(\underline{t})$	parameters of temperature
$\underline{A}(\underline{t})$, $\underline{I}(\underline{t})$	parameters of geometry representing time dependent values in the modification process (e.g., cross sections, dimensions of the system, location of the reinforcement, and the prestressing elements)
$\underline{E}(\underline{t})$	material parameters
$\underline{t} = (\underline{\theta}, \tau, \underline{\varphi})$	spatial coordinates $\underline{\theta} = \theta_1, \theta_2, \theta_3$, time τ, further parameters $\underline{\varphi}$, e.g. temperature

The parameters of the load and modification process are usually uncertain parameters. The following mathematical models are available to describe uncertainty (see also Figure 1):

- Randomness
- Fuzziness
- Fuzzy randomness

whereas fuzziness and randomness are considered as special cases of the general model fuzzy randomness [Möller & Beer 2004]. The choice of the model depends on the available data.

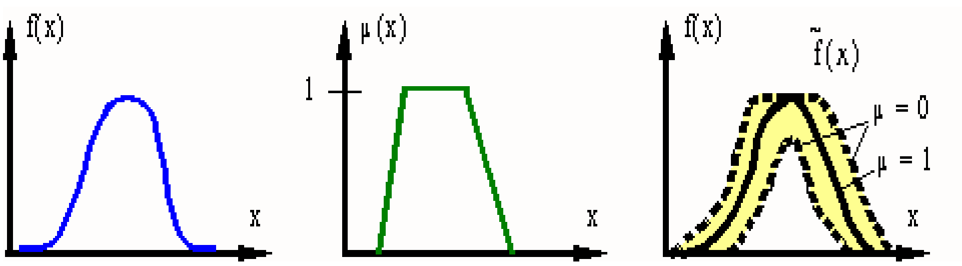

Figure 1. Mathematical models of uncertainty

If sufficient statistical data exist for a parameter the parameter may be described stochastically. Thereby the choice of the type of the probability distribution function affects the result considerably. Often statistically not ensured samples exist for a parameter. Then the description by the uncertainty model fuzziness is recommended. The model comprehends both objective and subjective information. The uncertain parameters are characterized by aid of a membership function $\mu(x)$, see eq. (1). The membership function assesses the gradual membership of elements to a set [Möller & Beer 2004].

$$\tilde{x}=\{(x;\mu_x(x))\ |\ x\in X\};\quad \mu_x(x)\geq 0\ \ \forall\ \ x\in X \tag{2}$$

The uncertainty model fuzzy randomness is a superordinate model that both stochastic and non-stochastic properties of parameters enclose. Fuzzy random variables are used if, e.g., reproduction conditions vary during the period of observation, or if expert knowledge complements the statistical material. A fuzzy random variable is the fuzzy set of their originals, see eq. (3). The originals are probability functions of random variables.

$$\tilde{f}(x)=\{(f(x);\mu_f(f(x)))\ |\ f\in f\};\qquad \mu_f(f(x))\geq 0\ \ \forall\ \ f\in f \tag{3}$$

Due to the predominantly informal and lexical fuzziness of the parameters of the load and modification process the uncertain parameters are described by the mathematical model fuzziness. As the parameters are time dependent they are considered as fuzzy functions $\tilde{x}(\underline{\tilde{t}})=\tilde{x}(\underline{\tilde{\theta}},\tilde{\tau},\underline{\tilde{\varphi}})$ or fuzzy processes $\tilde{x}(\tilde{\tau})$.

2. Formal Description of Uncertain Discontinuous Processes

A fuzzy vector $\underline{\tilde{x}}$ describes uncertain parameters at discrete points. A fuzzy function $\tilde{x}(\underline{\tilde{t}})$ enables the formal description of at least piecewise continuous uncertain parameters in $\mathbb{R}^1$, $\mathbb{R}^2$, or $\mathbb{R}^3$. The following definition of fuzzy functions is introduced. Given are

- the fundamental sets $\mathbf{T}\subseteq\mathbb{R}$ and $\mathbf{X}\subseteq\mathbb{R}$
- the set **F(T)** of all fuzzy variables $\underline{\tilde{t}}$ on the fundamental set **T**
- the set **F(X)** of all fuzzy variables $\tilde{x}$ on the fundamental set **X**.

