Imprecise Probabilities with a Generalized Interval Form

Yan Wang
Department of Industrial Engineering and Management Systems
University of Central Florida
Orlando, FL 32816, USA
wangyan@mail.ucf.edu

Abstract

Different representations of imprecise probabilities have been proposed, such as behavioral theory [1], evidence theory [2, 3], random set [4], possibility theory [5, 6], probability bound analysis [7], *F*-probabilities [8], and clouds [9]. These methods combine classical intervals with probabilities to distinguish uncertainty from variability. In this paper, we proposed a new form of imprecise probabilities based on generalized or modal intervals [10].

Given a sample space Ω and a σ-algebra $\mathcal{A}$ of random events over Ω, we define the generalized interval probability $\mathbf{p}: \mathcal{A} \to \mathbb{KR}$ which obeys the axioms of Kolmogorov. Generalized intervals are algebraically closed under Kaucher arithmetic, which provides a concise representation and calculation structure as a natural extension of precise probabilities. For instance, the probability of the complement of an event E is $\mathbf{p}(E^c) := 1 - \mathrm{dual}\,\mathbf{p}(E)$.

With the separation between proper and improper interval probabilities, *focal* and *non-focal* events are differentiated based on the associated logical semantics of generalized intervals in system analysis. Focal events have the semantics of critical, uncontrollable, specified, etc. in probabilistic analysis, whereas the corresponding non-focal events are complementary, controllable, and derived.

A generalized imprecise conditional probability is defined based on unconditional interval probabilities such that the algebraic relation between conditional and marginal interval probabilities is maintained. A generalized interval Bayes' rule (GIBR) is also proposed. The GIBR allows us to interpret the logical relationship between interval prior and posterior probabilities.

---

[1] Walley, P., 1991, *Statistical Reasoning with Imprecise Probabilities*, Chapman & Hall, London.

[2] Dempster, A., 1967, "Upper and lower probabilities induced by a multivalued mapping," *Annals of Mathematical Statistics*, **38**(2), pp.325-339.

[3] Shafer, G., 1976, *A Mathematical Theory of Evidence*, Princeton University Press, Princeton, NJ.

[4] Molchanov, I., 2005, *Theory of Random Sets*, Springer, London.

[5] Zadeh, L.A., 1978, "Fuzzy sets as a basis for a theory of possibility," *Fuzzy Sets Systems*, **1**(1), pp.3-28.

[6] Dubois, D. and Prade, H., 1988, *Possibility Theory: An Approach to Computerized Processing of Uncertainty*, Plenum, New York.

[7] Ferson, S., Kreinovich, V., Ginzburg, L., Myers, D.S., and Sentz, K., 2002, "Constructing probability boxes and Dempster-Shafer structures," Sandia National Laboratories Technical report SAND2002-4015, Albuquerque, NM.

[8] Weichselberger, K., 2000, "The theory of interval-probability as a unifying concept for uncertainty," *International Journal of Approximate Reasoning*, **24**(2-3), pp.149-170.

[9] Neumaier, A., 2004, "Clouds, fuzzy sets, and probability intervals," *Reliable Computing*, **10**(4), pp.249-272

[10] Gardenes, E., Sainz, M.A., Jorba, L., Calm, R., Estela, R., Mielgo, H., and Trepat, A., 2001, "Modal intervals," *Reliable Computing*, **7**(2), pp.77-111.