An uncertain mapping of **F(T)** to **F(X)** that assigns exactly one $\tilde{x}\in$ **F(X)** to each $\underline{\tilde{t}}\in$ **F(T)**, respectively, is referred to as a fuzzy function denoted by

$$\tilde{x}(\underline{\tilde{t}}):\quad \mathbf{F(T)}\ \overset{\sim}{\longrightarrow}\ \mathbf{F(X)} \tag{4}$$

$$\tilde{x}(\underline{\tilde{t}})=\{(\tilde{x}_f=x(\underline{\tilde{t}})\quad\forall\quad \underline{\tilde{t}}\ |\ \underline{\tilde{t}}\in\mathbf{F(T)}\} \tag{5}$$

In system modification the fundamental set **T** may contain both the uncertain time coordinate $\tilde{\tau}$ and the crisp spatial coordinate $\underline{\theta}$. In this case the assigned fuzzy function is denoted by $\tilde{x}(\underline{\tilde{t}})=\tilde{x}(\underline{\theta},\tilde{\tau})$ with $\underline{\tilde{t}}=(\underline{\theta},\tilde{\tau})$. The fuzzy function $\tilde{x}(\underline{\theta},\tilde{\tau})$ enables the modeling of processes with uncertain time points. This is of interest if the system is modified at non-precise known points in time. If the time points are crisp, the special case

$$\tilde{x}(\underline{\theta},\tau)=\tilde{x}(\underline{t})=\{(\tilde{x}_t=\tilde{x}(\underline{t}))\quad\forall\quad \underline{t}\ |\ \underline{t}\in\mathbf{T})\} \tag{6}$$

is obtained [Möller & Beer 2004]. Figure 2 shows a fuzzy process $\tilde{x}(\underline{\theta}_j,\tau)$ for a specific point with the coordinate $\underline{\theta}_j$.

For the numerical simulation of system modifications the bunch parameter representation of a fuzzy function is applied.

$$x(\underset{\sim}{s}, \underline{t}) = \{(\tilde{x}_t = x(\underset{\sim}{s}, \underline{t})) \quad \forall \quad \underline{t} \mid \underline{t} \in \mathbf{F}(\mathbf{T})\} \tag{7}$$

For each crisp bunch parameter vector $\underline{s} \in \underset{\sim}{s}$ with the assigned membership value $\mu(\underline{s})$ a crisp function $x(\underline{t}) = (x(\underline{s}, \underline{t})) \in \tilde{x}(\underline{t})$ with $\mu(x(\underline{t})) = \mu(\underline{s})$ is obtained. The fuzzy function $\tilde{x}(\tilde{\underline{t}})$ may thus be represented by the fuzzy set of all real valued functions $x(\underline{t}) \in \tilde{x}(\underline{t})$ with $\mu(x(\underline{t})) = \mu(x(\underline{s}, \underline{t})) = \mu(\underline{s})$

$$x(\underset{\sim}{s}, \underline{t}) = \{(x(\underline{t}), \mu(x(\underline{t}))) \mid x(\underline{t}) = x(\underline{s}, \underline{t})\}; \tag{8}$$
$$\mu(x(\underline{t})) = \mu(\underline{s}) \ \forall \ \underline{s} \mid \underline{s} \in \underset{\sim}{s}$$

which may be generated from all possible real vectors $\underline{s} \in \underset{\sim}{s}$. For every $\underline{t} \in \underline{\mathbf{T}}$ takes values which are simultaneously contained in the associated fuzzy functional values $\tilde{x}(\underline{t})$. The real functions x($\underline{t}$) of $\tilde{x}(\underline{t})$ are defined for all $\underline{t} \in \underline{\mathbf{T}}$. These are referred to as trajectories.

Numerical processing of fuzzy functions $\tilde{x}(\underline{t}) = (x(\underset{\sim}{s}, \underline{t}))$ demands the discretization of their arguments $\underline{t}$ in space and time.

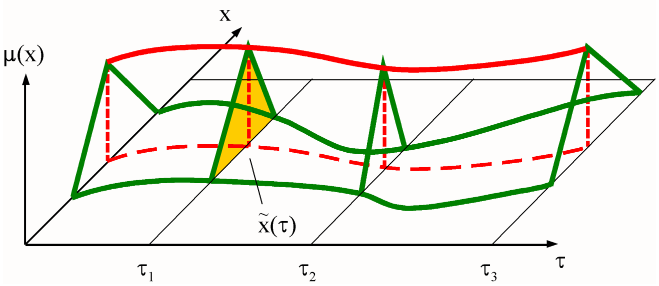

Figure 2. Fuzzy process

3 Numerical Processing of Uncertain Discontinuous Processes

In deterministic structural analysis crisp structural input vectors $\underline{x}$ containing parameters, for example, for loads, geometrical and material properties are mapped with the aid of a computational model to structural responses such as stresses, internal forces, and displacements. This mapping may be denoted as

$$\underline{x} \quad \rightarrow \quad \underline{z} \tag{9}$$

in which the arrow indicates the computational model as the mapping model. This deterministic computational model is subsequently referred to as deterministic fundamental solution within the framework of an uncertain analysis.

If the structural parameters possess uncertainty in the form of fuzziness, eq. (9) may be rewritten as

$$\underset{\sim}{x} \quad \rightarrow \quad \underset{\sim}{z} \tag{10}$$

representing a fuzzy structural analysis. The input vectors $\underset{\sim}{x}$ are then formed by fuzzy structural parameters $\tilde{x}_i$; and the fuzzy structural response vectors $\underset{\sim}{z} = (\ldots, \tilde{z}_j, \ldots)$ are determined on the basis of fuzzy set operations. For processing fuzzy quantities through structural computations in a general and numerically efficient manner a global optimization scheme referred to as α-level optimization has been developed [Möller, Graf & Beer 2000]. This includes a modified evolution strategy as the kernel solution technique.

The concept of α-discretization is applied to numerically represent the fuzzy structural parameters $\tilde{x}_i$ as a set of α-level sets for a sufficiently high umber of α-levels. All fuzzy input parameters are discretized using the same number of α-levels α_k, k = 1 … r. With the aid of the deterministic fundamental solution (mapping model) crisp elements from the fuzzy input vectors, $\underline{x} \in \tilde{\underline{x}}$, are processed to obtain crisp elements of the fuzzy structural response vectors, $\underline{z} \in \tilde{\underline{z}}$. In terms of α-level optimization this means the mapping of $\underline{x} \in \underline{X}_{\alpha k}$ to $\underline{z} \in \underline{Z}_{\alpha k}$, in which $\underline{X}_{\alpha k}$ and $\underline{Z}_{\alpha k}$ are crisp input and result subspaces, respectively, for each α-level. The mapping of all elements of $\underline{X}_{\alpha k}$ yields the crisp subspace $\underline{Z}_{\alpha k}$. Once the largest element $z_{j,\alpha k_r}$ and the smallest element $z_{j,\alpha k_l}$ of the dimension j of the crisp subspace $\underline{Z}_{\alpha k}$ have been found, two points of the membership function $\mu(z_j)$ of the fuzzy result z_j are known. The search for these extreme elements $z_{j,\alpha k_r}$ and $z_{j,\alpha k_l}$ on each α-level represents an optimization problem and is referred to as α-level optimization, see Figure 3. For the detection of $z_{j,\alpha k_r}$ and $z_{j,\alpha k_l}$ with a high probability in general cases with no restrictions regarding the properties of the mapping model, which represents the objective function in the optimization procedure, the modified evolution strategy according to [Möller, Graf & Beer 2000] is employed. This procedure possesses a simple structure, exhibits a reasonable robustness with regard to numerical noise in the mapping model, and can be applied very flexibly in dependence on the problem by adjusting several effective control parameters. The computational costs of the modified evolution strategy increases approximately linearly with the number of dimensions of the problem. For a further improvement of the performance of the procedure a post-computation is carried out after the completion of all optimizations for all α-levels. This includes a recheck of all $z_{j,\alpha k_r}$ and $z_{j,\alpha k_l}$ with the aid all information gathered during all individual optimizations and a re-optimization of those results, which are identified as being not yet optimum. The features robustness, numerical efficiency, and general applicability of the modified evolution strategy enable an application of α-level optimization in combination with arbitrary nonlinear algorithms as mapping models for structural analysis.

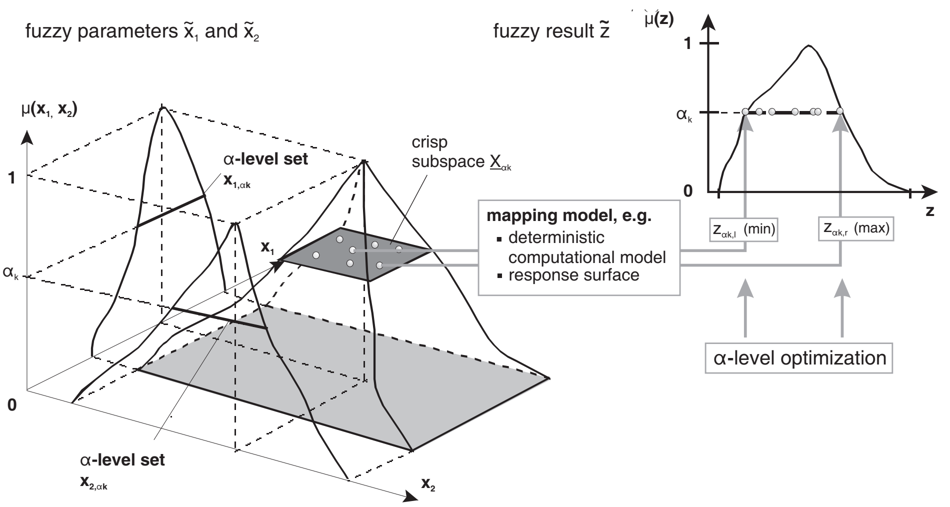

Figure 3a. α-level optimization

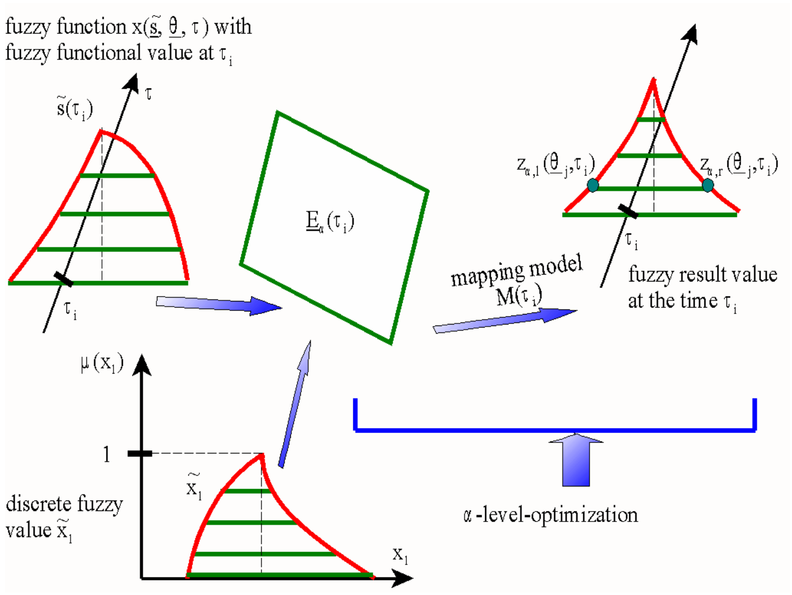

Figure 3b. α-level optimization

The deterministic fundamental solution represents the respective analysis algorithm and is selectable. In this paper the geometrically and physically nonlinear analysis of plane reinforced concrete, prestressed concrete, and steel bar structures [Bartzsch 2006] is chosen as deterministic fundamental solution. The bars are subdivided into integration sections, the cross sections are subdivided into layers. On this basis an incrementally formulated system of second order differential equations for the straight or imperfectly straight bar is obtained. The slip at the bond joint is regarded as an additional degree of freedom s.

$$\left[\frac{d\Delta z(\theta_1)}{d\theta_1}\right]_{(n)}^{[k]} = \underline{A}(\theta_1,\underline{z})_{(n-1)}\cdot\Delta\underline{z}(\theta_1)_{(n)}^{[k]} + \Delta\underline{b}(\theta_1,\underline{z})_{(n)}^{[k-1]} + \\ \ldots + \underline{d}(\theta_1,\underline{z})_{(n-1)}\cdot\Delta\dot{\underline{z}}_1(\theta_1)_{(n)}^{[k]} + \underline{m}(\theta_1,\underline{z})_{(n-1)}\cdot\Delta\ddot{\underline{z}}_1(\theta_1)_{(n)}^{[k]} \tag{11}$$

with

[k]	counter of iteration steps
(n)	counter of increments
θ_1	bar coordinate
Δ	increment
$\underline{z}$	vector of structural response, $\underline{z} = \{\underline{z}_1; \underline{z}_2\} = \{u\ w\ \varphi\ s;\ N\ Q\ M\ N_s\}$
$\underline{A}$	matrix of coefficients (constant within the increment)
$\underline{b}$	"right hand side" of the system of differential equations with loads and varying parts resulting from geometrically nonlinearities, with physically nonlinear correction forces, as well as with forces from unbonded prestressing
$\underline{d}$	damping matrix
$\underline{m}$	mass matrix

The implicit nonlinear system of differential equations for the differential bar sections is linearized by increments. All geometrically and physically nonlinear components in the $\Delta\underline{b}$-vector are recalculated after every iteration step, and the $\underline{A}$-, $\underline{d}$-, and $\underline{m}$-matrix are recalculated after the completion of the iteration within the increment. The solution of the system of differential equations by a Runge-Kutta integration results in the system of differential equations of the unknown incremental displacements $\Delta\underline{v}$, velocities $\Delta\dot{\underline{v}}$, and accelerations $\Delta\ddot{\underline{v}}$ of the nodes.

$$\underline{K}_{T(n-1)} \cdot \Delta\underline{v}^{[k]}_{(n)} + \underline{D}_{(n-1)} \cdot \Delta\underline{\dot{v}}^{[k]}_{(n)} + \underline{M} \cdot \Delta\underline{\ddot{v}}^{[k]}_{(n)} = \Delta\underline{P}_{(n)} - \Delta\,\overset{o}{\underline{F}}{}^{[k]}_{(n)} + \Delta\Delta\underline{F}_{(n-1)} \qquad (12)$$

Due to the system modification components of the systems of differential equations (11) and (12) is changes. A special modification increment is adopted for the numerical processing of these changes. Layers of cross sections or structural members which are added to the system within a system modification are inserted stress-free and strain-free into the system. This is numerically processed by modifications of the corresponding components of eqs. (11) and (12). If additionally layers of cross sections or structural members are removed from the structure, the stresses of those components are transferred to the residual system.

4. Examples

4.1 STEEL CONCRETE STRUCTURE

For the steel-concrete-composite beam that is displayed in Figure 4, the process of manufacturing and loading is analyzed numerically.

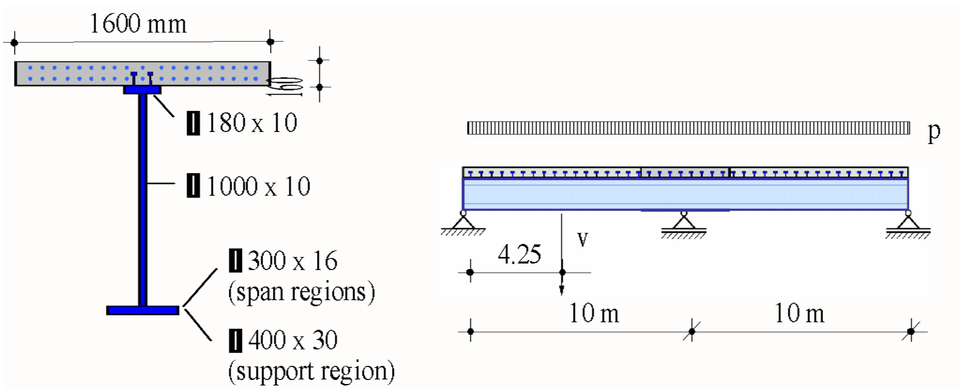

Figure 4. Cross section, system

In the states of manufacturing first the span region of the composite beam are concreted, after it the support region. According this in the numerical analysis first the fresh concrete load is considered and afterwards the respectively concrete layers are taken into consideration within a specific system modification increment. Finally the traffic load of p = 400 kN/m (about 60% of the ultimate load) is applied.

concrete C35/45	f_{ctm}= 3,2 N/mm²
	$f_{cm,cyl}$ = 43 N/mm²
construction steel S355	f_y = 360 N/mm^2
	f_u = 510 N/mm^2
reinforcement steel	f_y = 500 N/mm^2
	f_u = 550 N/mm^2

Between concrete and steel a nonlinear shear stress slip dependency is regarded, see continuous lines in Figure 5. It is considered as fuzzy function with the likewise in Figure 5 displayed bunch parameter. In comparison the structure is analyzed additionally with a linear shear stress slip dependency with the same initial stiffness (dashed lines) and with a rigid bond (dotted line). The linear shear stress slip dependency is also considered as fuzzy function with the bunch parameter in Figure 5.

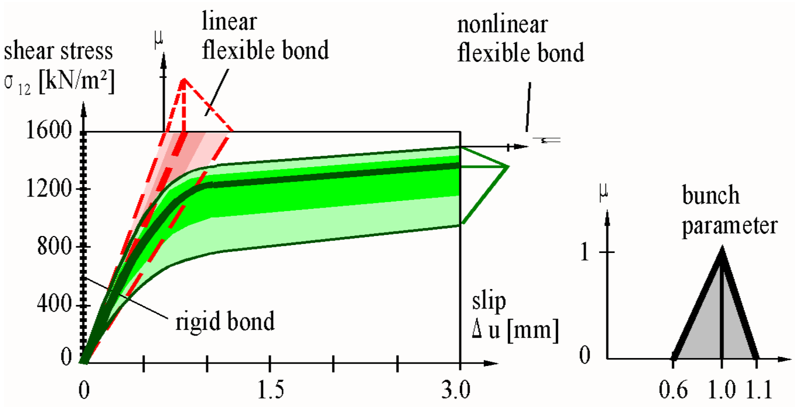

Figure 5. Fuzzy functions of shear stress slip dependency, bunch parameter s

The alteration of the vertical displacement of the girder in the span region at the longitudinal bar coordinate 4.25 m is a selected fuzzy result. The fuzzy displacement is shown in Figure 6 for the three cases of shear stress slip dependencies.

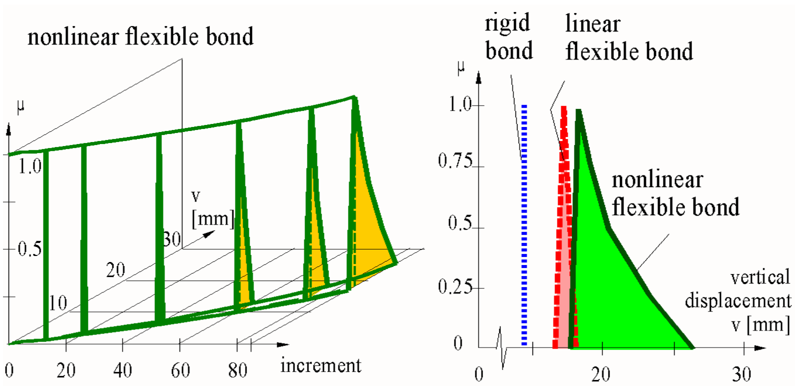

Figure 6. Fuzzy vertical displacements

4.2 NATURAL STONE ARCH BRIDGE

The second example regards the Syratal bridge in Plauen (Germany) built 1903, world wide the widest span natural stone arch bridge at that time. The span is ninety meters, see Figure 7.

Seven years ago (in 2000) the bridge was reconstructed and the masonry was grouted. The main parts of the bridge are the arch and the lateral masonry.

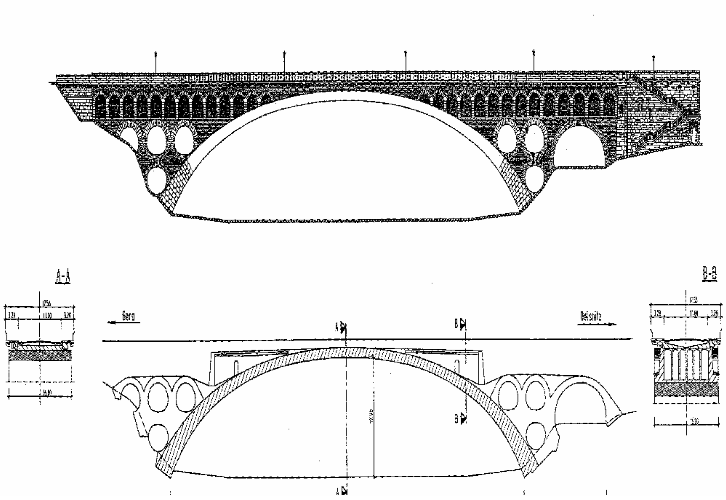

Figure 7. System, see [Schmiedel & Setzpfand 1999]

The system takes into consideration the interaction between the arch and the masonry on the right and left side of the arch. The horizontal displacements of the arch activate the stiffness of the lateral masonry. This effect is modeled by nonlinear node springs, see Figure 8.

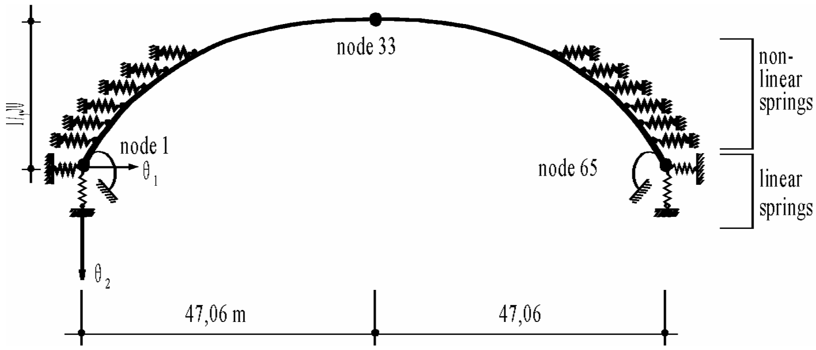

Figure 8. Computational model

In Figure 9 is shown the nonlinear force displacement dependency for the nodes springs and the fuzzy stiffness factor f_{KF}.

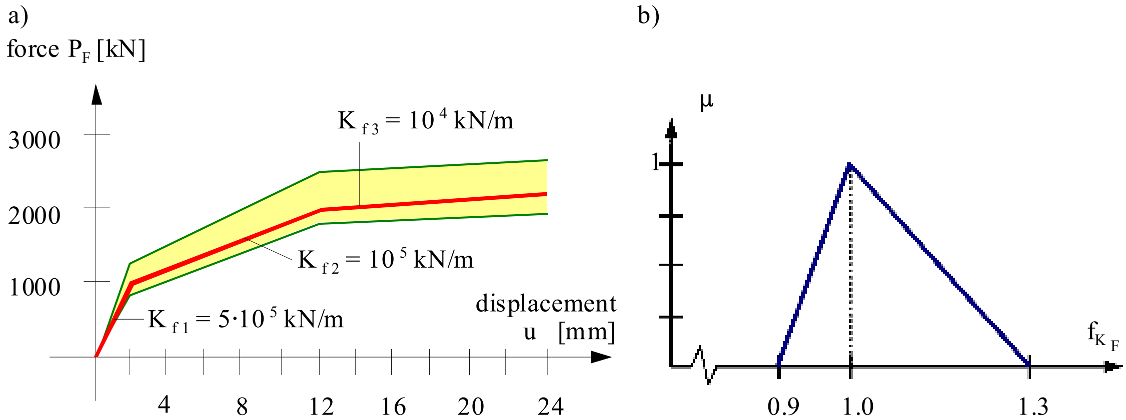

Figure 9. Uncertain force displacement dependency as fuzzy function

The system modification is caused by grouting of masonry. The modification process has a discontinuity as consequence of the rehabilitation. A representative load process is shown in Figure 10.

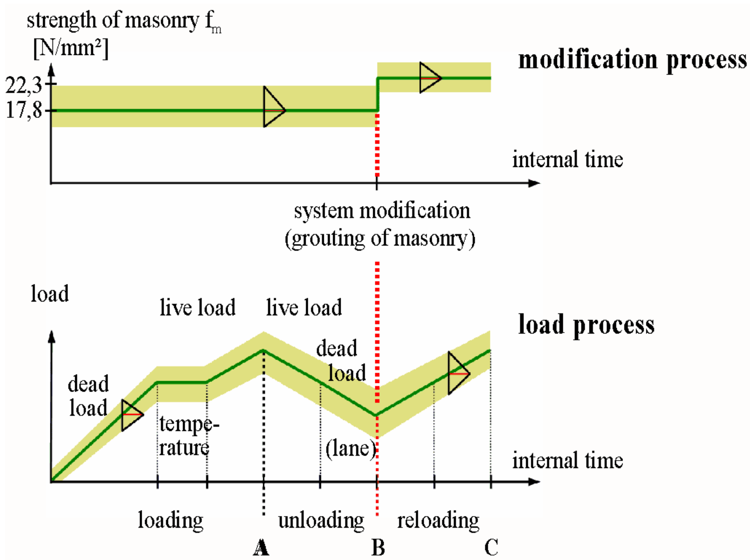

Figure10. Load and modification process

The masonry rehabilitation causes a modification of the constitutive relationship. The curve I in Figure 11 stands for the original constitutive law. The curve III shows the modified constitutive law, and the curve II is a specific sigma-epsilon-path for the modification.

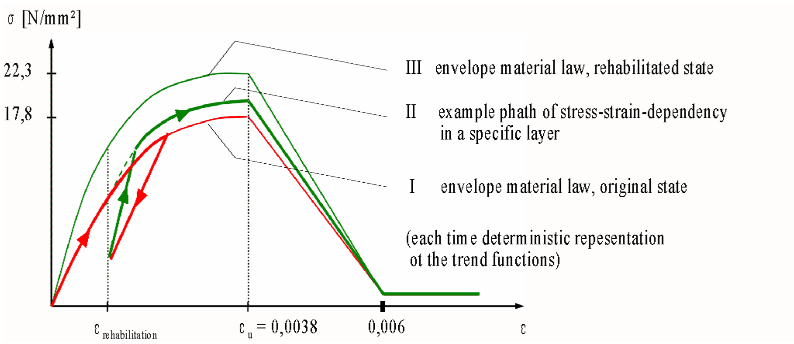

Figure 11. Trend functions of constitutive laws

The failure load factor is computed. That characterizes the ultimate traffic load, and leads to system failure. The ultimate traffic load is equal given live load multiplied by failure load factor η. In Figure 12 is given the fuzzy failure load factor η. Case I investigates the arch without system modification, case II with the unrehabilitated and rehabilitated masonry strength for the system modification process. Case III leads to overestimation of the load bearing capacity.

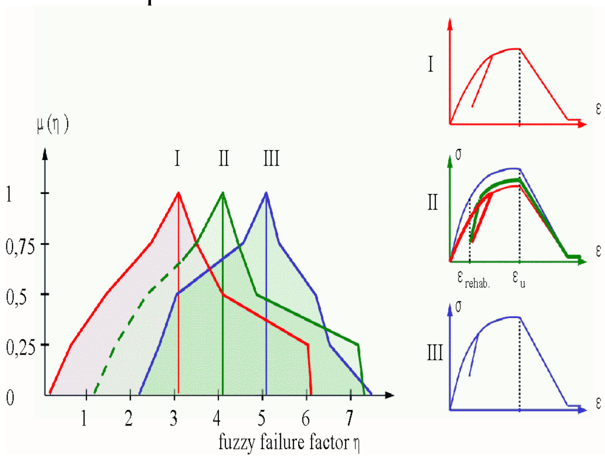

Figure 12. Fuzzy failure factor

In Figure 13 are results of the numerical monitoring, the fuzzy results for the vertical displacement of the crown of the arch (node 33) at the internal time points A, B, and C.

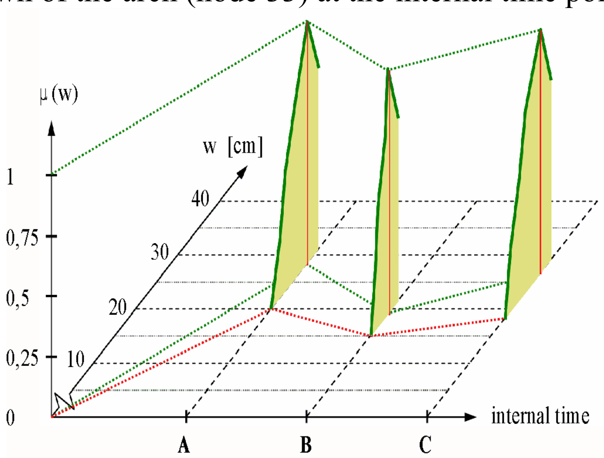

Figure 13. Fuzzy vertical displacements

Conclusions

Analyzing a structure close to reality requires consider the complete load and modification process. The parameters of the load and modification process are generally uncertain. They may be described by fuzzy processes for a numerical monitoring.

Acknowledgement

The authors gratefully acknowledge the support of the German Research Foundation (DFG).

References

Bartzsch, M. 2006. *Tragwerksmodifikation als unstetiger und unscharfer Prozeß.* Technische Universität Dresden, Institut für Statik und Dynamik der Tragwerke, H. 12

Bartzsch, M., Graf, W., Möller, B. & Sickert, J.-U. 2004. Modification of structures with uncertain parameters. In *ECCOMAS*, Jyväskylä, CD-ROM

Möller, B. & Beer, M. 2004. *Fuzzy Randomness - Uncertainty in Civil Engineering and Computational Mechanics.* Springer, Berlin, Heidelberg

Möller, B. & Graf, W. 2005. Tragwerksprozesse in der Baustatik. In *Baustatik-Baupraxis 9*, TU Dresden, Bericht, S. 381-393

Möller, B., Graf, W. & Beer, M. 2000. Fuzzy structural analysis using α-level optimization. *Computational Mechanics*, 26(2000), pp. 547-565

Möller, B., Graf, W. & Beer, M. 2003. Safety assessment of structures in view of Fuzzy randomness. *Computers & Structures*, 81(2003), pp. 1567-1582

Möller, B., Graf, W., Liebscher, M., Pannier, S. & Sickert, J.-U. 2007. An inverse solution of the lifetime-oriented design problem. In *3th ICLODC*, Ruhr-Universität Bochum

Möller, B., Graf, W. & Nguyen S. H. 2004. Modeling the life-cycle of a structure using fuzzy processes. *International Journal of Computer-Aided Civil and Infrastructure Engineering* 19(2004), pp. 157-169, Blackwell Publ., Malden, Cambridge, Oxford

Schmiedel, J. & Setzpfandt, G. 1999. Syratalbrücke Plauen (Friedensbrücke). In *Bundesministerium für Verkehr, Bau- und Wohnungswesen (Hrsg.),* Steinbrücken in Deutschland, Teil 2: Berlin, Brandenburg, Mecklenburg-Vorpommern, Sachsen-Anhalt, Sachsen, Thüringen. Verl. Bau + Technik, Düsseldorf, S. 335-338